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The Design of Linear Magnetic Negative Stiffness Element for Engineering Application using Rectangular Permanent Magnets

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Negative stiffness element (NSE) has extensive uses in engineering applications, such as vibration isolation, energy harvesting, and mechanical metamaterial. However, to realize a linear negative stiffness characteristic is still a challenging task. In this paper, we present a compact magnetic negative stiffness element (MNSE) that composed of three rectangular permanent magnets and configured as repelling configuration in horizontal to realize linear negative stiffness characteristic. The effects of the MNSE configuration parameters on the negative stiffness characteristic are analyzed in detail. The results demonstrate that the magnitude of the negative stiffness characteristic can be adjusted by changing the height ratio and width ratio between the central and outer magnets. The height difference between the central and outer magnets can be used to tune the degree of nonlinearity of the negative stiffness characteristic and to get the uniformity stiffness characteristic in the equilibrium position. The procedure to realize the linear negative stiffness characteristic with the expected magnitude and displacement range is developed and confirmed. The proposed MNSE and the design procedure offer an engineering application foundation for the magnetic linear negative stiffness.

Keywords : index terms—permanent magnet, linear negative stiffness characteristic, negative stiffness element, vibration isolation

1. Introduction

It is well known that when a force deforms an elastic element, the resulting displacement will be in the same direction as the applied force. This property is known as positive stiffness. Less familiar is the concept of negative stiffness, where the deforming force and the resulting displacement are in opposite directions [1]. The mechanical instability of NSE has been considered as an unwanted behavior or a weakness, but the idea of negative stiffness has been used to improve the performance of isolator [2-4], performance optimization for sensor [5, 6], seismic protection of structures [7, 8], the band gap frequency tailor for acoustic metamaterial [9], realize hyper-damping properties of the material [1, 10], increasing the electromechanical coupling of the energy harvesting device [11-13] and so on.

In recent years, many methods have been proposed to obtain a negative stiffness characteristic, such as using the elaborately designed mechanical structure [14-16], geometric nonlinearity structure configuration [17, 18] and material mechanical property [19-21]. Recently, using the magnets with different configurations to produce negative stiffness characteristic has been considerable interest in various engineering applications due to their unique and beneficial properties such as more intensive negative stiffness magnitude, compact size, low-cost and many structural design methods can be used to optimize configuration parameters [22, 23]. A MNSE composed of three cuboidal magnets configured as repelling in horizontal was proposed by Wenjiang wu [2]. Similarly, a MNSE is made up of four identical magnets, assemble as two repelling configurations to improve the damping property of the material that was put forwarded by O.Akintoye [24]. Carrella *et al*. proposed a MNSE that composed of three-disc magnets and configured as

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attracting in vertical for realizing the high-static–low-dynamic stiffness characteristic of isolator [25]. Compared with using cuboidal or disc magnets, the greater magnitude of negative stiffness characteristics can be obtained by using magnet rings. Thus using the magnet rings to realize the negative stiffness has growing interests in the engineering field. Two magnetic rings magnetized radially are arranged in the repulsive configuration as a MNSE was developed by Shan. Y. to reduce the volume and improve the performance of the pneumatic vibration isolator [26]. Jia xi Zhou *et al*. proposed a MNSE that consist of a pair of mutually repelling permanent magnet rings and connected in parallel with a coil spring to attenuate the transmission of vibration from the ambulance floor to an infant [27].

In general, the negative stiffness characteristic of the NSE can be evaluated from two aspects: the magnitude of negative stiffness in the equilibrium position; the degree of nonlinearity of the negative stiffness in the adjacent area of the equilibrium position. Obviously, the magnitude and degree of nonlinearity of the negative stiffness characteristic are critical for the engineering applications. For example, the magnitude of negative stiffness has a great influence on the isolation frequency scope of vibration isolator, the degree of nonlinearity of the negative stiffness have a negative impact on the performance of the isolator and the amplitude-frequency characteristic of the sensor. Unfortunately, nonlinear negative stiffness characteristic appears in the equilibrium position of the MNSE mentioned above. It is convenient to obtain a nonlinear negative stiffness characteristic using permanent magnets, but to realize a linear one is cumbersome. By now, only a few studies have been conducted to attain the linear negative stiffness characteristic in the narrow range near the equilibrium position. Y. Zheng *et al*. [4] proposed a MNSE that composed of a pair of coaxial ring radial magnetization permanent magnets and a designing procedure to obtain a linear negative stiffness in a narrow displacement range near the equilibrium position. G. Dong *et al*. [28] exploited three magnetic rings configured in attraction as MNSE to obtain a flatten negative stiffness characteristic near the equilibrium position. While Zhenhua Zhou *et al*. [29] investigated a MNSE that compose of two axial-magnetized permanent magnetic rings to obtain the linear negative stiffness characteristic. It is feasible to use permanent magnet rings with different configurations to get a linear negative stiffness characteristic. However, the displacement range of the linear negative stiffness is very narrow in previous studies, meanwhile, the magnet rings are hard to fabricate and expensive. So, using permanent magnets to realize a wide range linear negative stiffness characteristic is still a challenging task.

In this paper, we present a MNSE with low-cost magnets and simple configuration to realize the linear negative stiffness characteristic. The effects of the configuration parameters on the negative stiffness characteristic are analyzed in detail, and the procedure to obtain a linear negative stiffness characteristic with the expected magnitude and linear negative stiffness range is developed. The remainder of this paper is organized as follows. The configuration and modeling of the MNSE are described in Section 2. In Section 3, the effects of configuration parameters on the negative stiffness characteristic are evaluated. The magnitude and degree of nonlinearity are discussed in Section 4. The design procedure for the linear negative stiffness characteristic is proposed and validated in Section 5. Finally, some conclusions are summarized in Section 6.

2. The Configuration and Modeling of the MNSE

2.1. The configuration of the MNSE

The MNSE is composed of three rectangular permanent magnets and the 3-D structural drawing shown in Fig. 1(a). As shown in Fig. 1(b), these rectangular permanent magnets configured as repelling configuration in horizontal. When the MNSE applied in the engineering practice, the central magnet can move only in the vertical direction, and the other degrees of freedom will be constrained by extra-mechanism.

As shown in Fig. 1(a), the dimension of the central magnet is $2a_1 \times 2b_1 \times 2c_1$, and the dimensions of the outer magnets are $2a \times 2b \times 2c$. The center distance between the central and the outer magnets in the horizon direction is h, the air-gap width is η, and the relationship between the air-gap width and the center distance is $\eta = h - c - c_1$. The equilibrium position is defined as the position where the relative displacement x is zero. It is assumed that the

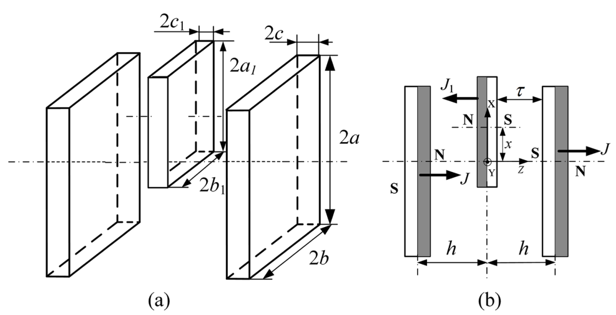

Fig. 1. (a) 3-D structural drawing and (b)configuration of the proposed MNSE.

magnetic polarizations are rigid and uniform in each magnet and not demagnetized, the magnetic polarization vectors are J and J_1 that shown in Fig. 1(b). Based on the structural symmetry of the MNSE, we define a_1, b_1 and c_1 is the height, width and thickness of the central magnet; a, b and c is the height, width and thickness of the outer magnets. To describe the configuration, two aspect ratios are defined for the MNSE: the "height ratio" is given by the ratio between the height of central and outer magnets, $\kappa = a_1/a$; the "width ratio" is given by the ratio between the width of central and outer magnets, $\gamma = b_1/b$. Besides, we defined the height difference between the central magnet height and out magnet height as τ.

2.2. Semi-analytical expression of negative stiffness for the MNSE

The analytical expression of the force interaction between two magnets was proposed by Akoun and Yonnet [30], the magnetic force of the outer magnets acting on the central magnet in the vertical is used to calculate the stiffness of the MNSE. Therefore, the magnetic force of the MNSE can be expressed as:

$$F_x = \frac{J\bullet J_1}{2\pi\mu_0}\sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1}\sum_{l=0}^{1}\sum_{p=0}^{1}\sum_{q=0}^{1}(-1)^{i+j+k+l+p+q}\phi\left(U_{ij},V_{kl},W_{pq},r\right) \quad (1)$$

$$\phi(U,V,W,r) = \frac{\left(V^2-W^2\right)}{2}\ln\left(r-U\right)+UV\ln(r-V)+VW\arctan(\frac{UV}{rW})+\frac{r}{2}U \quad (2)$$

$$U_{ij} = x+(-1)^j a-(-1)^i a_1 \quad (3)$$

$$V_{kl} = (-1)^l b_1-(-1)^k b \quad (4)$$

$$W_{pq} = h+(-1)^q c_1-(-1)^p c \quad (5)$$

$$r = \sqrt{U_{ij}^2+V_{kl}^2+W_{pq}^2} \quad (6)$$

In (1), μ_0 is the permeability of vacuum. As shown in Fig. 1(b), the magnetic polarization vectors J and J_1 are parallel to the horizontal direction, but in the opposite direction. Accordingly, the dot product of J and J_1 can be written as:

$$J\bullet J_1 = -JJ_1 \quad (7)$$

The stiffness of the MNSE can be calculated by differentiating the force expression in (1). with respect to the vertical relative displacement x, and the expression of the negative stiffness can be derived as

$$K = -\frac{\partial F_x}{\partial x} = \frac{JJ'}{2\pi\mu_0}\sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1}\sum_{l=0}^{1}\sum_{p=0}^{1}\sum_{q=0}^{1}(-1)^{i+j+k+l+p+q}\varphi\left(U_{ij},V_{kl},W_{pq},r\right) \quad (8)$$

where

$$\varphi\left(U,V,W,r\right) = r+V\ln(r-V) \quad (9)$$

where the variables U, V, W, and r are the same as in (1).

2.3. The evaluation index for the negative stiffness characteristic of the MNSE

To select appropriate indexes to evaluate the two aspects of the negative stiffness characteristic that mentioned in section 1, the negative stiffness characteristic of the MNSE that the dimensions of the central magnet and outer magnets are 20 mm×100 mm×10 mm with the center distance h is 20 mm is investigated. As shown in Fig. 2, the red solid line denotes the analytical result that calculated by using (8), and an approximation represented by quadratic polynomial fitting is performed in the range of ± 5 mm in the equilibrium position represented by the blue dotted line. It is evident that the approximate negative stiffness has a good agreement with the analytical one. That indicates the negative stiffness characteristic in the vicinity of the equilibrium position can be approximated by using a quadratic polynomial with respect to the relative displacement x, and expressed as $k_x = k_0+k_1x^2$, where the parameter k_0 can be used to represent the linear component or magnitude of the negative stiffness and k_1 can be used to describe the nonlinear component of the negative stiffness.

The degree of nonlinearity of the negative stiffness in the relative displacement x is the absolute value of the ratio that the difference between stiffness magnitude k_x in the relative displacement x and k_0 to the linear component k_0, which can be expressed as

$$\delta_x = \left|\frac{k_x-k_0}{k_0}\right| \quad (10)$$

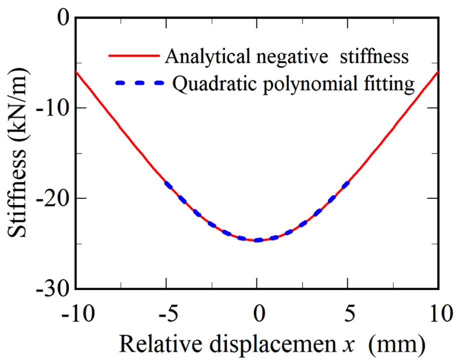

Fig. 2. Comparison between the analytical and quadratic polynomial fitting approximation of the negative stiffness of the MNSE that the dimension of the central magnet and outer magnets are 20 mm×100 mm×10 mm, and the center distance is 20 mm.

Noticeably, using k_0 and δ_x or k_2, we can give a reasonable and comprehensive evaluation for the negative stiffness characteristic of the MNSE.

3. Effects of Configuration Parameters on the Negative Stiffness Characteristic

In this paper, we focus on the realization of linear negative stiffness characteristics by adjusting the configuration parameters of the MNSE. According to the relations among the air-gap width η, center distance h, the magnets thickness c and c_1, different air-gap width and the thickness of the magnets, all these factors influence the center distance. Hence, these parameters which we only consider is the center distance h, the value of c and c_1 are fixed at 5 mm. It is noted that all the illustrative calculations are done with J=J_1=1.34 T.

3.1. Effects of central and outer magnets height on the negative stiffness characteristic of the MNSE

To disclose the effect of magnets height on the negative stiffness characteristic, the negative stiffness characteristic of the MNSE is considered that the height and width of central magnet is 10 mm and 50 mm, the width of the outer magnets is 50 mm and the height change from 2 mm to 100 mm. (i.e., the height ratio κ change from 0.2 to 10).

As is shown in Fig. 3(a), the shape of the negative stiffness curve–is convex when the height of the outer magnets is 2 mm, then concave and finally change to convex with the increment of the height of the outer magnet. That means the nonlinear component of the negative stiffness start from negative, then to positive, and to negative finally. The impact of the height ratio κ on the linear component and the nonlinear component of the negative stiffness characteristic is shown in Fig 3(b), the k_0 and k_1 is get using the quadratic polynomial fitting in the relative displacement range of ± 5 mm. It is evident that the minimum of linear component and the maximum of the nonlinear component appears when the heights of the central and outer magnets are the same (i.e., κ=1). Based on the nonlinear component sign variation with the outer magnets height increment, we can conclude that the nonlinear component will be zero at some certain height ratio point, just as shown in the partial enlarged detail in Fig. 3(b). Obviously, in the points 'I' and 'II', the nonlinear component is zero, and indicated that the linear negative stiffness characteristic realizes in the equilibrium position. In the points 'I' and 'II', the height difference τ is 3.3 mm and -3.3 mm, respectively. That is to say, a linear negative stiffness characteristic in the equilibrium position can be

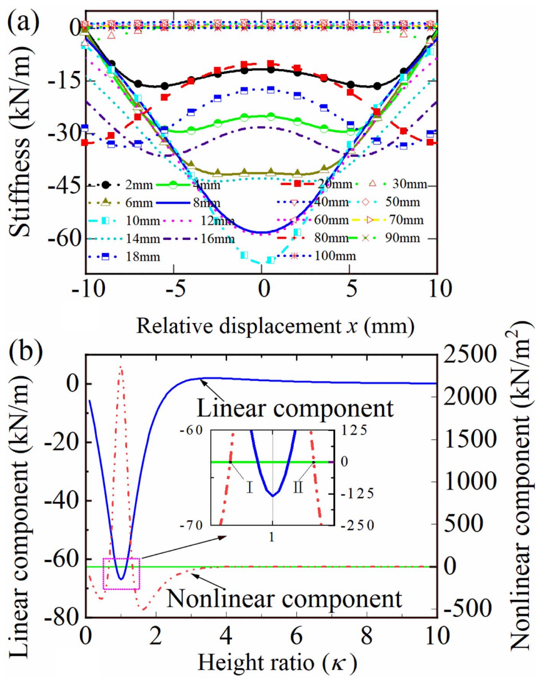

Fig. 3. (a) The negative stiffness curve of the MNSE, each curve represents the MNSE with the selected outer magnets height. (b) The linear and nonlinear components of the negative stiffness characteristic of the MNSE with different height ratios. (The height, width and thickness of the central magnet is 10 mm, 50 mm and 5 mm respectively, the width and thickness of the outer magnets be the same as the central magnet, and center distance is 14 mm)

obtained by tuning the height of the outer magnets to 6.7 mm or 13.3 mm.

With the selected central magnet height, we tune the height ratio by changing the height of the outer magnets. To further analyze the effect of magnets height on the negative stiffness, the height of the central magnet is selected from 10 mm to 100 mm. It can be seen from Fig. 4(a) and (b) that the minimum of linear component and maximum of nonlinear component occurred when the height ratio is one. Although the selected height is different, the conclusion is consistent with the previous analysis. The minimum-linear component of the negative stiffness is increased with the increment of the selected central magnet height, and the maximum nonlinear component is just the opposite.

As shown in Fig. 4(b), all of the nonlinear components of the negative stiffness is positive when the height ratio is one. When height ratio increases or decreases in the area that adjacent to the height ratio is one, the nonlinear component will be varied from positive to negative. It is indicated that increases or decreases the height ratio to a specific value, the degree of nonlinearity of the negative

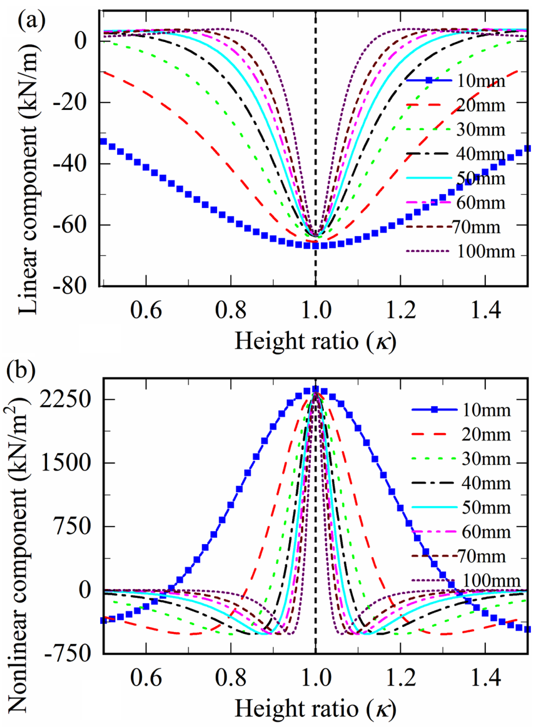

Fig. 4. (a) The linear and (b) nonlinear component of the negative stiffness characteristic of the MNSE under different height ratio with the selected central magnet height, each curve represents the MNSE with the selected central magnet height. (The width and thickness of the central and outer magnets are 50 mm and 5 mm, center distance is 20 mm.)

stiffness characteristic will be zero, and the uniformity negative stiffness characteristic near the equilibrium position can be realized. To illustrate the influence rule of height ratios for the realization of linear negative stiffness more clearly, the height ratio of negative stiffness with zero nonlinear component converts to the height difference,

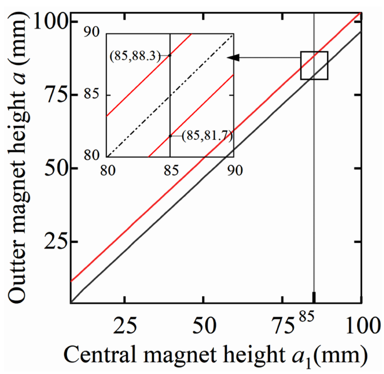

Fig. 5. The outer magnets height for zero nonlinear component of the negative stiffness characteristic with the selected central magnet height. (The width and thickness of the central and outer magnets is 50 mm and 5 mm, center distance is 20 mm)

Table 1. The absolute value of the height difference for the linear negative stiffness of the MNSE with the selected center distance. Unit (mm)

Center distance	12	14	16	18	20	22	24	26
Height difference	1.8	3.3	4.5	5.6	6.7	7.6	8.6	9.5
Center distance	28	30	32	34	36	38	40	
Height difference	10.4	11.3	12.2	13.1	14	14.8	15.7	

then the result shown in Fig. 5. We can conclude that no matter what specified value of the central magnet height is, the absolute value of the height difference τ keeps the same value (3.3 mm).

We may conclude that the absolute value of height difference for the linear negative stiffness characteristic is only decided by the center distance. The detailed height difference absolute value for linear negative stiffness characteristic realization with the given center distance list in Table 1.

The relationship between the absolute value of height difference and center distance for linear negative stiffness characteristic realization in the equilibrium position can be fitting with polynomial using the least square method and expressed as

$$\tau = 5.976\exp(0.02519h)\text{-}16.68\exp(-0.0822h) \tag{11}$$

Obviously, when the central and outer magnets have the same width, we can use (11) to calculate the height difference for the linear negative stiffness.

3.2. Effect of central and outer magnets width on the negative stiffness characteristic

To find the effect of width ratio on the negative stiffness characteristic, the negative stiffness characteristic of the MNSE with the fixed central magnet width and tuning the width ratio by changing the width of outer magnets, while central and outer magnets height and center distance remain unchanged investigated. The linear and nonlinear components of the negative stiffness of the MNSE with the selected central magnet width with different width ratio is shown in Fig. 6(a) and (b). That the height of the central and outer magnets is 10 mm, the center distance is 14 mm, and the selected central magnet width is changed from 10 mm to 100 mm with the 10 mm increment step.

According to Fig. 6(a) that with the given central magnet width, the linear component of the negative stiffness linearly decreases with the increment of the width ratio when the width ratio is smaller than one, then almost remains constant when the width ratio greater than one. The behavior of the nonlinear component shows the

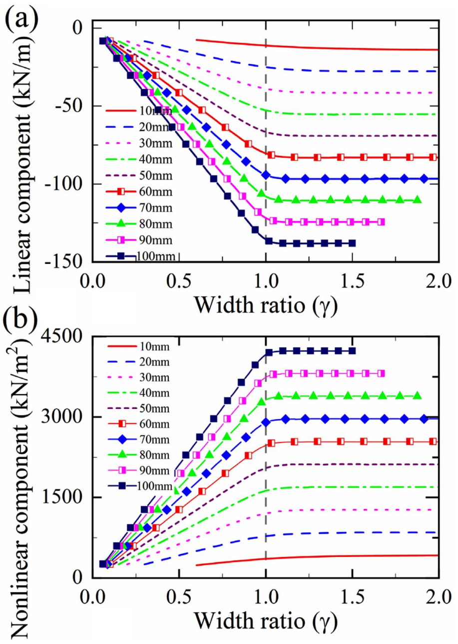

Fig. 6. (a) Linear component and (b) nonlinear component of the negative stiffness characteristic of the MNSE with the selected central magnet width under difference width ratio, each curve represents the MNSE with the selected central magnet width. (The thickness and height of the central and outer magnets is 5 mm and 10 mm, and center distance is 14 mm).

opposite pattern. With the larger specified central magnet width, the absolute value of the slope of the linear and nonlinear component curve will be larger, also the absolute value of the linear and nonlinear component when the width ratio greater than one. And an interesting conclusion can be found that the linear and nonlinear components will be changed with the variation of the given central magnet width and the height ratio, but the degree of nonlinearity of the negative stiffness is 0.48 and almost unchanged with the specified central and outer magnet height. Furthermore, the conclusion is still standing with other center distance value, and this means that the degree of nonlinearity of the negative stiffness of the MNSE has no relationship with the width of the central and outer magnets only decided by the center distance, the height of the central magnet and outer magnets.

In summary, change the width ratio of the MNSE with other parameters unaltered, only the linear and nonlinear components of the negative stiffness characteristic will be changed, but the degree of nonlinearity is almost unchanged. So, with the consideration of the size constraint of MNSE, we can hold the width ratio as one, and change the width of central and outer magnets tuning the negative stiffness magnitude to the expected value and keep the degree of nonlinearity unchanged simultaneously.

4. Analysis on the linear Negative Stiffness Characteristic of MNSE

In the previous section, the effects of MNSE configuration parameters on the negative stiffness characteristic have been discussed thoroughly. We found that the central and outer magnets' height, width and center distance can be used to adjust the magnitude of negative stiffness, and the height difference can be utilized to realize the linear negative stiffness characteristic in the equilibrium position. In this section, the magnitude and displacement range of the linear negative stiffness characteristic will be analyzed in detail.

4.1. Magnitude of the linear negative stiffness characteristic of the MNSE

Based on the analysis of section 3.1, with the given center distance center, the linear negative stiffness characteristic can be obtained by adjusting the height difference between the central magnet and outer magnets. And with one given center magnet height, two kinds of linear negative stiffness characteristics can be obtained. One is the height difference is positive and the other one is negative. When the width of central and outer magnets is 50 mm (i.e. the width ratio is one), the comparison of the linear negative stiffness magnitude of the MNSE with the give distance center under different central magnet height is shown in Fig. 7(a), and in one given center distance, the blue and red color of the lines represent the magnitude of linear negative stiffness with the negative and positive height difference, respectively. It is obvious in Fig. 7(a), when the central magnet height is small, the magnitude of the negative stiffness with negative height difference is higher than the negative stiffness with positive height difference. Due to physical constraint, when the central magnet height is smaller than the height difference for the linear negative stiffness characteristic realization, only the negative height difference can be realized. And with the given center distance, the negative stiffness magnitude with positive or negative height difference decreases initially and then increases with the increment of the central magnet height, and then becomes the same and almost no change with the variation of the central magnet height. With the given center distance, the magnitude of the linear negative stiffness characteristic almost remains constant when the central magnet height more than 60 mm. When central magnet height is 60 mm, the magnitude of linear negative stiffness characteristic of

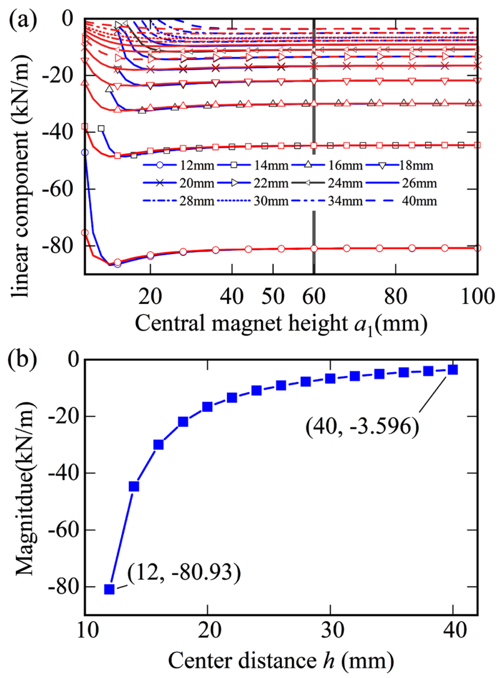

Fig. 7. (a) The comparison of the linear negative stiffness magnitude of the MNSE with the given center distance under different central magnet height, each line type represents MNSE under different center distance with a given center distance value. (b) The magnitude of the linear negative stiffness characteristic of the MNSE with central magnet height is 60 mm, central and outer magnets 50 mm.

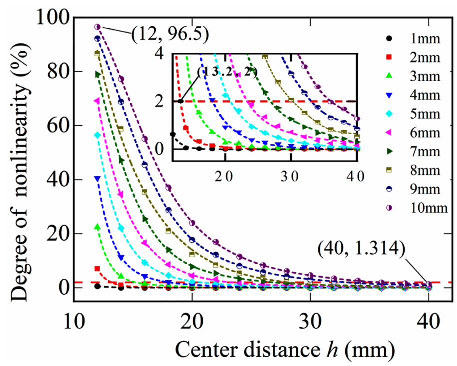

Fig. 8. The degree of nonlinearity of linear negative stiffness in the selected relative displacement of the MNSE with different center distance, each curve represents the negative stiffness with a selected relative displacement rang, and the dotted line represents the fitting curve between the degree of nonlinearity of the linear negative stiffness and the center distance. (The center magnet height is 60 mm, the central magnet and outer magnets width is 50 mm).

the MNSE with difference center distance is shown in Fig. 7(b), and the relationship between linear negative stiffness magnitude K_0 and the given center distance h is

$$K_0 = -1.326 \cdot 10^4 \exp(-0.4628h) - 77.68\exp(-0.08082h) \quad (12)$$

4.2. Range of the linear negative stiffness characteristic of the MNSE

Base on the analysis of section 3.1, we know that with the given center distance, the linear negative stiffness characteristic in the equilibrium position can be obtained by adjusting the height ratio, but the rang of the linear negative stiffness is not considered in the previous analysis. Refer to (10), the degree of nonlinearity of the negative stiffness of the MNSE is closely related to the relative displacement x. With the selected relative displacement range, the comparison of the degree of nonlinearity of the linear negative stiffness of MNSE with the selected central magnet height (60 mm) under different center distance is shown in Fig. 8.

It obvious in Fig. 8 that the degree of nonlinearity of linear negative stiffness is inverse proportion to the center distance with the selected displacement range: the bigger of center distance, the smaller degree of nonlinearity of the linear negative stiffness is. With the selected center distance, the degree of nonlinearity of linear negative stiffness is proportional to the selected relative displacement range x near the equilibrium position: the smaller of the relative displacement range x, the stronger degree of nonlinearity of the linear negative stiffness is. For quantitative analysis,-we suppose that when the degree of nonlinearity of the negative stiffness in the relative displacement range is smaller than 2%, the negative stiffness characteristic in the displacement range near the equilibrium position can be regarded as linear negative stiffness. When the selected relative displacement x range is 2 mm, and the degree of nonlinearity is 2%, the corresponding center distance h is 13.2 mm that we can found in the partial enlarged detail in Fig. 8. That means when the center distance of MNSE more than 13.2 mm, the linear negative stiffness characteristic can be realized in the range of ± 2 mm, and the corresponding value of the center distance of the MNSE with the selected linear negative stiffness rang is list in Table 2.

And the relationship between linear negative stiffness range and the given center distance can be fitted by using

Table 2. The center distance of the MNSE with the selected linear negative stiffness range. Unit(mm)

Linear negative stiffness range	2	3	4	5	6
Center distance	13.2	15.3	18.1	20.8	23.3
Linear negative stiffness rang	7	8	9	10	
Center distance	26.9	30.3	33.1	36.1	

the least square method, and expressed as:

$$h = 2.984\rho + 6.146 \tag{13}$$

5. Design procedure for the linear negative stiffness of the MNSE

The effects of configuration parameters of the MNSE on the negative stiffness have been discussed separately. Based on the preceding analysis, to acquire the linear negative stiffness with the expected magnitude and displacement range can be obtained by the following designing procedure: Firstly, according to the anticipate displacement range of the linear negative stiffness and using (13), the center distance of the MNSE can be determined; Secondly, based on the value of the center distance obtained in the first step, the height difference can be calculated by utilizing (11). Thirdly, adjusting the width of central and outer magnets under the width ratio is one to realize the expected magnitude of the negative stiffness. To validate the proposed designing procedure, we suppose the anticipate magnitude and linear range of the negative stiffness is -8000 N/m and 5.5 mm, respectively. Firstly, we use (13) and the value of expected displacement range of linear negative stiffness, the center distance can be obtained, and the value is 22.6 mm; Secondly, based on the center distance get in the first step and using (11), the height difference can be calculated, and the absolute value of the height difference is 7.9 mm. That means the heigh of outer magnets t is 67.9 mm or 52.1 mm, and the linear negative stiffness characteristic can be obtained. In this validation, we only consider the situation that the height difference is positive.

Based on the designing procedure, the configuration parameters of the MNSE can be determined: the central magnet height is 60 mm, the height of outer magnets is 67.9 mm, the central and out magnets width and thickness are 50 mm and 5 mm, and the center distance is 22.6 mm. The corresponding negative stiffness characteristic is represented by the curve 'α' in Fig. 9. The flat negative stiffness curve in the adjacent area of the equilibrium position demonstrates the linear negative stiffness characteristic is realized. The degree of the nonlinearity of the negative stiffness in the relative displacement range 5.5 mm is 1.66% that smaller than the expected value the degree of nonlinearity, but the magnitude of the negative stiffness is -12541.6 N/m. Thirdly, to obtain the expected magnitude, we adjust the width of the central and outer magnets to 32.7 mm, the magnitude of the linear negative stiffness is -7794 N/m, and the degree of the nonlinearity of the negative stiffness the relative displacement range 5.5 mm is 1.59%, the negative stiffness is represented by curve 'β' in Fig. 9. There is a minimum deviation between the expected and design magnitude, but it is entirely acceptable in engineering applications. Obviously, the linear negative stiffness characteristic with the expected magnitude and displacement range is obtained using the proposed design procedure.

6. Conclusion

In this paper, a MNSE with low-cost magnets and simple configuration to realize the linear negative stiffness characteristic is proposed. The analytical expression of negative stiffness is established, and quadratic polynomial approximation fitting is performed to select the appropriate index to evaluate the negative stiffness characteristic of the MNSE. The effects of MNSE configuration parameters on the negative stiffness characteristic are investigated in detail. In summary, the height, width of central and outer magnets, the center distance have played an essential role in the design of linear negative stiffness characteristic: the height ratio and width ratio can be used to regulate the magnitude and degree of the nonlinearity of the negative

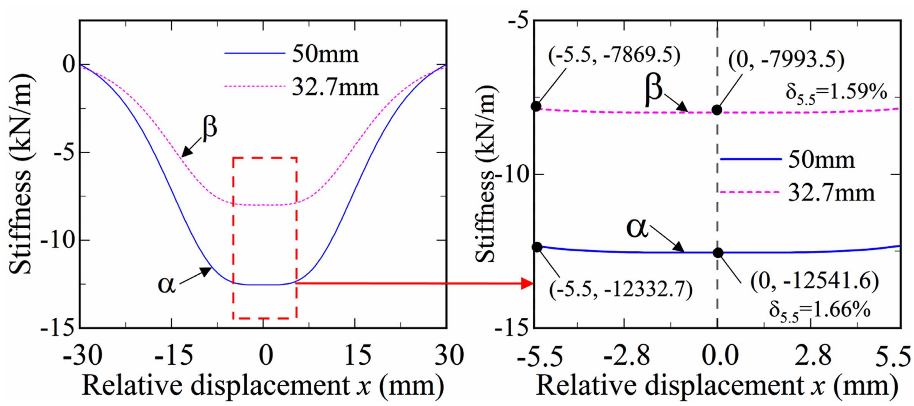

Fig. 9. The negative stiffness of the MNSE with the proposed design procedure.

stiffness characteristic, and the height difference between the central and outer magnets can be used to realize the linear negative stiffness of the MNSE and to get the uniformity stiffness characteristic in the equilibrium position. The design procedure to achieve the linear negative stiffness with the expected magnitude and displacement range is developed and validated**.** In this paper, we suppose that the degree of nonlinearity of the negative stiffness in the relative displacement range is less than 2%, the negative stiffness characteristic in this relative displacement range can be regarded as linear negative stiffness. In fact, we can suppose a smaller value of the degree of nonlinearity to get a more rigorous linear negative stiffness, and the proposed design procedure is still valid.

The proposed MNSE and the linear negative stiffness design procedure can be used in various engineering applications, such as extend the vibration isolation range in low and ultra-low frequency vibration isolation, improve the low-frequency vibration energy harvesting efficiency, and sensor bandwidth adjustment. And the application of the MNSE and the linear negative stiffness design procedure will be investigated in our future work.

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