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Theory of Generation Linewidth in Spin-torque Nano-sized Auto-oscillators

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Theory of the generation linewidth of a current-driven spin-torque magnetic nano-oscillator in the presence of thermal fluctuations has been developed and a simple analytical formula for the generation linewidth in the supercritical regime of generation has been derived. It is shown that the strong dependence of the oscillator frequency on the precession power leads to substantial broadening of the generation linewidth of a spin-torque oscillator compared to the case of a linear oscillator, i.e. an oscillator with power-independent generation frequency. The relation between the nonlinearity-induced broadening of the generation linewidth and the nonlinearity-induced increase of the phase-locking band of a spin-torque oscillator to an external microwave signal has been revealed. The derived expression for the generation linewidth predicts a linewidth minimum when the nano-contact is magnetized at a certain angle to its plane, at which the nonlinear frequency shift vanishes. This result is in good agreement with recent experiments.

Keywords : spin-torque, nano-oscillator, generation linewidth

The problem of determination of the generation linewidth of an auto-oscillator where the energy losses are compensated by some external source of energy is an important problem which attracted a lot of interest since the development of first radio-frequency auto-oscillating circuits [1, 2]. It was established that the generation linewidth $\Delta\omega$ in a typical auto-oscillator is determined, for the most part, by the thermal phase noise (see e.g. Eq. (9.36) in [1]), and can be expressed in the following general form:

$$\Delta\omega = \Gamma_0 \frac{k_B T}{E(a)}. \quad (1)$$

Here Γ_0 is the linewidth of the oscillator in the passive regime (for example, in an auto-oscillator with a standard linear oscillating circuit $\Gamma_0 = R/2L$, where R is the circuit resistance and L is its inductance), k_B is the Boltzmann constant, T is the absolute temperature, and $E(a) = \beta|a|^2$ is the averaged energy of the auto-oscillation having the complex amplitude a, and β is the coefficient relating the averaged energy to the squared modulus of the auto-oscillation amplitude. For a standard electrical auto-oscillating circuit $\beta = C/2$, where C is the capacitance of the oscillating circuit and a is the amplitude of the voltage on this capacitance. Eq. (1) is rather general and is equally applicable to any type of conventional auto-oscillator (transistor, vacuum tube, tunnel diode, laser, etc.) in which the *oscillation frequency* ω *is independent of the auto-oscillation amplitude*, i.e. in the limit $d\omega/d|a|^2 \to 0$.

There exist, however, auto-oscillators for which the *oscillation frequency exhibits a strong nonlinearity* $N \equiv d\omega/d|a|^2$ that is too large to be neglected. In such systems, one expects that even the small fluctuations in the amplitude at a steady state can give important contributions to the phase noise. In the present work we consider a pertinent example of such an oscillator, namely, magnetic spin-torque (ST) nano-oscillator [3-7], which consists of a nano-sized metallic contact attached to a magnetic multilayer. Direct electrical current passing through the magnetized nano-contact can lead to the transfer of spin angular momentum between the magnetized magnetic layers in the stack [3, 4], which in turn creates an effective negative damping for the magnetization of the thinner ("free") magnetic layer.

This negative damping plays the role that is analogous to the role played by an active element in any other auto-oscillator, and can lead to self-sustained oscillations of magnetization in the free layer. The frequency of these

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auto-oscillations is determined by the applied magnetic field, static magnetization, etc., and is, in general, close to the ferromagnetic resonance (FMR) frequency in this layer.

In contrast to traditional quasi-linear auto-oscillators, the frequency of the ST nano-oscillator strongly depends on the amplitude of the magnetization precession a: $\omega(a) = \omega_0 + N|a|^2$. The sign and magnitude of the nonlinear frequency shift coefficient N depend on the direction and magnitude of the bias magnetic field (see [6-9] for details), and can be varied over the range that is comparable to the oscillation frequency ω_0 itself.

In the case of such a large nonlinear frequency shift it is not clear weather the classical result (1), derived for a quasi-linear auto-oscillator, can describe quantitatively the generation linewidth in an ST oscillator, and, therefore, a new theory of the generation linewidth that explicitly takes into account the nonlinear frequency shift should be developed for the ST auto-oscillator.

In this paper we develop a theory of the generation linewidth of a spin-torque auto-oscillator with nonlinear frequency shift, and show that this nonlinearity leads to the significant linewidth broadening. We demonstrate that the correct treatment of such nonlinearities is essential for even the *qualitative* description of strongly nonlinear spin-torque auto-oscillators.

It has been shown previously [9-11] that the dynamics of the magnetization in a spin-torque nano-oscillator can be adequately described by the complex dimensionless amplitude $a(t)$ of spin wave mode excited by the direct current. The phase $\phi \equiv \arg(a)$ of the complex amplitude is equal to the azimuthal angle of the magnetization precesion, while the power $p \equiv |a|^2$ determines the polar precession angle $\theta \equiv \arccos(M_z/M_0) = \arccos(1\text{-}2p)$. Here M_0 is the length of the magnetization vector in the "free" magnetic layer, and M_z is the projection of this vector on the equilibrium magnetization direction **z** (see [9] for details). The equation of motion for the spin wave amplitude $a(t)$ can be written in the form

$$\frac{da}{dt} + i(\omega_0 + N|a|^2)a + \Gamma_0(1 + Q|a|^2)a - \sigma I(1 - |a|^2)a = f_n(t)\,. \qquad (2)$$

Here ω_0 is the ferromagnetic resonance (FMR) frequency in the free layer (see Eq. (37) in [9]) and N is the nonlinear frequency shift coefficient (see Eq. (38) in [9]). Γ_0 is the damping rate of a small-angle precession (see Eq. (31) in [9]) that characterizes the equilibrium line-width in the passive regime, and Q is a dimensionless phenomenological coefficient characterizing the nonlinearity of the magnetization damping (see [12]). σ is the spin-polarization efficiency defined in Eq. (2) of [11] and I is the bias charge current. $f_n(t)$ is a stochastic term that accounts for the influence of the thermal fluctuations (noise). The function $f_n(t)$ is a white Gaussian noise with zero mean value and the second-order correlator given by:

$$\langle f_n(t) f_n^*(t')\rangle = 2\Gamma_0 P_n \delta(t - t')\,. \qquad (3)$$

Here P_n is the oscillator power at thermal equilibrium, i.e. $\langle |a|^2 \rangle_{I=0} = P_n$. Note that the energy of spin-torque oscillator can be written as $E(a) = \beta|a|^2$, with the constant β given by $\beta = (M_0/\gamma)\,\omega_0 V_{\text{eff}}$, where γ is the gyromagnetic ratio and V_{eff} is the effective volume of the magnetic material of the free layer involved in the auto-oscillation. Then, the equilibrium noise power P_n can be written as $P_n = k_B T/\beta$, where k_B is the Boltzmann constant and T is the absolute temperature of the system.

We would like to stress that while Eq. (2) is obtained for the case of a spin-torque oscillator, it can adequately describe an auto-oscillator of *any* nature under the influence of white noise $f_n(t)$, provided that this oscillator has a nonlinear frequency dependence ($\omega_0+N|a|^2$), non-linear natural positive damping $\Gamma_0(1+Q|a|^2)$, and non-linear negative damping $\sigma I(1-|a|^2)$.

It is convenient to rewrite one complex equation (2) as two real equations for the power $p = |a|^2$ and phase $\phi = \arg(a)$ of the oscillations:

$$\frac{dp}{dt} - 2(\zeta + Q)\Gamma_0(P_0 - p)p = 2\sqrt{p}\,\mathrm{Re}(\tilde{f}_n)\,, \qquad (4a)$$

$$\frac{d\phi}{dt} + \omega_0 + N_p = \frac{1}{\sqrt{p}}\mathrm{Im}(\tilde{f}_n)\,. \qquad (4b)$$

Here $\zeta \equiv I/I_{\text{th}}$ is the supercriticality parameter of the bias current, $I_{\text{th}} \equiv \Gamma_0/\sigma$ is the threshold current at which self-sustained magnetization oscillations start, P_0 is the stationary power of generation,

$$P_0 = \frac{\zeta - 1}{\zeta + Q}\,, \qquad (5)$$

and $\tilde{f}_n(t) \equiv f_n(t)e^{-i\phi(t)}$. Note, that the statistical properties of $f_n(t)$ and $\tilde{f}_n(t)$ are identical. Therefore, tilde will be omitted in the following text for simplicity.

In the supercritical region $\zeta > 1$ there exists non-zero stationary solution of Eqs. (4) in the absence of thermal noise ($f_n(t) = 0$):

$$p(t) = P_0, \quad \phi(t) = -\omega t + \Phi, \qquad (6)$$

where

$$\omega = \omega_0 + N\,P_0 = \omega_0 + N\frac{\zeta - 1}{\zeta + Q} \qquad (7)$$

is the auto-oscillation frequency and Φ is a constant initial phase of the oscillations.

In all practical application we are interested mainly in a substantially supercritical regime, when the generated power P_0 is much larger then the thermal equilibrium value P_n, $P_0 >> P_n$. In this regime one can assume that the power fluctuations will be much smaller than the mean power level P_0 and the phase Φ will be a slow function of time. Substituting the anzatz

$$p(t) = P_0 + \delta p(t), \quad \phi(t) = -\omega t + \Phi(t) \tag{8}$$

in Eqs. (4), and retaining only the terms of the first order in power δp fluctuations and noise amplitude f_n, one obtains the simplified system

$$\frac{d\delta p}{dt} + 2\Gamma_{\rm eff}P_0\delta p = 2\sqrt{P_0}\mathrm{Re}(f_n), \tag{9a}$$

$$\frac{d\Phi}{dt} + N\delta p = \frac{1}{\sqrt{P_0}}\mathrm{Im}(f_n), \tag{9b}$$

where $\Gamma_{\rm eff} = (\zeta + Q)\Gamma_0$.

It is clear from Eq. (9b) that in the case of a *nonlinear* oscillator $N \neq 0$ the power fluctuations δp become an additional source of the phase noise and, consequently, can lead to the broadening of the generation linewidth. Eq. (9a) for the power fluctuations δp is independent of the phase Φ and has the solution

$$\delta p(t) = 2\sqrt{P_0}\int_{-\infty}^{t}\mathrm{Re}(f_n(t'))\exp[-2\Gamma_{\rm eff}P_0(t-t')]dt'. \tag{10}$$

Using Eqs. (3) and (10), we can calculate the level of power fluctuations:

$$\langle \delta p^2\rangle = \frac{\Gamma_0}{\Gamma_{\rm eff}}P_n = \frac{P_n}{\zeta + Q}. \tag{11}$$

Then, using Eq. (5 and Eq (10) we can rewrite the condition of smallness of the power fluctuations $|\delta p| << P_0$ (under which our approach is valid) as:

$$\zeta - 1 >> \sqrt{(\zeta + Q)P_n}. \tag{12}$$

Taking into account that for typical spin-torque nano-oscillators $P_n \sim 10^{-4}$ and $Q \sim 1$, this condition is satisfied even for $\zeta \geq 1.03$, i.e. for the bias currents that are only a few percent larger than the threshold value. It should be noted, that the small power fluctuations $|\delta p| << P_0$ do not contribute *directly* to the generation linewidth, since the amplitude correlation function $K_A(\tau) \equiv \langle\sqrt{p(t)p(t+\tau)}\rangle$ remains finite even for $\tau \to \infty$, i.e. $K_A(\infty) = P_0 > 0$. The generation linewidth of the ST oscillator, like for any other auto-oscillator in a strongly supercritical regime, is determined only by the phase noise.

Using the solution (10) in Eq. (9b), it is possible to find a closed-form equation for the phase fluctuations $\Phi(t)$ and obtain the exact solution for the spectral linewidth of the auto-oscillator. There is, however, a simpler and more physically transparent approximate approach. Namely, we will assume that the generation linewidth $\Delta\omega$ is much smaller than the inverse correlation time of the power fluctuations, i.e. $\Delta\omega << \Gamma_{\rm eff}P_0$ (below we will determine the condition of validity of this approximation, see Eq. (17)). Then, in the frequency range of the order of the generation linewidth $|d\delta p/dt| \sim \Delta\omega|\delta p| << \Gamma_{\rm eff}P_0|\delta p|$, and the first (derivative) term in the left hand side of Eq. (9a) can be neglected compared to the second term, and, consequently, δp can be approximated as

$$\delta p = \frac{\mathrm{Re}(f_n)}{\Gamma_{\rm eff}\sqrt{P_0}}. \tag{13}$$

Substitution of this expression for δp in Eq. (9b) leads to a closed-form equation for the phase fluctuations $\Phi(t)$ in the system,

$$\frac{d\Phi}{dt} = \frac{1}{\sqrt{P_0}}\left[-\frac{N}{\Gamma_{\rm eff}}\mathrm{Re}(f_n) + \mathrm{Im}(f_n)\right],$$

$$= \frac{1}{\sqrt{P_0}}\sqrt{1+\left(\frac{N}{\Gamma_{\rm eff}}\right)^2}\,\mathrm{Im}(f_n e^{-i\alpha}), \tag{14}$$

where $\alpha = \mathrm{atan}(N/\Gamma_{\rm eff})$.

Eq. (14) is formally identical to the equation for the phase fluctuations in a system *without a nonlinear frequency shift* (see e.g. second equation (9.8) in [1]), but with the *increased noise level*

$$f_n(t) \to f_n'(t) = \sqrt{1+\left(\frac{N}{\Gamma_{\rm eff}}\right)^2}\,e^{-i\alpha}f_n(t). \tag{15}$$

Applying the general methodology of computation of oscillator linewidths in the *absence of nonlinear frequency shifts* (see, e.g., Chapter 9 in [1] or [13]) we can get the expression for the Lorentzian linewidth of the auto-oscillator *with a nonlinear frequency shift N*,

$$\Delta\omega = \Gamma_0\left(\frac{k_B T}{E_0}\right)\left[1+\left(\frac{N}{\Gamma_{\rm eff}}\right)^2\right]. \tag{16}$$

We have rewritten the ratio P_n/P_0 as $k_B T/E_0$, where $E_0 = \langle E(a)\rangle = \beta P_0$ is the average oscillator energy. A comparison of the classical result (1) with the generalized Eq. (16) shows clearly that the nonlinear frequency shift in the auto-oscillator leads to a significant linewidth broadening that is due to an effective renormalization of the phase noise (15) by the nonlinearity N.

The condition $\Delta\omega << \Gamma_{\rm eff}P_0$, under which the result Eq. (16) is valid, can be written as

$$\zeta - 1 >> (\zeta + Q)P_n\left[1 + \left(\frac{N}{\Gamma_{\rm eff}}\right)^2\right]. \tag{17}$$

This condition is stricter than Eq. (12) and, for a typical ST oscillator with $N/\Gamma_0 \sim 10$, is satisfied for $\zeta \geq 1.3$.

The result (16) is the principal result of this paper and illustrates the fact that three key parameters determine the linewidth of an auto-oscillator *with a nonlinear frequency shift*. First, the equilibrium relaxation rate of the oscillator Γ_0 determines the overall scale of the possible linewidth variations. Second, the generation linewidth is proportional to the ratio of the noise energy (which increases with temperature) to the average energy of the auto-oscillation. Third, the ratio of the nonlinear frequency shift coefficient N to the effective nonlinear damping $\Gamma_{\rm eff}$ gives a measure of the phase noise renormalization due to amplitude fluctuations.

It is interesting to note, that the factor $[1 + (N/\Gamma_{\rm eff})^2]$, describing nonlinear linewidth broadening in Eq. (16), is exactly the same as the factor describing increase of the bandwidth of phase-locking of a ST oscillator to an external microwave signal [15] (see Eq. (16) in [15] and note that the case of a linear damping $Q = 0$ was considered there). In [15] the authors considered the *deterministic* behavior of a ST oscillator, described by Eq. (2) with $Q = 0$ and the thermal noise term $f_n(t)$ replaced by the external microwave signal $f_n(t) \to F_e e^{-i\omega_e t}$. It was shown that the bandwidth $\delta\omega = |\omega_e - \omega|$ of the frequency detuning of the external signal, in which the ST oscillator can be phase-locked (i.e., starts to generate at the external frequency ω_e rather than at the free-running frequency ω), is given by the expression:

$$\delta\omega = \frac{F_e}{\sqrt{P_0}}\sqrt{1 + \left(\frac{N}{\Gamma_{\rm eff}}\right)^2}. \tag{18}$$

The similar structure of Eqs. (16) and (18) suggests that there exists a deep connection between the phenomenon of phase-locking of an oscillator to the external signal and the phenomenon of broadening of the generation linewidth in the presence of thermal fluctuations.

To show this more clearly we note that the effective amplitude $F_e(\delta\omega)$ of the random signal $f_n(t)$, *acting in the frequency range* $\delta\omega$, can be calculated with the help of Eq. (3) as

$$F_e^2(\delta\omega) = \Gamma_0 P_n \delta\omega. \tag{19}$$

Substituting this expression for the effective signal amplitude in Eq. (18), one obtains the following equation for the self-consistent determination of the phase-locking bandwidth for a random external signal:

$$\delta\omega = \sqrt{\frac{\Gamma_0 P_n \delta\omega}{P_0}\left[1 + \left(\frac{N}{\Gamma_{\rm eff}}\right)^2\right]}. \tag{20}$$

The solution of this equation coincides with Eq. (16). Thus, the phenomenon of the broadening of the generation linewidth of an oscillator under the influence of thermal noise can be considered as a *phase-locking of the oscillator to the thermally-induced random external signal* $f_n(t)$.

Generation linewidth Eq. (16) is inversely proportional to the energy of oscillation, which, in the case of a ST nano-oscillator, is proportional to the effective volume $V_{\rm eff}$ of the magnetic material of the free layer involved in the auto-oscillation, $E_0 = (M_0/\gamma)\omega_0 V_{\rm eff} P_0$. The effective volume $V_{\rm eff}$ for a ST auto-oscillator based on a magnetic nanopillar is equal to the volume of the nanopillar free layer itself, $V_{\rm eff} = V$, which is substantially smaller than the effective volume for a similarly-sized magnetic nanocontact. In the latter case the effective volume is defined by Eq. (4) in [11] and is several times larger than the physical volume of the free magnetic layer *under the contact* due to the propagation of spin waves radiated from the nano-contact. Thus, our theory finds a natural explanation for a well-known experimental fact (see [6-8]) that the auto-oscillation linewidths associated with magnetic nanopillars are, in general, several times broader that those in magnetic nano-contacts.

Another important result that follows from Eq. (16) is the prediction of a linewidth minimum that follows from a change in sign in the frequency shift (e.g., from "red" ($N < 0$) to "blue" ($N > 0$)) as the magnetization is tilted out of the film plane. When the out-of-plane magnetization angle is increased the nonlinear frequency shift coefficient N passes through zero (see, e.g., Fig. 8 in [9]) at which one recovers the smallest value of the phase linewidth. A change in magnetization angle can be achieved in practice by applying a large external magnetic field at different orientations out of the film plane. This linewidth minimum has been recently observed in experiment (see Fig. 6 in [4]).

To illustrate the qualitative behavior of the generation linewidth of a ST auto-oscillator with the variation of the external control parameters, such as the magnitude of the bias current I and the magnitude H_e and direction θ_e of the external bias magnetic field, we present below results of the linewidth calculations obtained from Eq. (16). We consider a circular nano-contact oscillator with a permalloy free layer ($4\pi M_0 = 7.5$ kG, Gilbert damping constant $\alpha_G = 0.01$, nonlinear damping parameter $Q = 3$) having the thickness $d = 5$ nm and radius of the current-carrying region $R_c = 25$ nm. In our calculations we used the spin-

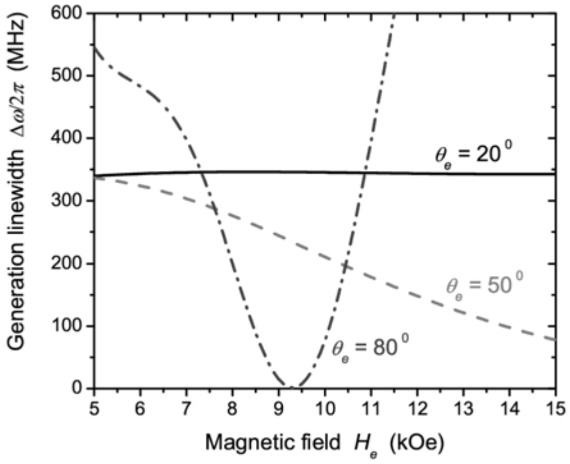

Fig. 1. (Color online) Generation linewidth of a spin-torque auto-oscillator calculated from Eq. (16) as a function of the applied magnetic field H_e for $I = 9$ mA and for several angles θ_e between the bias field and the plane of the magnetic free layer of the nano-contact.

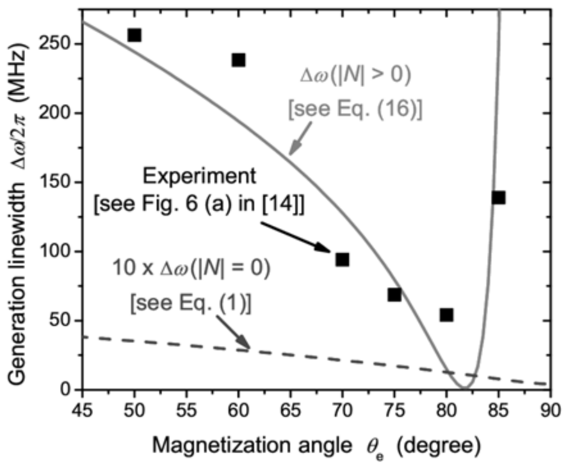

Fig. 2. (Color online) Generation linewidth as a function of the applied field angle θ_e for $H_e = 9$ kOe and $I = 9$ mA: solid line-calculation using Eq. (16) with the account of the nonlinear frequency shift coefficient N; dashed line-calculation using the classica result Eq. (1) multiplied by 10. Points show experimental data for the averaged generation linewidth [14] (see Fig. 6a in [14]).

polarization efficiency $\varepsilon = 0.2$ and assumed that the nano-contact is kept at a room temperature $T = 300$ K. The chosen parameters of our model ST oscillator are close to the parameters of the experimental system used in [14].

While the linewidth narrowing with the increase of the bias current is clearly seen from Eq. (16), the linewidth behavior as a function of the bias magnetic field is more complex than what might be expected. As it is shown in Fig. 1, for small magnetization angles θ_e the generation linewidth is large and almost constant in a wide range of applied magnetic fields H_e (see curve for $\theta_e = 20^{\circ}$ in Fig. 1). With the increase of the magnetization angle (curve for $\theta_e = 50^{\circ}$ in Fig. 1) generation linewidth decreases with the increase of the magnetic field magnitude H_e. Such a narrowing of the generation linewidth in caused by the decrease of the absolute value of the nonlinear frequency shift coefficient $|N|$, when the equilibrium magnetization is tilted away from the film plane. Finally, for the almost normal direction of the external magnetic field ($\theta_e = 80^{\circ}$ in Fig. 1), one observes a non-monotonous variation in the linewidth around the field magnitude that corresponds to the vanishing of the nonlinear frequency shift coefficient $N \to 0$. In such regions there is a rapid linewidth narrowing followed by a comparable broadening as the field is increased through the critical region in which the linewidth minimum occurs.

If Fig. 2 we showed the dependence of the generation linewidth Eq. (16) on the applied field orientation angle θ_e. The data in Fig. 2 are calculated for $H_e = 9$ kOe and I = 9 mA, which corresponds to the central part of the region of parameters experimentally studied in [14]. Solid line in Fig. 2 was obtained using Eq. (16) and demonstrates a deep minimum near the point at which the nonlinear frequency shift coefficient N vanishes. Points in Fig. 2 show the experimental data from Fig. 6a in [14]. A good agreement between the developed theory and the experiment [14], both in qualitative behavior and quantitative values of the generation linewidth, is evident from Fig. 2. At the same time, the classical result (1) (see dashed line in Fig. 2 and note that these data were multiplied by 10) predicts generation linewidth two orders of magnitude smaller than the experimental value and does not reproduce characteristic dip at the magnetization angles $\theta_e \approx 80^{\circ}$.

In conclusion, we have developed a theory of the generation linewidth of a nonlinear spin-torque auto-oscillator with a nonlinear frequency shift which generalizes the classical result (1). We have shown that far above the generation threshold the nonlinearity in the oscillator frequency leads to the renormalization of the phase noise. The developed theory explains a number of characteristic, but previously unexplained, features of ST oscillators observed in experiment: (i) General linewidth narrowing with the increase of the bias current and the oscillation amplitude (see Fig. 4 in [14]); (ii) Presence of a sharp minimum in the dependence of the generation linewidth on the external bias magnetic field orientation (see Fig. 6 in [14]); (iii) Lower values of the generation linewidth in magnetic nano-contacts compared to magnetic nano-pillars (due to the difference in the effective magnetic volumes involved in the auto-oscillations).

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