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Validity of the Analytic Expression for the Temperature of Joule Heated Nano-wire

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We confirm the validity of the analytic expression for the temperature of the Joule heated nano-wire [C.-Y. You *et al.* Appl. Phys. Lett. 89, 222513 (2006)] with finite element method. The temperature of the Joule heated nano-wire is essential information for the research of the current induced domain wall movement. The analytic expression includes an adjustable parameter which must be determined. Since the physical origin of the adjustable parameter is simplification of the heat source profile, the validity of the analytic expression must be examined for wide range of the nano-wire structure. By comparison with this analytic expression with the results of full numerical finite element method, the adjustable parameter has been determined. The numerically confirmed adjustable parameter values are in the range of 0.60~0.69, which is well matched with the theoretically expected one. Furthermore, it is found that the adjustable parameter is a slow varying function of the nano-wire geometry. Based on this numerical confirmation, we can apply the analytic expression for the wide range of the nano-wire geometry with proper adjustable parameters.

Keywords : nano-wire, CIDWM, Joule heat, FEM, spin transfer torque

1. Introduction

Recently, the spin transfer torque has been intensively studied both theoretically [1-3] and experimentally [4-6]. The spin transfer torque generates novel physical phenomena such as current induced magnetization switching [7] and magnetic domain wall movement (CIDWM) in the magnetic nano-wire by flowing spin polarized currents [10-13]. Since the magnetic configurations can be controlled by a spin polarized electric current rather than by an applied magnetic field, it has a great potential for the application of spintronic devices such as spin transfer torque RAM (random access memory) [8] or race-track memory [9] which is based on CIDWM. In particular, for the case of CIDWM, it is reported that required current density is more than 5.0×10^{11} A/m^2, while the current density of 7.5×10^{11} A/m^2 generates enough Joule heat to increase the temperature of the nano-wire near or exceeding the Curie temperature of the nano-wire [11]. Therefore, the heating problem is a crucial factor for the applications of CIDWM. In the previous work [15], an analytic expression has been obtained for the temperature of nano-wire for a given geometry and thermal properties of the substrate. However, for the mathematical simplicity, the heat generated by Joule heating at the nano-wire was assumed as a Gaussian profile. Furthermore, it was assumed that the heat is generated only at the interface between the nano-wire and the substrate as shown in Fig. 1(a)~(c). Those two main assumptions are sources of an adjustable parameter α. Even though the validity of the analytic expression is already examined for various thickness of the nano-wire with fixed width [15], more rigorous confirmation is required before we apply the analytic expression for the general cases. We perform a numerical calculation with realistic heat source as shown in Fig. 1(b), the heat is generated in the whole nano-wire by the Joule heating. A finite element method is employed with commercial software, COMSOL Multiphysics [16]. We obtain numerical results for various thicknesses and widths of the nano-wire, and compare them with the analytic expression in order to find the adjustable parameter α. With these heavy numerical studies, the validity of the analytic expression is confirmed. And the adjustable parameter α is found to have a value of 0.60~0.69 for wide range of the width and thickness of the nano-wire and is shown to depend only weakly on the nano-wire geometry. Therefore, we can apply this analytic expression to the nano-wire for the wide range of geometries.

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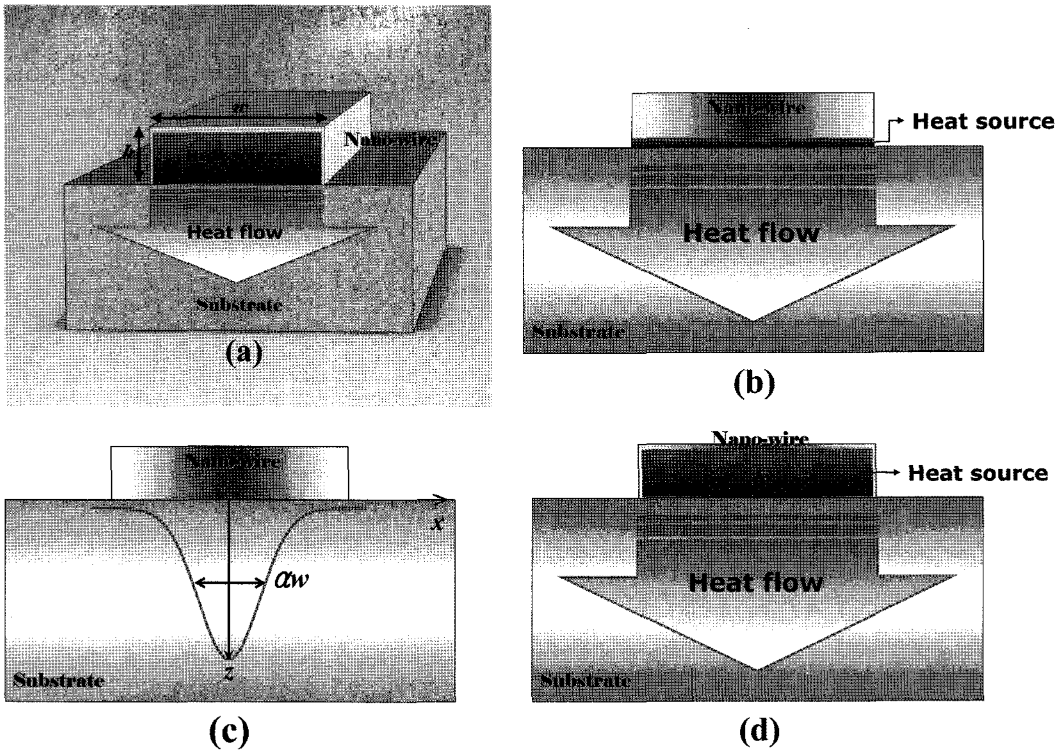

Fig. 1. (a) The heat is generated by the current in the whole cross-section of the nano wire. (b) Nano wire system is regarded as two-dimensional problem. We assumed that the position of heat source is at the interface between nano-wire and substrate for analytic solution. (c) Gaussian profile of the heat source and the geometrical meaning of the adjustable parameter α. (d) We assumed that heat source is distributed over all cross section of the nano-wire and heat profile has rectangular shape for numerical calculations.

2. Theory

In the previous study [15], the analytic expression for the temperature of a magnetic nano-wire with applied current has been obtained. Brief explanation of the process and result of the previous work will be helpful for the reader. The heat conduction equation is given by

$$\rho C\frac{\partial T(\bar{r},t)}{\partial T}=K\nabla^{T}T(\bar{r},t)+S(\bar{r},t) \tag{1}$$

where ρ, C, and K are the density, specific heat, and thermal conductivity of the medium. $T(\bar{r},t)$ and $S(\bar{r},t)$ are the temperature and heat source at the position of $\bar{r}$ and time t. The heat source term $S(\bar{r},t)$ which consists of power density per unit area $S_J(x,t)$, power absorption rate $S_A(z)$ and pulse function, $p(t)$, $S(\bar{r},t)=S_J(x,t)$ $S_A(z)p(t)$. In Ref. [15], the Green's function method [17] is employed in order to find the analytical solution of the Eq. (1) for the nano-wire system with some assumptions. Most important assumption is that the heat generated in the nano-wire only passes through the interface to the substrate. Due to this assumption, we can handle the heat source as confined one at the interface as shown in Fig. 1(b) and let $S_A(z)=\delta(z)$. The $S_J(x,t)$ should be rectangle as like Fig. 1(b), however, a Gaussian shape has an advantage of the mathematical simplicity. Therefore we can choose Gaussian type heat source as shown in Fig. 1(c) with an adjustable parameter α as follows:

$$S_J^G(x,t)=\frac{P_0}{\sqrt{\pi}\alpha wL}\exp\left(-\frac{x^2}{(\alpha w)^2}\right) \tag{2}$$

Here, $P_0(=RI^2=LwhJ^2/\sigma_W)$, σ_W, h, w and $\theta(x)$ are Joule heat due to current density J, electric conductivity, thickness and width of the wire and step function. The adjustable parameter α is expected to have a value of around 0.5 due to its geometrical meaning, and it must be determined numerically. With this adjustable parameter α, the temperature of the nano-wire at the origin, $T(x=0, z=0, t)$ is given by increase of temperature ΔT and 273.15 K,

$$T(t)=\frac{whJ^2}{\pi\sigma_W K_S}\left(\ln\left(\frac{4\sqrt{\mu_S t}}{\alpha w}\right)-\theta(t-t_p)\ln\left(\frac{4\sqrt{\mu_S(t-t_p)}}{\alpha w}\right)\right)+273.15 \tag{3}$$

for $t\gg w^2/\mu_S$. Here μ_S, ρ_S, t_p and K_S are the diffusivity, density, pulse duration time and thermal conductivity of the substrate. The Eq. (3) gives similar tendency of temperature for the given material condition with rectangular heat profile as already proved in Ref. [15]. The goal of this paper is finding the value of α for the various nano-wire geometries in order to show the validity of the Eq. (3).

3. Finite Element Method Calculations

In order to compare with the analytic expression, the temperature profile of nano-wire was numerically calculated by finite element method with COMSOL Multiphysics [16]. Symmetry of nano-wire along the direction of the nano-wire allows us to consider the nano-wire system as a two dimensional problem as shown in Fig. 1(d). We select Ni as nano-wire's material. The material parameters for SiO_2 were taken as μ_S=8.27×10^{-7} m^2/s, C_S=730 J/(kg K), and K_S=1.4 W/(mK). Since heat is conducted mainly through the substrate and air convection is negligible, we assume that the boundaries which are exposed to the air, are thermally insulating. Also we assume the temperature of boundaries of SiO_2, the bottom and both sides, were fixed as 273.15 K, since the size of substrate is much lager than the wire and we can handle the substrate as a heat reservoir. Joule heating power per volume is given by J^2/σ_W ($J = 10^{12}$ A/m^2, σ_W=1.38×10^7 (Ωm)$^{-1}$). Heat profile is investigated while width and thickness of nano-wire are varied.

4. Results and Discussions

First, we refer to Yamaguchi's experimental conditions [14] for the geometric conditions and material parameters for reality of our numerical results. Figure 2 shows temperature as a function of time for the nano-wire whose cross section is 240 nm×10 nm when the current density of 10^{12} A/m^2 is applied during 5 μs. The rectangles denote the result of the numerical calculation and the gray solid line represents the analytic one with α=0.5. Two curves almost coincide, which means the analytic solution well agrees with the numerical result.

Figure 3 and 4 show the peak temperature $T(t = t_p)$ dependence on the thickness and width of the nano-wire.

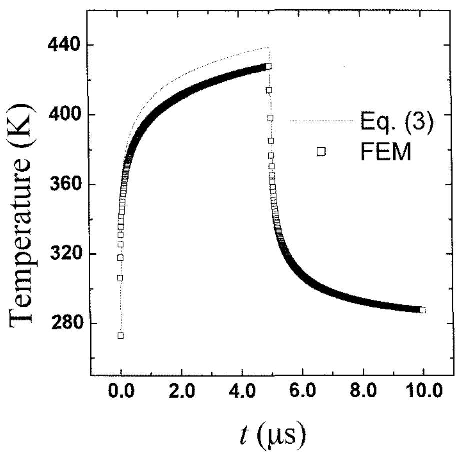

Fig. 2. Temperature curves of nano-wire with pulse current applied for t_p=5 μs. Eq. (3) is represented by the solid line and FEM result by the symbol □.

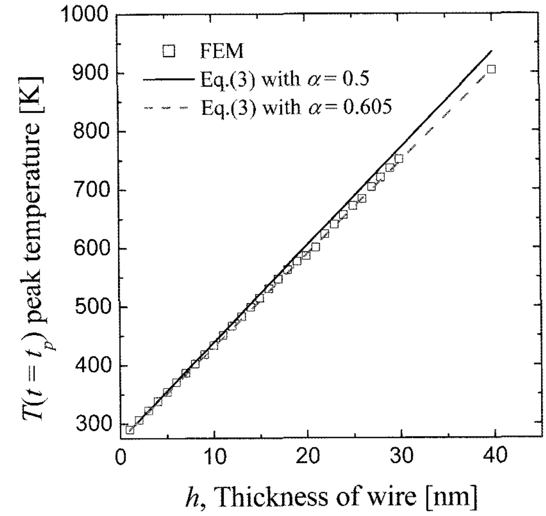

Fig. 3. Peak temperature (t=t_p) vs thickness of nano-wire for FEM result (□), Eq. (3) with α = 0.5 (solid line) and Eq. (3) with α = 0.605 (gray dashed line) for fixed width of the nano-wire as 240 nm.

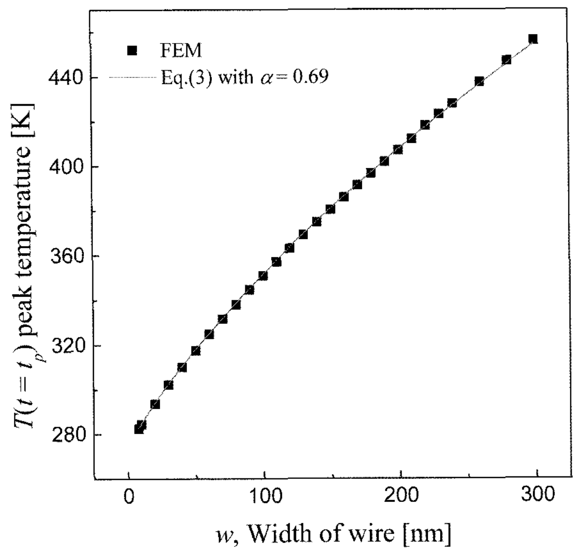

Fig. 4. The dependence of the peak temperature (t=t_p) on the width of nano-wire. FEM result are denoted by the solid symbol and and the results of Eq. (3) with α = 0.69 by the solid gray line for fixed thickness of the nano-wire as 10 nm.

The peak temperature increases linearly with the thickness of the nano-wire, while it increases in a slightly nonlinear way with the width of the wire. According to the analytic result, as shown in Eq. (3), the peak temperature of nano-wire $T(t = t_p)$ is function of h and $w(\ln(1/w))$. The heat flow is proportional to the surface area contacted to the substrate. The more current flows in the larger cross-sectional area for a given current density, and amount of heat generated by current per unit length, J^2wh/σ_W, is proportional to the cross-section area of the nano-wire. Therefore, the increase in the thickness of the wire enhances the amount of the heat and makes the peak temperature increase as shown in Fig. 3. Whereas, to enlarge the width of wire makes, not only Joule heating power increase, but also enlarges the contact area which the heat flow out through. It is the origin of the logarithm term.

We find the adjustable parameter α by fitting the numerical results with Eq. (3) for various h and w. The results are shown in Fig. 3 and 4. These values of a are 0.605±0.004 and 0.69±0.0034 for each case. Furthermore, we yield α for various thicknesses and widths of nano-wire. Figure 5 shows the peak temperatures for various sizes of wires. The ranges of the width and thickness of nano-wire are 50 nm~400 nm and 2.5 nm~40 nm, respectively. Each point of the peak temperature is yielded by numerical calculation by using pulse duration time t_p=5 μs with current density J=10^{12} A/m^2.

The value of α is evaluated by substituting peak temperature of Fig. 5 to T of Eq. (3). Corresponding t=t_p (=5 μs) is represented in Fig. 6. The value of α falls within the range of 0.63~0.69, which is quite reasonable values. The contour lines (white curves of Fig. 6) indicate that α weakly depends on the thickness and width of nano-wire. As wire becomes wider and thinner, α becomes

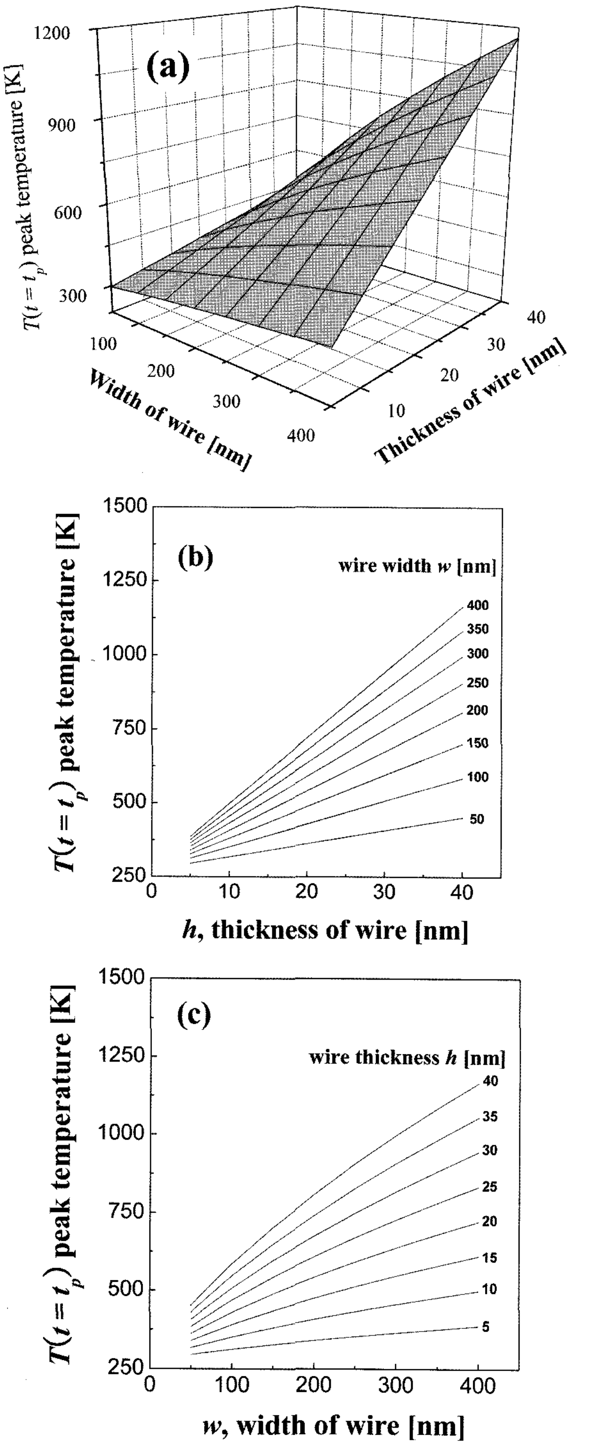

Fig. 5. (a) Peak temperature as a function of nano-wire width and thickness. (b) Peak temperature as a function of nano-wire thickness for a given width. (c) Peak temperature as a function of nano-wire width for a given thickness.

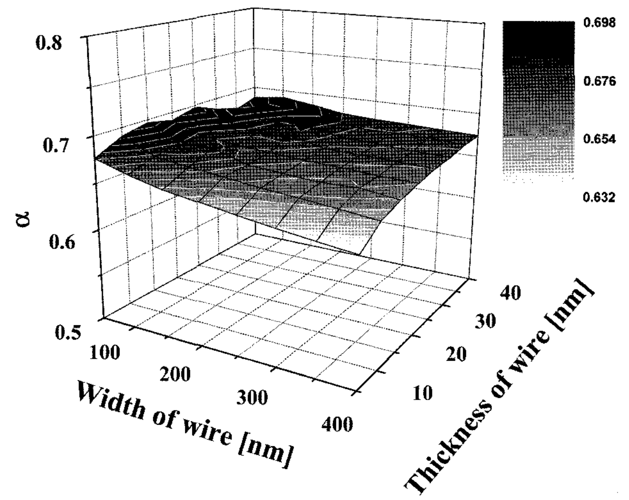

Fig. 6. The values of the adjustable parameter α of the nano-wire for various dimension of nano-wire. The contour curves show that α weakly depends on the geometry of the nano-wire.

smaller. The analytic approach can not pick such behavior of α.

5. Conclusion

We investigate the temperature of a magnetic nano-wire using a finite element method. We numerically calculate the temperature as a function of the time, and it shows the same tendency with the analytic expression. The dependence of the peak temperature on the cross-sectional dimension (width and thickness) of nano-wire is also calculated for wide range of the geometry, and it is well explained with the analytic expression with the proper choice of the adjustable parameter α. The value of α is in the range of 0.60~0.69, which is reasonable. The analytic expression well describes the temperature of the Joule heated nano-wire with the proper adjustable parameter α, which is obtained in this study.

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References

[1] J. C. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996).
[2] L. Berger, J. Appl. Phys. **49**, 2156 (1978).
[3] L. Berger, Phys. Rev. B **54**, 9353 (1996).
[4] M. Tsoi, A. G. M. Jansen, J. Bass, W.-C. Chiang, M. Seck, V. Tsoi, and P. Wyder, Phys. Rev. Lett. **80**, 4281 (1998).
[5] S. I. Kiselev, J. C. Sankey, I. N. Krivorotov, N. C. Emley, R. J. Schoelkopf, R. A. Buhrman, and D. C. Ralph, Nature **425**, 380 (2003).
[6] J. Z. Suna, D. J. Monsmab, T. S. Kuan, M. J. Rooks, D. W. Abraham, B. Oezyilmaz, A. D. and Kent, R. H. Koch, J. Appl. Phys. **93**, 6859 (2003).
[7] J. Y. Lee, S. Choi, and S.-K. Kim, J. of Magnetics **11**, 74 (2006).
[8] M. Hosomi, H. Yamagishi, T. Yamamoto, K. Bessho, Y. Higo, K. Yamane, H. Yamada, M. Shoji, H. Hachino, C. Fukumoto, H. Nagao, and H. Kano, Electron Devices Meeting, 2005. IEDM Technical Digest. IEEE International **2005**, 4 (2005).
[9] S. S. P. Parkin, U. S. patent 6834005 (2003).
[10] E. Saitoh, H. Miyahima, T. Yamaoka, and G. Tatara, Nature **432**, 203 (2004).
[11] A. Yamaguchi, T. Ono, S. Nasu, K. Miyake, K. Mibu, and T. Shinjo, Phys. Rev. Lett. **92**, 077205 (2004).
[12] M. Kläui, et al. Phys. Rev. Lett. **94**, 106601 (2003).
[13] Z. Li and S. Zhang, Phys. Rev. B **70**, 024417 (2004): S. Zhang and Z. Li, Phys. Rev. Lett. **93**, 127204 (2004).
[14] A. Yamaguchi, S. Nasu, H. Tanigawa, T. Ono, K. Miyake, K. Mibu, and T. Shinjo, Appl. Phys. Lett. **86**, 012511 (2005).
[15] C.-Y. You, I. M. Sung, and B.-K. Joe, Appl. Phys. Lett. **89**, 222513 (2006).
[16] http://www.comsol.com
[17] H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, 2nd ed. (Clarendon, Oxford, 1959); J. V. Beck, K. D. Cole, A. Haji-Sheikh, and B. Litkouhi, Heat Conduction Using Green's Functions (Hemisphere, Washington, DC, 1992).