

## Irreversible Magnetization of MgB<sub>2</sub> Superconductor

Mun-Seog Kim\*, Kyu-Tae Kim, Wan-Seop Kim, Po Gyu Park, Kyu Won Lee, and Sung-Ik Lee<sup>1</sup>

Division of Electromagnetic Metrology, Korea Research Institute of Standards and Science, P. O. BOX 102, Yuseong, Daejeon 305-600, Korea

<sup>1</sup>Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Korea

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We report the magnetic-field dependence of the irreversible magnetization of binary superconductor MgB<sub>2</sub>. For the temperature region of  $T < 0.9T_c$ , the contribution of the bulk pinning to the magnetization overwhelms that of the surface pinning. This was evident from the fact that the magnetization curves,  $M(H)$ , were well described by the critical-state model without considering the reversible magnetization and the surface pinning effect. It was also found that the  $M(H)$  curves at various temperatures scaled when the field and the magnetization were normalized by the characteristic scaling factors  $H^*(T)$  and  $M^*(T)$ , respectively. This feature suggests that the pinning mechanism determining the hysteresis in  $M(H)$  is unique below  $T = T_c$ .

**Key words :** MgB<sub>2</sub>, irreversible magnetization, the exponential critical-state model, flux jumping, irreversibility line, surface barrier

### 1. Introduction

In the mixed state, the magnetization of superconductors is a combination of two different contributions,  $M_{eq}$  and  $M_{irr}$ .  $M_{eq}$  is the equilibrium or reversible magnetization [1] and  $M_{irr}$  is the irreversible magnetization. The former is caused by the equilibrium surface current. The latter arises from the surface (Bean-Livingston) barrier effect [2], as well as the bulk pinning due to the interaction between vortices and various defects within the superconductor. The surface barrier originates from the competition between two forces, an attractive interaction between a vortex and its image vortex and a repulsive interaction between a vortex and the surface-shielding current. For high- $T_c$  cuprate superconductors, the irreversible magnetization at low temperatures is dominated by the bulk pinning. However, the role of the surface-barrier effect becomes significant as the temperature approaches toward  $T_c$  [3].

In this work, we measured the magnetization  $M(H)$  of MgB<sub>2</sub> superconductor [4] as a function of the external magnetic field to elucidate its pinning properties in detail. A prominent flux-jumping effect at low temperatures due

to a low thermal conductivity of the material [5] was observed, which was reflected by abrupt changes of magnetization when external magnetic field varies.

Except for the flux-jumping region, the  $M(H)$  curves for various temperatures were successfully described by the exponential critical-state model [6]. The  $M(H)$  curves up to  $T/T_c = 0.9$  were scaled to a single universal curve with the characteristic scaling factors,  $H^*(T)$  and  $M^*(T)$ , implying a significant role of bulk pinning in our experimental window. In addition, we found that the characteristic field  $H^*(T)$ , proportional to the irreversible line  $H_{irr}(T)$  in vortex-phase diagram, was not described by the  $H_{irr} \sim (1 - T/T_c)^n$  ( $n \geq 1.5$ ) [7], which was in sharp contrast to the cuprates.

### 2. Experiments

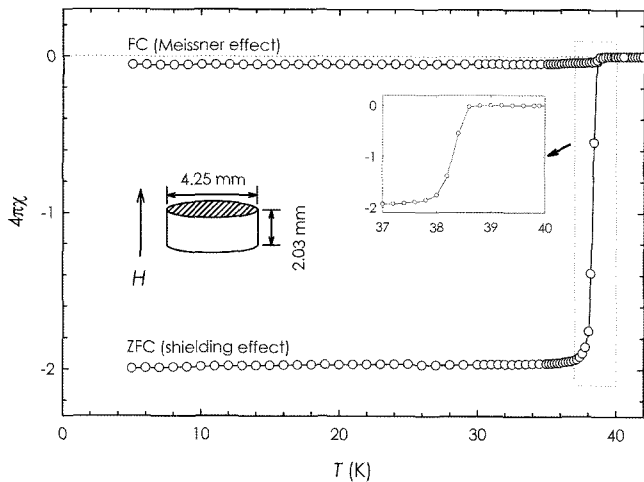
Details on the sample preparation are given elsewhere [8]. A stoichiometric mixture of Mg and B (99.97%, Alfa Aesar) was ground in a glove bag filled with Ar gas. The resulting precursor was palletized, wrapped in Ta foil, and put into a high-pressure cubic cell. It was pressed up to 3.5 GPa at 950°C for 2 hours, and then quenched to room temperature. The dimensions of the sample were 2.03 mm long and 4.25 mm in diameter. The magnetization curves were measured by using a superconducting quantum inter-

\*Corresponding author: Tel: +82-42-868-5725,  
e-mail: msk2003@kriss.re.kr

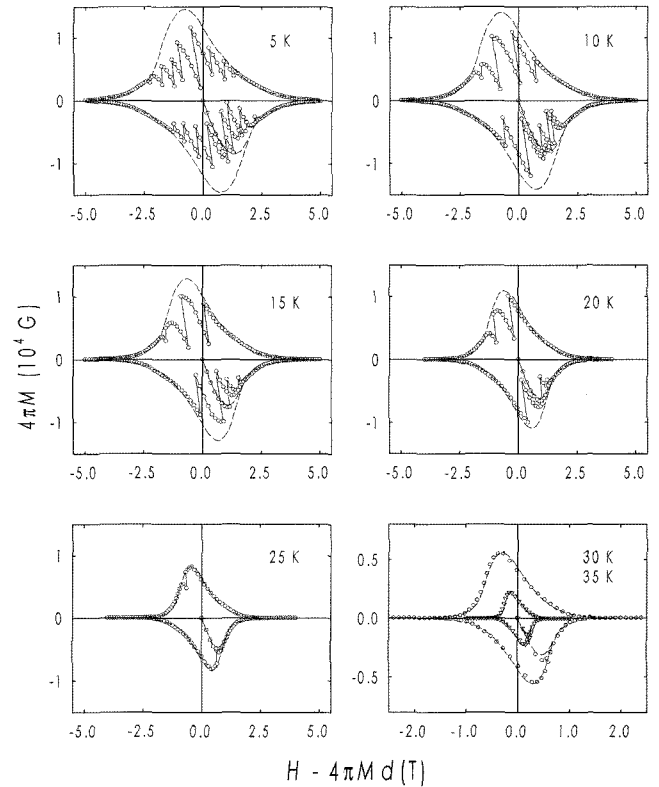
ference device magnetometer (Quantum Design, MPMS-XL).

### 3. Results and Discussion

Figure 1 shows the temperature dependence of the magnetic susceptibility,  $4\pi\chi(T)$ , which is measured in external field,  $H_{\text{ext}} = 5$  Oe, parallel to principle axis of the cylindrical sample. This figure reveals that the superconducting transition occurs at 38.6 K and the transition width is less than 1 K. At low temperatures, the absolute value of  $4\pi\chi$  for ZFC curve is close to 2.0, while the value for FC curve is about 0.1. This pronounced difference reflects a strong vortex pinning in this material. For s superconductor, the low-field susceptibility is written by  $4\pi\chi = 4\pi M/H_{\text{ext}} = 1/(d-1)$ , where the demagnetization factor,  $d$ , can vary with relative direction of the external field to the principle axis of the sample. From this formula and the low-temperature susceptibility for ZFC curve, the demagnetization factor of the sample is deduced to be  $d = 0.51$ . This value is quite close to 0.54 calculated from the sample dimensions, 2.03 mm long and 4.25 mm in diameter, using the Orborn formula [9]. This accordance says that the coupling between grains consisting of the sample is quite strong, and thus the demagnetization effect is governed by the outward appearance of the sample, not by the size and the shape of each grains. In other words, the sample shows a bulk superconductivity, even though the grain boundaries exist. As we mentioned in previous section, our sample was synthesized by the high-pressure technique under a high



**Fig. 1.** Temperature dependence of the magnetic susceptibility,  $4\pi\chi(T)$ , measured in external field,  $H_{\text{ext}} = 5$  Oe, parallel to principle axis of the cylindrical sample.  $4\pi\chi$  data were taken in both zero-field-cooled (shielding) and field-cooled (Meissner) conditions. Inset:  $4\pi\chi(T)$  (ZFC) around transition region.



**Fig. 2.** Magnetization curves,  $4\pi M(H)$ , measured in the region of  $5 \text{ K} \leq T \leq 35 \text{ K}$  and  $-5 \text{ T} \leq H \leq 5 \text{ T}$ . The dashed lines represent the theoretical curve for the exponential critical-state model.

temperature, which might make the strong grain coupling possible.

Figure 2 shows  $4\pi M(H)$  curves measured at various temperatures. In the figure, the  $x$ -axes for  $H$  were corrected by the demagnetization factor  $d = 0.51$  obtained from Fig. 2. Below 25 K, the saw-tooth behavior is clearly seen at relatively low-field region due to the flux jumps [5, 10]. In the critical state, a small perturbation of temperature could give rise to increase the sample temperature. As a result, the screening-current density is diminished, which is accomplished by redistribution of vortices inside the superconductor. This process results in the perturbation of electric field and thereby generation of an additional heat. Since the thermal conductivity of  $\text{MgB}_2$  is not so high [5], the additional heat release due to the flux motion is not removed effectively. Thus the critical state becomes unstable and the flux jumps occur, causing instability of magnetization.

The irreversible magnetization can be described by various critical-state models [6, 11, 12]. Bean's critical-state model [12] has been widely used to describe the magnetization and to calculate the critical-current density for superconducting materials. The model assumes that

the slope  $dh(r)/dr$  is a constant and field independent, where  $h(r)$  denotes the local magnetic induction inside a sample. Therefore, the critical-current density or the irreversible magnetization should also be field independent, which is contrary to the experiments. Here, we employ the exponential critical-state model [6] to describe the magnetization properly, which takes into account the field dependence of the critical-current density.

In the frame of the exponential model, the critical-current density,  $J_c(h(r))$ , is given by

$$J_c(h(r)) = J_0 \exp(-|h(r)|/H_0), \quad (1)$$

where  $J_0$  and  $H_0$  are adjustable parameters which depend on material. According to Ampere's law, the field gradient inside a sample is given by

$$\frac{dh(r)}{dr} = -\text{sgn}(J) \frac{4\pi}{c} J_c(h(r)), \quad (2)$$

where  $\text{sgn}(x)$  is the sign function and  $c$  is the speed of light.

For an infinitely long cylindrical superconductor with radius  $a$ , we solve Eq. (2) and calculate the irreversible magnetization. In the calculation, the demagnetization effect is irrelevant, since the applied field  $H$  is parallel to principle axis of a long cylinder. To get a complete picture of the critical states, five different cases according to the external-field regime should be considered.

*Case I:  $0 \leq H \leq H_p$  in increasing-field branch*

As the external magnetic field increases from 0 to  $H_p$ , the vortices can penetrate into the sample. (Here, we assume the lower critical field,  $H_{c1}$ , to be zero, which is valid when  $|H_{\text{irr}}| \gg |H_{\text{eq}}|$ .) The penetrated vortices are distributed in the region between  $r_1$  and  $a$  ( $0 \leq r_1 \leq a$ ), where the  $r_1$  is zero at  $H = H_p$ . The solution for  $h(r)$  is given by

$$h(r) = \begin{cases} 0 & \text{for } 0 \leq r \leq r_1 \\ H_0 \ln[(J_0 r + c_0)/H_0] & \text{for } r_1 \leq r \leq a \end{cases}.$$

The boundary conditions  $h(a) = H$  and  $h(r_1) = 0$  give the integration coefficient  $c_0$ , the field boundary  $r_1$ , and the  $H_p$ ,

$$\begin{aligned} c_0 &= H_0 \exp(H/H_0) - J_0 a \\ r_1 &= (H_0 - c_0)/J_0 \\ H_p &= H_0 \ln(J_0 a/H_0 + 1). \end{aligned}$$

*Case II:  $H_p \leq H \leq H_{\text{max}}$  in increasing-field branch*

For  $H \geq H_p$ , the vortices are distributed in the whole region of  $0 \leq r \leq a$ . The solution for this case is the same as that in case I, if the  $r_1$  is replaced by zero. The  $H_{\text{max}}$

denotes the maximum value of the applied field.

*Case III:  $H_{\text{max}} \geq H \geq H_r$  in decreasing-field branch*

When the external field decreases from  $H_{\text{max}}$ , the distribution of vortices in the sample is given by

$$h(r) = \begin{cases} H_0 \ln[(J_0 r + c_{\text{max}})/H_0] & \text{for } 0 \leq r \leq r_2 \\ H_0 \ln[(-J_0 r + c_1)/H_0] & \text{for } r_2 \leq r \leq a \end{cases}.$$

While the  $h(r)$  in the region of  $0 \leq r \leq r_2$  is determined by  $H_{\text{max}}$  in the previous case, the external field determines  $h(r)$  in  $r_2 \leq r \leq a$ . The signs of slope  $dh(r)/dr$  in the two regions differ from each other. For  $H = H_r$ , the  $r_2$  becomes zero. From the boundary condition  $h(a) = H$  and the continuity condition at  $r = r_2$ , the parameters  $c_{\text{max}}$ ,  $c_1$ ,  $r_2$ , and  $H_r$  are given by

$$\begin{aligned} c_{\text{max}} &= H_0 \exp(H_{\text{max}}/H_0) - J_0 a \\ c_1 &= H_0 \exp(H/H_0) + J_0 a \\ r_2 &= (c_1 - c_{\text{max}})/2J_0 \\ H_r &= H_0 \ln[\exp(H_{\text{max}}/H_0) - 2J_0 a/H_0]. \end{aligned}$$

*Case IV:  $H_r \geq H \geq 0$  in decreasing-field branch*

In this case, the external field decreases from  $H_r$  to zero. Even at zero field, however, the supercurrent flows through the entire region of  $0 \leq r \leq a$ . This current determines the residual (remanent) magnetization. The solution is given by

$$h(r) = H_0 \ln[(-J_0 r + c_2)/H_0] \text{ for } 0 \leq r \leq a.$$

The boundary condition  $h(a) = H$  gives the parameter  $c_2$ ,

$$c_2 = H_0 \exp(H/H_0) + J_0 a.$$

*Case V:  $0 \leq H \leq -H_p$  in decreasing-field branch*

For  $r \geq r_3$ , the flux distribution is described by the same manner as in the case I. However, for  $r \leq r_3$ , the distribution is governed by the residual vortices. The solution is given by

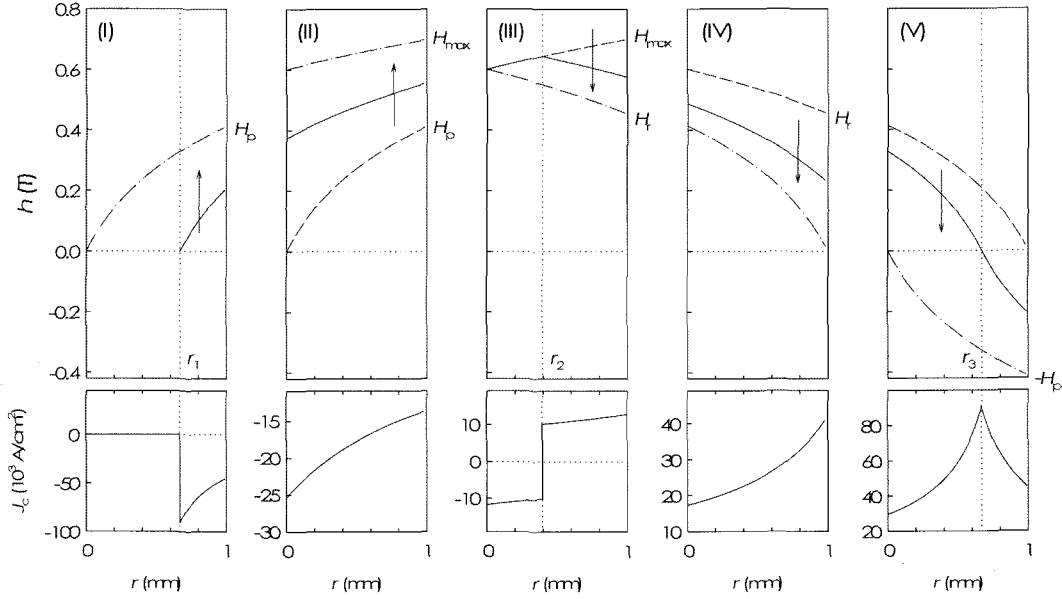
$$h(r) = \begin{cases} H_0 \ln[(-J_0 r + c_3)/H_0] & \text{for } 0 \leq r \leq r_3 \\ -H_0 \ln[(J_0 r - c_4)/H_0] & \text{for } r_3 \leq r \leq a \end{cases}.$$

From the boundary condition  $h(a) = H$  and the continuity condition at  $r = r_3$ , the parameters  $c_3$ ,  $c_4$ , and  $r_3$  are given by

$$\begin{aligned} c_3 &= H_0 \exp(H/H_0) - J_0 r_3 \\ c_4 &= -H_0 \exp(-H/H_0) + J_0 a \\ r_3 &= (H_0 + c_4)/J_0. \end{aligned}$$

*Other Cases*

There are additional cases for a complete hysteresis



**Fig. 3.** Distributions of vortices,  $h(r)$ , and supercurrent density,  $J_c(r)$ , inside a cylindrical superconductor in the frame of the exponential critical-state model. Parameters used for the visualization are  $J_0 = 7200 \text{ A/m}^2$ ,  $H_0 = 0.3 \text{ T}$ , and  $H_{\text{max}} = 0.7 \text{ T}$ (see text).

curve. However, the  $M(H)$  for those cases can be described by the solutions obtained above. In fact, the solutions for  $h(r)$  at the field branches of  $-H_p \rightarrow -H_{\text{max}}$ ,  $-H_{\text{max}} \rightarrow -H_r$ ,  $-H_r \rightarrow 0$ , and  $0 \rightarrow H_p$  are the same as in cases II, III, IV, and V, respectively, if the sign of external magnetic field changes. (We name the additional branches as cases II\*, III\*, IV\*, and V\*.)

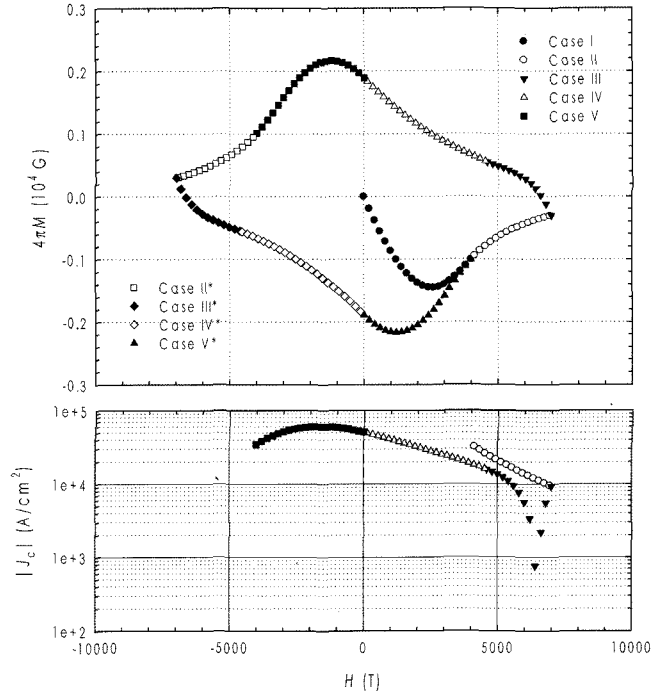
In cylindrical coordinates, we obtain an average magnetic induction  $\langle h(r) \rangle$  of a sample with radius  $a$

$$\langle h \rangle = B = H + 4\pi M = \frac{1}{\pi a^2} \int_0^{1/2} \int_0^{2\pi} h(r) d\theta dr.$$

As shown in Fig. 3, we visualize the field (flux) distribution inside a cylindrical superconductor with  $a = 1 \text{ mm}$  and the corresponding profile of the critical-current density in the case of  $J_0 = 7200 \text{ A/m}^2$ ,  $H_0 = 0.3 \text{ T}$ , and  $H_{\text{max}} = 0.7 \text{ T}$ . The resulting  $M(H)$  curve is represented in Fig. 4.

The dashed lines in Fig. 2 show our attempt to fit  $M(H)$  with the exponential critical-state model. The fitting parameters,  $J_0 (H_0)$  are  $31800 \text{ A/cm}^2 (0.97 \text{ T})$ ,  $31700 \text{ A/cm}^2 (0.88 \text{ T})$ ,  $30800 \text{ A/cm}^2 (0.75 \text{ T})$ ,  $29500 \text{ A/cm}^2 (0.69 \text{ T})$ ,  $20400 \text{ A/cm}^2 (0.46 \text{ T})$ ,  $14200 \text{ A/cm}^2 (0.30 \text{ T})$ , and  $7400 \text{ A/cm}^2 (0.10 \text{ T})$  at 5 K, 10 K, 15 K, 20 K, 25 K, 30 K, and 35 K, respectively.

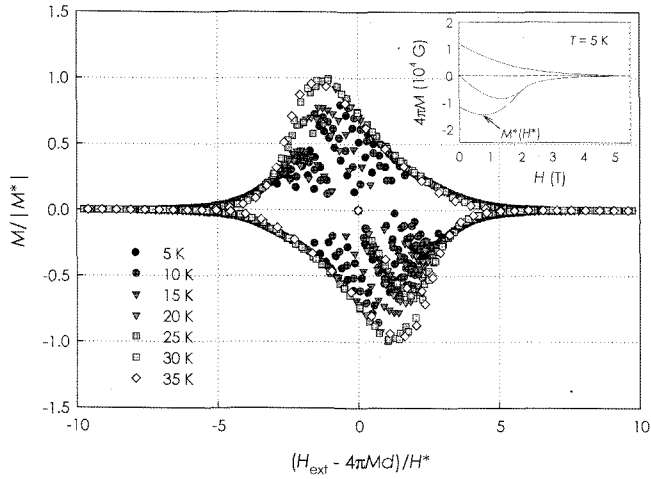
The comparisons are carried out in the region where the flux jumps do not occurs. As one can see, the data are well described by the critical-state model even without considering the contribution of the reversible magneti-



**Fig. 4.** Theoretical curve of magnetization  $4\pi M(H)$  (top panel) and absolute value of average critical-current density  $|J_c(H)|$  for  $|H| \geq H_p$  (bottom panel) deduced from  $h(r)$  in Fig. 3.

zation and the surface barrier effect. This implies that, in the mixed state in the  $\text{MgB}_2$  superconductor, the magnetization mainly comes from the contribution of the bulk pinning effect.

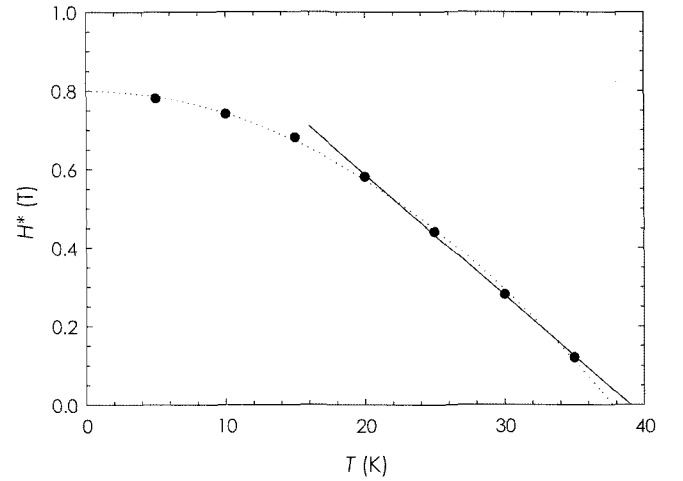
We note that the shapes of the  $M(H)$  curves shown in



**Fig. 5.** Scaling of the magnetization curves,  $M(T)/M^*(T)$  vs.  $H/H^*$ , in the temperature region  $5 \text{ K} \leq T \leq 35 \text{ K}$ . The inset illustrates the definitions of  $M^*$  and  $H^*$  (see text).

Fig. 2 are remarkably similar to each other, if the flux jumping regions are excluded. This suggests that the vortex pinning mechanism in  $\text{MgB}_2$  does not change even as the temperature is varied. More concrete evidence for this can be found from the scaling analysis of the  $M(H)$  curves. For the scaling of the experimental  $M(H)$  curves, we get two phenomenological parameters from the theoretical  $M(H)$  curves as shown in the inset of Fig. 5.  $H^*(T)$  means the field where the magnetization in the increasing field branch reaches its maximum value  $M^*(T)$ . We divided the  $M$  and the  $H$  in each curve of Fig. 2 by  $|M^*(T)|$  and  $H^*(T)$ , respectively. The result is shown in Fig. 5. Except for the flux jumping regions, all the curves collapse on a single universal curve. In the case of high- $T_c$  cuprates, such scaling behavior of the  $M(H)$  curves is established in a limited low-temperature region, because the pinning mechanism determining the magnetic hysteresis changes as the temperature increases toward  $T_c$ . For  $\text{Bi}_2\text{Sr}_2\text{Ca}_1\text{Cu}_2\text{O}_8$  (Bi-2212), while the bulk pinning is dominant at low temperatures, the contribution of the surface or geometrical barrier effects to the magnetization becomes more important as the temperature increases [3]. Thus, universal scaling of  $M(H)$  is not seen in Bi-2212.

Figure 6 shows the temperature dependence of the scaling parameters  $H^*(T)$  used in the above analysis. Since the  $M(H)$  curves at various temperatures were successfully scaled with  $H^*$ , one can expect that  $H^*(T)$  is proportional to the irreversibility line  $H_{\text{irr}}(T)$  where the magnetic hysteresis disappears. For  $T \geq 20 \text{ K}$ ,  $H^*(T)$  is linear with respect to temperature, but the tendency deviates from the linearly as the temperature decreases. On the whole, the  $H^*(T)$  shows a quadratic behavior in temperature with the downward curvature as represented



**Fig. 6.** Temperature dependence of phenomenological parameter  $H^*$ . Solid line represents linear least-squares fit of the data at  $T \geq 20 \text{ K}$ , and dotted line shows  $\sim 1 - (T/T_c)^2$  curve fitted to the data in  $5 \text{ K} \leq T \leq 35 \text{ K}$ .

by the dotted line in Fig. 6. This feature differentiates  $\text{MgB}_2$  from the cuprates materials with  $H_{\text{irr}} \sim (T_c - T)^n$  ( $n \geq 1.5$ ) with the upward curvature.

#### 4. Summary

In summary, we measured the magnetization curves  $M(H)$  for metallic  $\text{MgB}_2$  superconductor, which has a  $T_c \approx 39 \text{ K}$ , in the region  $5 \text{ K} \leq T \leq 35 \text{ K}$  and  $-5 \text{ T} \leq H \leq 5 \text{ T}$ . The magnetic hysteresis in our experimental region was well described by the exponential critical-state model, without considering the reversible magnetization and the surface barrier effect. Also, we found that all the magnetization curves collapsed onto a single universal curve when the field and the magnetization were normalized by the characteristic scaling factors  $H^*(T)$  and  $M^*(T)$ , respectively. These results lead us to the conclusion that the irreversible magnetization of  $\text{MgB}_2$  is dominated by bulk pinning and that the pinning mechanism does not change even when the temperature is varied up to  $T/T_c \approx 0.9$ .

#### Acknowledgements

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