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Misfit Strain Induced Reflection of Light from Magnetic-Nonmagnetic Interfaces

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We have theoretically investigated changes in reflection coefficients induced by misfit strain located near the interface between an iron-yttrium garnet magnetic film and a nonmagnetic gadolinium-gallium garnet substrate in a transverse magneto-optical configuration.

1. Introduction

It is well known that elastic strains are generally induced near the interface between crystalline media (so-called pseudomorphic strain), caused by the lattice misfit of the different materials in layered structures [1]. Thicknesses of the resulting deformed layers, which are determined by the elastic parameters of the materials, can reach values up to a few hundred angstroms. Such deformed layers can lead to optical effects at interfaces, for example, changes in the spectra of photoluminescence in semiconductor heterostructures [2]. Deformations near the interface can also change the reflection properties via the photoelastic interaction [3], [4]. The photoelastic contribution to the reflection of light has been investigated both theoretically and experimentally (by ellipsometric methods) for nonmagnetic metals [5]. Influence of the surface-strain effects via photoelastic refractive-index changes on photorefractive gratings has also been investigated [6]. The reflectance of an inhomogeneous layer with arbitrary refractive-index profile near an interface has been calculated by numerical solution of Maxwells equations [7]. It should be expected that similar effects of the photoelastic changes in the reflection of light can arise in the case of magnetic films on nonmagnetic substrates.

The aim of the present work is to investigate the contribution of the strained layers near interfaces between magnetic and nonmagnetic media to the optical reflection coefficients.

2. General Relationships

Let us consider a bilayer structure formed by a magnetic film of yttrium-iron garnet (YIG) $Y_3Fe_5O_{12}$ with thickness d_1 on a nonmagnetic substrate of gadolinium-gallium garnet (GGG) $Gd_3Ga_5O_{12}$ with thickness d_2. The interface between YIG and GGG ($z=d_1$) is parallel to the XY plane and the reflection plane is XZ. For simplicity, we shall investigate the transverse magneto-optic configuration, with the magnetization vector **M** oriented along the x-axis. For this geometry, the electromagnetic waves in a magnetic medium can be presented as pure TM- and TE- (or s- and p-) modes. In the following we shall investigate a thin YIG film on a thick GGG substrate. Usually, $d_1 << d_2$, for example, $d_1 \approx 1$ μm and $d_2 \approx 10$ μm. The critical thickness h_{cf} for the pseudomorphic structure of YIG film on GGG substrate can be estimated using the equation [1]:

$$h_{cf} \approx \frac{b(1-\nu\cos^2\beta)}{8\pi(1+\nu)f_m}\ln\frac{2h_c}{b} \tag{1}$$

where ν is Poisson's ratio, b is the magnitude of the Burgers vector, β is the angle between a dislocation line and the Burgers vector. Pseudomorphic strains induced by the misfit $f_m = (a_f - a_s)a_s^{-1}$ (a_f and a_s are lattice parameters of the magnetic film and the substrate, respectively) are present in films with thickness h_{cf} above the interface [1]. It should be noted that for a thick substrate, the strained layer will be very thin [1]. On the other hand, for a thin substrate, the strained layer can be comparable with h_{cf}. Using typical values for a YIG film on a GGG substrate in Eq. (1), the critical thickness h_{cf} is estimated to be about 0.1 μm. The dielectric permeability tensor $\varepsilon_{ij}(z)$ of such system has the form

$$\varepsilon_{ij}(z) = \begin{cases} \delta_{ij}, & z<0 \\ \delta_{ij}^{(1)}, & 0<z<d_1 \\ \delta_{ij}^{(2)}, & d_1<z<d_1+d_2 \\ \delta_{ij}, & z<d_2 \end{cases} \tag{2}$$

The tensors $\varepsilon_{ij}^{(1,2)}(z)$ have the following structure:

$$\varepsilon_{ij}^{(1,2)}(z) = \varepsilon_{(0)ij}^{(1,2)} + \Delta\varepsilon_{ij}^{(1,2)}(z) \tag{3}$$

where $\varepsilon_{(0)ij}^{(1,2)}$ are the dielectric permeabilities of the non-deformed media. We have:

$$\varepsilon_{(0)ij}^{(1,2)} = \begin{pmatrix} \varepsilon_{(0)}^{(1)} & 0 & 0 \\ 0 & \varepsilon_{(0)}^{(1)} & i\varepsilon' \\ 0 & -i\varepsilon' & \varepsilon_{(0)}^{(1)} \end{pmatrix}, \quad \varepsilon_{(0)ij}^{(2)} = \begin{pmatrix} \varepsilon_{(0)}^{(2)} & 0 & 0 \\ 0 & \varepsilon_{(0)}^{(2)} & i\varepsilon' \\ 0 & -i\varepsilon' & \varepsilon_{(0)}^{(2)} \end{pmatrix} \tag{4}$$

In Eq. (4), the nondiagonal components of the dielectric permeability are linear in the magnetization: $\varepsilon' = fM_x$, where f is the linear magneto-optical constant. The additional terms $\Delta\varepsilon_{ij}^{(1,2)}(z)$ in Eq. (3) are induced by strain and can be written in the well-known form [3, 4]:

$$\Delta\varepsilon_{ij}^{(1,2)} = p_{ijkl}^{(1,2)} u_{kl}^{(1,2)} \tag{5}$$

In Eq. (5), $p_{ijkl}^{(1,2)}$ and $u_{kl}^{(1,2)}$ are the photoelastic and strain tensors in the two media. Following the approach proposed in [8], we present elastic strains in the YIG-film on the GGG-substrate in the form.

$$u^{(1)}(z) = f_m\left[\theta(d_1 - z) - \theta(d_1 - h_{cf} - z) + \frac{h_{cf}}{z - d_1}\theta(d_1 - h_{cf} - z)\right] \tag{6}$$

$$u^{(2)}(z) = f_m \frac{d_1}{d_2}[\theta(d_2 - z) - \theta(d_2 - h_{cs} - z)] \tag{7}$$

where $\theta(x)$ is the Heaviside step function. Taking into account biaxial stresses and the form of $p_{ijkl}^{(1,2)}$ for a system with cubic symmetry, from Eq. (4) we find that tensor $\Delta\varepsilon_{ij}^{(1,2)}(z)$ is characterized by the following nonzero components

$$\Delta\varepsilon_{xx}^{(1,2)} = \Delta\varepsilon_{yy}^{(1,2)} = \frac{u^{(1,2)}(z)}{1 - v^{(1,2)}}[(p_1^{(1,2)} + 3p_2^{(1,2)}) - v^{(1,2)}(p_1^{(1,2)} + p_2^{(1,2)})] \tag{8}$$

$$\Delta\varepsilon_{zz}^{(1,2)} = \frac{2u^{(1,2)}(z)}{1 - v^{(1,2)}}[p_2^{(1,2)} - v^{(1,2)}(p_1^{(1,2)} + p_2^{(1,2)})] \tag{9}$$

In Eqs. (8) and (9) $p_1^{(1,2)}$ and $p_2^{(1,2)}$ are nonzero components of photoelastic tensors $v^{(1,2)}$ and are Poissons ratios for the two media.

3. Reflection Matrix

In the general case, the electric field of reflected light $\mathbf{E}^R$ can be presented via the reflection matrix $\widehat{R}$ [3].

$$E_\alpha^{(R)} = R_{\alpha\beta} E_\beta^I \tag{10}$$

where E_β^I is the electric field of the incident light and

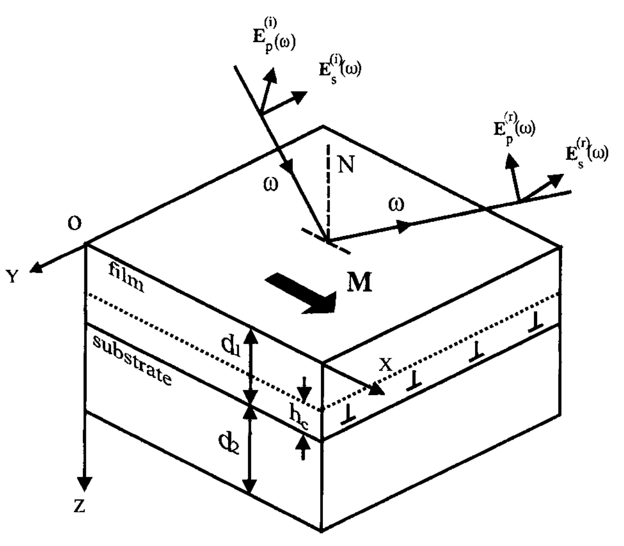

Fig. 1. The schematic image of the reflection of light from the film-substrate interface. The big arrow shows the direction of the magnetization in the magnetic film. N is the normal to the film surface.

β =s or p. The electric field of the reflected electromagnetic wave (EMW) and the reflection matrix are determined by the expressions.

$$E_\alpha^R = E_{\alpha 0}^R(z) + E_{\alpha S}^R(z) \tag{11}$$

$$R_{\alpha\beta} = R_{\alpha\beta}^{(0)} + \Delta R_{\alpha\beta} \tag{12}$$

In Eq. (11), $E_{\alpha 0}{}^R$ is the contribution to the total electric field of the EMW reflected from a non-strained interface.

$$\begin{pmatrix} E_{s0}^R \\ E_{p0}^R \end{pmatrix} = \begin{pmatrix} R_{ss}^{(0)} & 0 \\ 0 & R_{ss}^{(0)} \end{pmatrix} \begin{pmatrix} E_s^I \\ E_p^I \end{pmatrix} \tag{13}$$

In Eq.(12), $\widehat{R}^{(0)}$ is the usual reflection matrix for a sharp interface [3] and $\Delta\widehat{R}$ is the change of reflection matrix induced by the strained layer. The strain-induced part of the electric field of the reflected EMW E_{iS}^R can be determined from the expression.

$$E_{iS}^R(z) = \frac{k_0^2}{2\pi}\left(\int_0^{d_1} G_{ik}^{(1)}(z - z')\Delta\varepsilon_{kl}^{(1)}(z')dz'\right) \left(\int_{d_1}^{d_1+d_2} G_{ik}^{(2)}(z - z')\Delta\varepsilon_{kl}^{(2)}(z')E_l^{(2)}(z')dz'\right) \tag{14}$$

The Green's functions $G_{ik}^{(1,2)}(z - z')$ are solutions of operator equations for the two media.

$$L_{ik}^{(1,2)}(\partial_z) G_{kl}^{(1,2)}(z - z') = \delta_{il}\delta(z - z') \tag{15}$$

where $L_{ik}^{(1,2)}(\partial_z)$ are the operators of the wave equation for the total electric field in the magnetic film and the nonmagnetic substrate, respectively.

$$L_{ik}^{(1)}(\partial_z) = \begin{pmatrix} k_0^2\varepsilon_0^{(1)} - \partial_z^2 & 0 & i(k_x\partial_z - k_0^2\varepsilon') \\ 0 & k_x^2 - \partial_z^2 - k_0^2\varepsilon_0^{(1)} & 0 \\ i(k_x\partial_z + k_0^2\varepsilon') & 0 & k_x^2 - k_0^2\varepsilon_0^{(1)} \end{pmatrix} \quad (16)$$

$$L_{ik}^{(1)}(\partial_z) = \begin{pmatrix} k_0^2\varepsilon_0^{(1)} - \partial_z^2 & 0 & ik_x\partial_z \\ 0 & k_x^2 - \partial_z^2 - k_0^2\varepsilon_0^{(1)} & 0 \\ ik_x\partial_z & 0 & k_x^2 - k_0^2\varepsilon_0^{(1)} \end{pmatrix} \quad (17)$$

where k_0 is the wave number of light in vacuum and k_x is the x-component of the wave vector of light in a medium.

The solutions of Eqs. (15) are determined by the following expressions for the Greens function components.

i) TM-mode

$$G_{xk}^{(1)}(z-z') = \frac{k_{0z}\exp(-k_{0z}z)}{Dk_{TE}^2}$$
$$\{\lambda_{xk}^{(1)+}\exp[ik_{TM}(d_1-z)]K_1^+ - \lambda_{xk}^{(1)-}\exp[-ik_{TM}(d_1-z)]K_1^-\} \quad (18)$$

$$G_{xk}^{(2)}(z-z') = -\frac{k_{0z}\exp(-k_{0z}z)}{Dk_{TE}^2}$$
$$\{\lambda_{xk}^{(2)+}\exp[ik_{2z}(d_1-z)]K_2^+ - \lambda_{xk}^{(1)-}\exp[-ik_{2z}(d_1-z)]K_2^-\} \quad (19)$$

$$G_{zk}^{(1,2)}(z-z') = \frac{k_x}{xk}G_{xk}^{(1,2)}(z-z') \quad (20)$$

where the index $k=x$ or z, k_{TM} is the wave number of the TM-mode, and the values D, $\lambda_{xk}^{(1,2)\pm}$ and $K_{1,2}^{\pm}$ are complex functions of dielectric permeability tensor components, wave vectors of normal EMW's, and the angle of incidence [9].

ii) TE-mode:

$$G_{yy}^{(1)}(z-z') = -\frac{\lambda_{yy}^{(1)}\exp(-k_{0z}z)}{D_0}$$
$$\{k_{TE}K_3\cos[k_{TE}(d_1-z)] - ik_{2z}K_4\sin[k_{TE}(d_1-z)]K_1^-\} \quad (21)$$

$$G_{yy}^{(1)}(z-z') = -\frac{k_{2z}\lambda_{yy}^{(2)}\exp(-k_{0z}z)}{D_0}$$
$$\{K_3\cos[k_{TE}(d_1-z)] - iK_4\sin[k_{TE}(d_1-z)]K_1^-\} \quad (22)$$

where k_{TE} is the wave number of the TE-mode, values D_0, $\lambda_{yy}^{(1,2)\pm}$ and $K_{3,4}$ are complex functions of dielectric permeability tensor components, wave vectors of normal EMW's, and the angle of incidence [9].

Substituting the expressions for the Green's functions Eqs. (18)-(22) into Eq. (14) we obtain the strain-induced contribution to the electric field of the reflected light. Then with Eq. (13) we find changes of the reflection matrix for both *s*- and *p*-polarizations of the reflected light. Numerical calculations the dependence of $\Delta\widehat{R}$ on the incidence angle for visible light of wavelength λ=0.63 μm show that the strain-induced contribution to the reflected light reaches a maximal value when the components of the usual reflection matrix approach zero. A comprehensive analysis of these dependencies will be published elsewhere [9].

4. Conclusion

In this article we showed by a simple phenomenological model that strained layers near the film-substrate interface contribute to the reflection of light. For a more adequate description of the light reflection from a real interface, it is necessary to take into account surface polarization, which reflects the symmetry of a surface or an interface. This fact leads to changes in the form of the photoelastic tensors and to the appearance of nondiagonal components in the reflection matrix. In a similar way, the reflection of light was studied by Zil'bershtein *et al.* [10], but without taking into account pseudomorphic strains which take place in real multilayered structures. It should be noted that quite recently, a surface-induced transverse magneto-optical Kerr effect was predicted theoretically [11, 12]. The origin of this phenomenon is the lowering of the symmetry of the magnetized surface, which allows observation of the surface-induced linear magneto-optical Kerr rotation in reflected light.

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