Effect of Distance on Optimum Transfer Efficiency for the Four-coil Magnetic Coupled Resonance System

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As known, the variation of distances between the adjacent coils will affect the transfer efficiency in magnetic coupled resonance system. In order to explore the influence of distance on the optimum transfer efficiency, a four-coil magnetic coupled resonance system is established. Based on the mutual inductance coupling theory, the relationship between the transfer efficiency and coupling coefficient is obtained. Supported by the simulation analysis and experimental evidences, it is found that the coupling coefficient \(k_{23}\) (between transmitter and receiver coils) is the most significant factor affecting the transfer efficiency. Under the optimal \(k_{23}\), the transfer efficiency will increase with the coupling coefficient \(k_{12}\) (between driver and transmitter coils). Besides, it will firstly increase and then decrease with the coupling coefficient \(k_{34}\) (between receiver and load coils). Moreover, the theoretical calculation formula of \(k_{23}\) is deduced, which can make a contribution to the optimization of parameters and improvement of transfer efficiency for the four-coil magnetic coupled resonance system.

Keywords: wireless power transfer, planar spiral coil, magnetic coupled resonance, coupling coefficient

1. Introduction

With the rapid development of various low power mobile appliances such as cell-phones, laptops and handheld equipment, there is substantial interests in the wireless power charging. In view of the drawbacks of traditional induction charging, the wireless power transmission (WPT) via magnetic coupled resonance displays its superiority due to its higher efficiency, longer range and greater power output [1, 2].

As a new WPT technology, the magnetic coupled resonant is based on the near-magnetic field coupling concept, which is the principle that two or more objects with the same resonant frequency can exchange energy efficiently and the coupled energy between non-resonant objects is very weak [3, 4]. The magnetic coupled resonance between coils is not only related to the position and direction [5-9], but also relates to the relay coil and the structures [10-15]. For instance, it was demonstrated that a high efficiency wireless power transfer system can be achieved by adjusting the coupling factors between the source and the internal resonator [6]. It was also verified that the change of addition of the relay coil between transmitter and receiver coils can affect the transfer distance and efficiency [13]. Besides, the size and structure of the resonant coils has an impact on the coupling coefficient, by comparing the three different structures (planar spiral, spherical and solenoid) coils, it is pointed out that the coil structure determines the coupling frequency and the distribution of magnetic field, affecting the effective coupling between coils [14].

Although many studies have been carried out in the relevant aspects mentioned above, but there are still few researches on the relationships between the optimum transfer efficiency and distances for the four-coil magnetic coupled resonance system. Thus, this article will be interesting and meaningful.

In this paper, the magnetic resonance coupled WPT system is established based on four single-layer planar spiral coils. Through the equivalent circuit, the relationship between the transfer efficiency and coupling coefficient between the adjacent coils is analyzed. Moreover, the expressions of transfer efficiency and optimum coupling coefficient have been derived. The simulation analysis and experiments are carried out, which indicate that the coupling coefficient \(k_{23}\) (between transmitter and receiver coils) is the most significant factor affecting the transfer efficiency. Under the optimal \(k_{23}\), the transfer efficiency will increase with the coupling coefficient \(k_{12}\) (between driver and transmitter coils). Besides, it will firstly increase and then decrease with the coupling coefficient \(k_{34}\) (between receiver and load coils). Moreover, the theoretical calculation formula of \(k_{23}\) is deduced, which can make a contribution to the optimization of parameters and improvement of transfer efficiency for the four-coil magnetic coupled resonance system.
calculated as:

The current in each coil can be regarded as the load. According to the classical circuit theory techniques, the current in each coil can be calculated as:

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4
\end{bmatrix} = \begin{bmatrix}
Z_{11} & Z_{12} & 0 & 0 \\
Z_{21} & Z_{22} & Z_{23} & 0 \\
0 & Z_{32} & Z_{33} & 0 \\
0 & 0 & Z_{43} & Z_{44}
\end{bmatrix}^{-1} \begin{bmatrix}
U_S \\
U_S \\
0 \\
0
\end{bmatrix}
\]  

(1)

In equation (2), \( R_i \) stands for the equivalent resistance of the resonator, \( L \) and \( C \) are the equivalent inductance and external tuning capacitor of the coil, respectively. \( M_j \) and \( k_{ij} \) represent the mutual inductance and coupling coefficient between resonant coils \((i \text{ and } j)\), respectively. The mutual inductance \( M \) between two coils can be expressed by a function of the transfer distance between coils [16], shown as:

\[
M = \frac{\pi \mu_0 (N_1 N_2)^{1/2} (r_1 r_2)^{3/2}}{2 D^{3/2}}
\]

Where \( \mu_0 \) is the vacuum permeability, \( N_1, N_2 \) is the number of turns of the coils, \( r_1, r_2 \) is the average radius and \( D \) is the axial distance between coils.

2.2. Analysis of coupling coefficient

2.2.1. Effect of coupling coefficient \( k_{12} \) on the transfer performance

The partial derivative of the \( k_{12}^2 \) for the equation (5) can be obtained as:

\[
k_{12}^{2\text{eff}} = \frac{k_{23}^2 k_{34}^2 Q_2 Q_3^2 Q_4 R_4}{[(1 + k_{12}^2 Q_1 Q_2)(1 + k_{34}^2 Q_3 Q_4) + k_{23}^2 Q_2 Q_3]^2}  > 0
\]

(6)

It is shown that the transfer efficiency is a monotonic function about \( k_{12} \), which means the transfer efficiency increases with the coupling coefficient \( k_{12} \).

Fig. 1. (a) The four-coil WPT system (b) The equivalent circuit.
Through the equation $\frac{\partial \eta}{\partial k_{23}} = 0$, the optimum transfer coefficient $k_{23}$ can be obtained as:

$$k_{23}^{\text{opt}} = \frac{(1 + k_{23} Q_1 Q_3)\sqrt{1 + k_{12} Q_1 Q_2}}{Q_2 Q_3}$$  \hspace{1cm} (7)

From equation (7) and (5), the maximum transfer efficiency can be obtained as:

$$\eta_{\text{opt},23} = \frac{k_{23}^2 Q_1 Q_2 Q_3}{(1 + \sqrt{k_{12} Q_1 Q_2})^2 (1 + k_{23}^2 Q_3^2 Q_4)}$$  \hspace{1cm} (8)

2.2.3. Effect of coupling coefficient $k_{34}$ on the transfer performance

Similarly, the optimum transfer coefficient $k_{34}$ can be obtained as:

$$k_{34}^{\text{opt}} = \sqrt{\frac{1 + k_{12}^2 Q_1 Q_2}{Q_3^2}} \left( \frac{1 + k_{12}^2 Q_2 Q_3 + k_{23}^2 Q_4^2 Q_3}{Q_4^2 + k_{23}^2 Q_1^2 Q_4^2} \right)$$  \hspace{1cm} (9)

From equation (9) and (5), the maximum transfer efficiency can be obtained as:

$$\eta_{\text{opt},34} = \frac{k_{12}^2 k_{23}^2 Q_1 Q_2 Q_3}{\sqrt{(1 + k_{12}^2 Q_1 Q_2)(1 + k_{23}^2 Q_4^2 Q_3) + k_{23}^2 Q_1^2 Q_4^2}}$$  \hspace{1cm} (10)

3. Simulation Analysis

The electromagnetic simulation modeling and analysis could be executed based on the finite element method [17-19]. All the geometry parameters of the coils are shown in Table 1. Select AC/DC module, magnetic field and frequency domain, the coil model is established according to that adopted in the experiment. The resonant frequency is set as $f = 4.60$ MHz, set up the boundary conditions such as the external circuit and excitation source.

The two-dimensional screen shot of the simulation model and the generation of magnetic flux density is shown in Fig. 2, where $d_{12} = 5$ mm, $d_{23} = 20$ mm, $d_{34} = 5$ mm.

### Table 1. Dimensions parameter of coils.

<table>
<thead>
<tr>
<th></th>
<th>driver Coil</th>
<th>Transmitter Coil</th>
<th>Receive Coil</th>
<th>Load Coil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner Diameter $d_1$/mm</td>
<td>8</td>
<td>3</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Outer Diameter $d_2$/mm</td>
<td>13.35</td>
<td>35.05</td>
<td>35.05</td>
<td>13.35</td>
</tr>
<tr>
<td>Wire Diameter $a$/mm</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>Layers</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Turns</td>
<td>10</td>
<td>62</td>
<td>62</td>
<td>10</td>
</tr>
<tr>
<td>Material</td>
<td>Copper</td>
<td>Copper</td>
<td>Copper</td>
<td>Copper</td>
</tr>
</tbody>
</table>

Fig. 2. (Color online) The distribution of the magnetic flux density.

Fig. 3. (Color online) The relationship between $\eta$, $d_{12}$ and $d_{23}$.
under the optimum $d_{23}$, the transfer efficiency decreases with the $d_{12}$.

### 3.2. Relationship between transfer efficiency, $k_{23}$ and $k_{34}$

When $d_{12}$ is fixed, for example 5 mm, by the parametric sweep of $d_{23}$ and $d_{34}$, the relationship between transfer efficiency, $d_{23}$ and $d_{34}$ can be described as Fig. 4.

It is found that the distance $d_{23}$ corresponding to the maximum transfer efficiency is also 20 mm under different $d_{34}$. And the transfer efficiency increases firstly and then decreases with the $d_{23}$ under the optimum $d_{23}$, the largest transfer efficiency is 67.60% when $d_{34}$ is 8 mm. From equation (2) and (3), it can be calculated that the optimum coupling coefficient $k_{23}^{opt}$ is 0.0269.

A common feature can be found from Fig. 3 and Fig. 4, the transfer efficiency is not varied obviously with the increase of $d_{12}$ and $d_{34}$ when $d_{23} \neq 20$ mm, but all those efficiencies are significantly lower than that when $d_{23} = 20$ mm. It can be concluded that the coupling coefficients $k_{12}$ and $k_{34}$ have a strong dependence on $k_{23}$.

### 4. Experimental Verification

In order to verify the effect of distance on optimum transfer efficiency, a magnetic coupled resonance WPT system is constructed as Fig. 5.

The high frequency supply for the system is composed by the signal generator and power amplifier. The impedance analyzer (LCR-8110G) is used to measure the equivalent circuit parameters, which are shown in Table 2. The voltage $U_L$ of the load is detected by oscilloscope and then the load power $P_L = \frac{U_L^2}{R}$ can be obtained. One resistance (1 Ω) is used in series with the driver coil to sense the current, and the input voltage value $U_S$ is measured by oscilloscope. So the input power can be obtained.

In order to verify the influence of $k_{23}$ on the transfer efficiency, the distances $d_{12}$ and $d_{34}$ are both set as 5 mm in experiment, the transfer efficiency with the change of $d_{23}$.

<table>
<thead>
<tr>
<th>Table 2. Components of the system parameters.</th>
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<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Circuit</td>
</tr>
<tr>
<td>$L_i$/nH</td>
</tr>
<tr>
<td>$C_i$/nF</td>
</tr>
<tr>
<td>$R_i$/Ω</td>
</tr>
<tr>
<td>$Q_i$</td>
</tr>
<tr>
<td>$f$/MHz</td>
</tr>
</tbody>
</table>

![Fig. 5. (Color online) The experimental setup.](image)

$d_{23}$ is measured as shown in Fig. 6.

The experimental results show that the transfer efficiency increases first and then decreases with $d_{23}$, the largest transfer efficiency $\eta = 50.72\%$ appears when $d_{23} = 15$ mm. The variation law of experimental results is basically consistent with the simulation analysis.

In order to verify the influence of $k_{12}$ and $k_{34}$ on the transfer efficiency, other five groups of experiment are carried out. The distance $d_{23}$ is set as 5, 10, 15, 20 and 25 mm, the relationships between transfer efficiency, $d_{12}$ and $d_{34}$ are shown in Fig. 7.

It can be found that under different $d_{12}$ and $d_{34}$, the maximum transfer efficiency all appears when $d_{23} = 15$ mm. Besides, the transfer efficiency decreases with $d_{12}$, increases first and then decreases with $d_{34}$. All these features are consistent with the simulation analysis, which verifies each other.

5. Conclusions

In this paper, the relationship between the transfer efficiency and coupling coefficient is analyzed. Supported by the simulation analysis and experimental evidences, it is found that the coupling coefficient $k_{23}$ (between the transmitter coil and receiver coil) is the decisive factor influencing the transfer efficiency. Under the optimum coupling coefficient $k_{23opt}$, the transfer efficiency increases with $k_{12}$, and increases firstly and then decreases with $k_{34}$.

Therefore, for the practical applications, some suggestions can be proposed by this research as follows:

Firstly, adjust the distance $d_{23}$ (between transmitter and receiver coil) to achieve the best coupling state to ensure that the transfer efficiency is a local optimum value.

Then, adjust the distance $d_{12}$ (between driver and transmitter coil) as close as possible on the premise of considering the system structure.

Finally, adjust the distance $d_{34}$ (between receiver and load coil) to an appropriate position, so as to achieve its optimum transfer efficiency.

6. Discussion

There are two very significant parameters in the magnetic coupled resonance WPT system, which are the transfer efficiency and transfer power. In this research, the transfer efficiency determines the amplitude of transfer power on account of the fixed input power. Thus, only one parameter transfer efficiency is analyzed and discussed in this paper.

In theoretical predictions, the optimum transfer efficiency is obtained at $d_{23} = 20$ mm, but in the case of experimental results it is obtained at $d_{23} = 15$ mm. The main reasons could be summarized as follows:

The actual experimental coils are tightly wound coils, not same as the ideal plane geometry in simulation. The difference of geometrical structure will affect the transmission characteristics of system.

The input voltage in simulation is ideal and stable, while the actual input voltage is probably not stable due to the loss of front circuit and interference from outside.

Although the values are different, the related conclusion and trends in theoretical predictions are still consistent with those in experiments, which means it can provide the guidelines for other systems.

Acknowledgements

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