Journal of Magnetics 28(1), 84-90 (2023)

ISSN (Print) 1226-1750
ISSN (Online) 2233-6656
https://doi.org/10.4283/JMAG.2023.28.1.084

Entropy Effect on Tri-Polar and Quadrupolar Magnetized Vortex in Electron-Positron and Ion (EPI) Plasma

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(Received 10 November 2021, Received in final form 25 December 2022, Accepted 2 January 2023)

The entropy of electron-positron and ion plasma plays a significant role in the transportation of particles, momentum, and energy. Drift waves in these plasmas produce central mechanisms for this transport. The effects on tri-polar and quadrupolar vortexes in epi plasma have been analyzed numerically under various approximations. As plasma consists of various elements, it is important to understand the effects of entropy on systems. In this article, we have discussed how entropy affects linear and non-linear equations in the presence of temperature, equilibrium density, electrostatic potential gradients, and a magnetic field. Additionally, by investigating tri-polar and quadrupolar vortexes without regard for entropy effect we can find information about this phenomenon.

Keywords : tri-polar vortex, drift waves, entropy, electron positron and ion plasma

1. Introduction

The entropy gradient affects the movement of energy, momentum, and mass in plasmas. It is also known that it plays a significant role in the transport of momentum, energy, and particles within the magnetic fields. Drift waves created by epi plasma are responsible for transporting these substances most efficiently [1]. The formation of spatial structures related with drift methods is one of the noticeable reasons for Low Heat transition in magnetic confinement system likely Tokamaks [2]. The most efficient means for the unusual transport of ions in magnetically confined devices are Ion Temperature Gradient (ITG) driven modes, Trapped Electron (TE) modes, and Pressure Gradient Ballooning (PGB) modes.

Entropy is a thermodynamic characteristic that defines the organization of systems. Essentially, entropy quantifies how many possible states a system has available to it. In 1989, Zhang *et al*. [3] were the first to observe entropy fluctuations in space. However, this has consequences that go beyond just thermodynamics-like processes like energy transfer or magnetic reconfiguration can also cause entropy changes. This research then led to investigations of how entropy affects different plasma models-something which is still ongoing today. By understanding entropy's effects on these various systems, we are able to improve our understanding of fusion plasmas as a whole [4].

The study of both linear and nonlinear drift waves which are generated by the electrostatic ion-temperature-gradient (ITG) is important for understanding fluctuations within Tokamak plasmas as well as the transport of ion energy [5, 6]. When ion and drift waves move in parallel to an externally applied magnetic field, a coupled linear mode of oscillation will occur. This can be observed when the plasma is heated by a neutral beam. In some experiments, it was found that density fluctuations increase with increasing temperature ($T_i > 4k$) [7, 8].

A number of scientists have looked into the driven mode of temperature gradient in Tokamak plasmas to better understand thermal and anomalous transport [9]. Tokamak plasmas have been studied in regards to anomalous transport. it has been found that small-level disturbances are related to the existence of ion temperature gradient modes [10]. The instability in the toroidal ITG mode is mainly due to impurity effects and magnetic field curvature [11, 12].

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In this article, we have examined and expanded the previous work on electrostatic vortices linked with Ion Temperature Gradient (ITG) driven drift modes in e-p-i plasmas by M. Azeem and Arshad M. Mirza [13]. The ion continuity equation and the potential for the creation of a monopolar vortex in epi plasma were first explored in our numerical model without the inclusion of entropy drift, and in the second half, these factors were calculated in our numerical model with the addition of entropy drift. The references [18, 19] non-diamond isoelectronic [20-22] Lithium-Ion, functional organic Zinc-chelate formation with nano-scaled granular structure and Fluorine-doped amorphous carbon layer [23, 24], non-thermal plasma, achieving giant piezoelectricity and on ferroelectric synapses with excellent conductance linearity is represented in [25-27].

2. Formulation

We examine non-uniform electron-positron-ion plasma (EPI) when equilibrium density, magnetic gradients and ion temperature are present in a uniform magnetic field. Ion pressure can be maintained by a body force $\vec{F}$, i.e., a gravitational force or acceleration of the equilibrium plasma, or an external electric field.

Mathematically, in the comparison of low frequency with ion gyro frequency [14]:

$$\omega_c = \left(\frac{e\vec{B}}{m_i c}\right), \tag{1}$$

In the above expression, e, m_i, c highlights respectively magnitude of electron charge, mass of ion and speed of light.

The ion fluid velocity in mathematical form is

$$\vec{v}_i = \vec{v}_{EB} + \vec{v}_{Di} + \vec{v}_\pi + \vec{v}_{pi} + \vec{v}_i \parallel \hat{b}, \tag{2}$$

where $\hat{b} = \frac{\vec{B}_0}{|B|}$ is a unit vector in the direction of magnetic field [15]. Where $\vec{v}_{EB}$ is $\vec{E} \times \vec{B}$ drift, $\vec{v}_{Di}$ is diamagnetic drift and $\vec{v}_{pi}$ is polarization drift. As $\vec{v}_{EB}$ is $\vec{V} \times \vec{B}$ drift which is defined as:

$$\vec{v}_{EB} = \frac{\vec{B}}{\vec{B}^2} \times \nabla\phi = \frac{c_0}{\vec{B}_0}\hat{b} \times \nabla\,\phi, \tag{3}$$

The diamagnetic drift $(\vec{v}_{Di})$ is addressed as

$$\vec{v}_{Di} = \left(\frac{c}{enB_0}\right)\hat{b} \times \nabla(\mathrm{n_i}T_i), \tag{4}$$

Also, $\vec{v}_{pi}$ is polarization drift found by an equation of motion, then by Newton's second law, we have

$$m\frac{d\vec{v}}{dt} = en\left(\vec{E} + \vec{v} \times \vec{B}\right) - \nabla P, \tag{5}$$

We take $\omega \ll \omega_c$, therefore, Eq. (5), becomes

$$0 = e\,n\left(\vec{E} + \vec{v} \times \vec{B}\right) - \nabla P, \tag{6}$$

Applying cross product of Eq. (6) with $\vec{B}$, we arrive

$$en\left(\vec{E} \times \vec{B} + \left(\vec{v} \times \vec{B}\right) \times \vec{B}\right) - \nabla P \times \vec{B} = 0, \tag{7}$$

Now by triple cross production, we have

$$(A \times B) \times C = B\,(C.A) - A\,(C.B), \tag{8}$$

$$\left(\vec{v} \times \vec{B}\right) \times \vec{B} = \vec{B}\left(\vec{v}_\perp.\vec{B}\right) - \vec{v}_\perp.\vec{B}^2, \tag{9}$$

$$\left(\vec{v} \times \vec{B}\right) \times \vec{B} = -\vec{v}_\perp.\vec{B}^2, \quad \therefore \vec{B}\left(\vec{v}_\perp.\vec{B}\right) = 0, \tag{10}$$

Invoking Eq. (10) in Eq. (7), we get

$$en\left(\vec{E} \times \vec{B} - \vec{v}_\perp.\vec{B}^2\right) - \nabla\,P \times \vec{B} = 0\,, \tag{11}$$

$$\vec{v}_\perp = \frac{\vec{E}\times\vec{B}}{\vec{B}^2} - \frac{\left(\nabla\,P\times\vec{B}\right)}{en\vec{B}^2}, \tag{12}$$

As $\vec{v}_{EB} = \frac{\vec{E}\times\vec{B}}{\vec{B}^2}$ and $\vec{v}_{Di} = -\frac{\nabla P\times\vec{B}}{en\vec{B}^2}$, then Eq. (12) reduce to the following equation

$$\vec{v}_\perp = \vec{v}_{ED} + \vec{v}_{Di}\,, \tag{13}$$

The equation of motion take the form

$$mn\frac{d\vec{v}}{dt} = en\left(\vec{E}_\perp + \vec{v}_\perp \times \vec{B}\right) - \nabla P, \tag{14}$$

Cross product of Eq. (14) with $\vec{B}$, we have

$$mn\frac{d\vec{v}}{dt} \times \vec{B} = \left[en\left(\vec{E}_\perp + \vec{v}_\perp \times \vec{B}\right) - \nabla P\right] \times \vec{B}, \tag{15}$$

$$\vec{v}_\perp = \vec{v}_{EB} + \vec{v}_{Di} + \left(\frac{mc^2}{e\vec{B}^2}\right)\left[\frac{\partial}{\partial t} + (\vec{v}_{EB} + \vec{v}_{Di})\,.\nabla\right]\vec{E}_\perp, \tag{16}$$

$$\vec{v}_{pi} = -\frac{c}{\vec{B}_0\omega_c}\left[\frac{\partial}{\partial t} + (\vec{v}_{EB} + \vec{v}_{Di}).\nabla\right]\nabla_\perp\phi, \tag{17}$$

Invoking, the above expressions in Eq. (2), we arrive

$$\begin{aligned}\vec{v}_i &= \frac{c}{\vec{B}_0}\hat{b} \times \nabla\phi + \left(\frac{c}{e\vec{B}_i n_i}\right)\hat{b} \times \nabla(n_i T_i) + \vec{v}_z \\ &\quad + -\left(\frac{c}{\vec{B}_0\omega_c}\right)\left(\frac{\partial}{\partial t} + \vec{v}_i.\nabla\right)\nabla_\perp\phi + \vec{v}_i \parallel \hat{b},\end{aligned} \tag{18}$$

Where $\vec{v}_z = \left(\frac{c}{e\vec{B}_0}\right)\hat{b} \times \nabla.T'$ ..

Note that, T' signifies collision less stress tensor, and $\vec{v}_i \parallel$ is parallel component of ion fluid velocity [14].

Putting equation (18), into ion continuity equation and getting $n_i = n_o + n_{i1}$ and $T_i = T_{io} + T_{i1}$, where $n_{i0} \ll n_0$ and $T_{i1} \ll T_{io}$.

Mathematically, the continuity equation is addressed as

$$\frac{\partial}{\partial t}+\nabla.(n_i\vec{v}_i)=0, \quad (19)$$

Now implementing

$$\frac{\partial n}{\partial t}+\nabla.(n_i\vec{v}_{EB})+\nabla.(n_i\vec{v}_{Di})+\nabla.(n_i\vec{v}_{pi}) + \nabla.(n_i\vec{v}_i \parallel)=0, \quad (20)$$

Solving each part individually and then invoking in the above one

$$\frac{\partial N}{\partial t}+n_o J\,\vec{v}_B.\nabla\phi-n_o\left(\frac{cT_e}{eB_o}\right)\hat{b}\times\left\{\frac{\nabla n_o}{n_o}+\nabla N\right\}.\nabla\Phi+n_o\vec{v}_B.\nabla T+\vec{v}_B.\nabla N-\delta_s^2\left[\partial t+\left(\frac{c}{B_o}\right)\hat{b}\times\nabla\phi.\nabla\right]\nabla_\perp^2\Phi-\delta_s^2\nabla.(\vec{v}_{Di}.\nabla)\nabla_\perp\Phi + \nabla_\parallel\left[(1+N)\vec{v}_{i_\parallel}\right]=0, \quad (21)$$

Finally, we have

$$(\partial t+\vec{v}_B.\nabla)N-\left(\frac{cT_e}{eB_o}\right)\hat{b}\times\nabla(\ln n_o+N).\nabla\phi+\vec{v}_B.\nabla T+J\,\vec{v}_B.\nabla\phi-\delta_s^2\left[\partial t+\frac{c}{B_o}\hat{b}\times\nabla\phi.\nabla+\vec{v}_{Do}\nabla\right]\nabla_\perp^2\Phi-\delta_s^2\nabla.[(\vec{v}_{Di}.\nabla)]\nabla\Phi+\nabla_\parallel(1+N)\vec{v}_\parallel=0, \quad (22)$$

Where $\vec{v}_{Di}=\left(\frac{cT_i}{eB_o}\right)\hat{b}\times\nabla(N+T)$ is perturbation in zeroth order ion diamagnetic drift.

3. Solution Methodology

We have examined how magnetic field, density, velocity gradient and temperature can cause improved ion-acoustic waves and electrostatic drift waves to develop unstable. At a certain point in the instability process, nonlinear effects appear to be significant for some of the perturbed quantities. To make calculations more straightforward, we use a dimensionless solution. Let us consider the following variables [16]

$$t'=\frac{c_s t}{L_{ni}},\ x'=\frac{x}{\rho_s},\ y'=\frac{y}{\rho_s},\ z'=\frac{z}{L_{ni}},\ \rho_s=\frac{c_s}{\omega_{ci}},$$

$$\Phi=\frac{(\phi e n_{io}L_{ni})}{\rho_s(n_{eo}+n_{po})T_0},\ T=\frac{T_{i1}L_{ni}}{\rho_s T_{io}},v'=\frac{L_{ni}v_{iz}}{\rho_s c_s}, \quad (23)$$

$$L_{ni}=[d_x(\ln n_{i0})]^{-1}, c_s=\sqrt{\frac{n_{io}T_o}{N_o m_i}},$$

For simplicity, we shall drop the s uperscripts prime [17].

$$D_t\vec{v}=\left[d_x\vec{v}_{i0}\partial_y-(1+\alpha\tau^{-1})\partial_z\right]\Phi-\alpha\tau^{-1}\partial_z T, \quad (24)$$

As $\vec{v}_B.\nabla=\alpha k_B\partial_y T$,

$$\left(D_t+\frac{5}{3}\alpha k_B\partial_y\right)T-\frac{2}{3}\partial_x(\ln N_o)\Phi + \left(\frac{5}{3}-\tau k_T\right)\partial_y\Phi-\frac{2}{3}D_i\Phi=0\,, \quad (25)$$

and

$$\partial_t(1-\nabla_\perp^2)\Phi-\alpha_o\Phi\partial_y\Phi+\alpha K_B\partial_y T-[\,1-(\alpha+\tau)K_B+\alpha K_T\nabla_\perp^2]\partial_y\Phi-(1+\alpha\tau^{-1})\,J\,(\Phi,\nabla_\perp^2\Phi)-\alpha\tau^{-1}\nabla.J\,(T,\nabla_\perp\Phi)+\partial_z\vec{v}=0 \quad (26)$$

$$\xi=y+n_o z-\mu_o t, \quad (27)$$

Considering the transformations

$$\frac{\partial}{\partial\xi}=\frac{\partial}{\partial y}+n_o\frac{\partial}{\partial z}-\mu_o\frac{\partial}{\partial t}, \quad (28)$$

$$\frac{\partial}{\partial\xi}=\frac{\partial}{\partial y}\,,\ \frac{\partial}{\partial\xi}=n_o\frac{\partial}{\partial z}\,,\ \frac{\partial}{\partial\xi}=-\mu_o\frac{\partial}{\partial t}, \quad (29)$$

$$D_t=\frac{\partial}{\partial t}+\left(\frac{\partial}{\partial n}\,\Phi\,\frac{\partial}{\partial\xi}-\frac{\partial}{\partial\xi}\,\Phi\frac{\partial}{\partial x}\right), \quad (30)$$

$$D_t=\,-\mu_o\frac{\partial}{\partial\xi}+\frac{\partial}{\partial x}\,\Phi\frac{\partial}{\partial\xi}-\frac{\partial}{\partial\xi}\,\Phi\frac{\partial}{\partial x}\,, \quad (31)$$

$$D_t=\,-\mu_o\left[\frac{\partial}{\partial\xi}-\frac{1}{\mu_o}\left\{\frac{\partial}{\partial x}\Phi\frac{\partial}{\partial\xi}-\frac{\partial}{\partial\xi}\Phi\,\frac{\partial}{\partial x}\right\}\right], \quad (32)$$

$$D_t=-\mu_o D_\xi \quad or \quad D_\xi=-\frac{1}{\mu_o}D_t \qquad \because \partial_\xi=\partial_y$$

Invoking above values in Eq. (24), we arrive

$$-\mu_o D_\xi\vec{v}=\left[\,d_x\vec{v}_{io}\frac{\partial}{\partial\xi}-(1+\alpha\tau^{-1})\,\eta_o\frac{\partial}{\partial\xi}\right]\Phi-\alpha\tau^{-1}\,\eta_0\frac{\partial}{\partial\xi}T, \quad (33)$$

$$D_\xi\vec{v}=-\frac{1}{\mu_o}\left[\,d_x\vec{v}_{io}\frac{\partial}{\partial\xi}-(1+\alpha\tau^{-1})\,\eta_o\frac{\partial}{\partial\xi}\right]\Phi-\alpha\tau^{-1}\,\eta_0\frac{\partial}{\partial\xi}T, \quad (34)$$

For more simplification, taking (–1), we arrives

$$\left(-\mu_o D_\xi+\frac{5}{3}\,\alpha k_B\partial_\xi\right)T-\frac{2}{3}\left(-\mu_o D_\xi\right)\Phi - \frac{2}{3}\,\partial_x(\ln N_o)\Phi\partial_\xi\Phi+\left(\frac{5}{3}-\tau k_T\right)\partial_\xi\Phi=0, \quad (35)$$

$$-\mu_o\left(D_\xi-\frac{5}{3\mu_o}\,\alpha k_B\partial_\xi\right)T+\frac{2}{3}\mu_o D_\xi\Phi - \frac{2}{3}\partial_x(\ln N_o)\Phi\partial_\xi\Phi+\left(\frac{5}{3}-\tau k_T\right)\partial_\xi\Phi=0, \quad (36)$$

For further simplifications, divide by $-\mu_o$

$$\left(D_\xi - \frac{5}{3\mu_o}\alpha k_B \partial_\xi\right) T - \frac{2}{3}\, D_\xi \Phi + \frac{2}{3\mu_o}$$
$$\partial_x (\ln N_o)\Phi \partial_\xi \Phi - \frac{1}{\mu_o}\left(\frac{5}{3} - \tau k_T\right)\partial_\xi \Phi = 0, \tag{37}$$

$$-\mu_o \partial_\xi (1-\nabla_\perp^2)\Phi - \alpha_o \Phi(\partial_\xi \Phi) + \alpha k_B \partial_\xi T$$
$$-[1-(\alpha+\tau)k_B + \alpha k_T \nabla_\perp^2]\partial_\xi \Phi - (1+$$
$$\alpha\tau^{-1})\, J(\Phi, \nabla_\perp^2 \Phi) - \alpha\tau^{-1}\nabla . J\,(T, \nabla_\perp \Phi) + \eta_o \partial_\xi \vec{v} = 0, \tag{38}$$

$$\partial_\xi (1-\nabla_\perp^2)\Phi + \frac{\alpha_o}{\mu_o}\Phi(\partial_\xi \Phi) - \frac{\alpha}{\mu_o} k_B \partial_\xi T$$
$$+\frac{1}{\mu_o}[1-(\alpha+\tau)k_B + \alpha k_T \nabla_\perp^2]\partial_\xi \Phi + \frac{1}{\mu_o}(1+$$
$$\alpha\tau^{-1})\, J(\Phi, \nabla_\perp^2 \Phi) + \frac{\alpha}{\mu_o}\tau^{-1}\nabla . J\,(T, \nabla_\perp \Phi) - \frac{\eta_o \partial_\xi \vec{v}}{\mu_o} = 0, \tag{39}$$

4. Tri-polar Vortex

We now seek a stationary solution of eqs. (37-39) known as tripolar vortex. To find this stationary vortex in an ion sheared flow in e-p-i plasma, we assume Gaussian profiles for density, temperature, and magnetic field. This allows us to approximate $K_B = K_{BO} + K_{B1}x$ and $K_T = K_{TO} + K_{T1}x$. The effect of the equilibrium perpendicular ion flow velocity, caused by a radial electric field $-\nabla\phi_o$, is included in the $E \times B$ drift. This is done by expressing the total electrostatic potential as the sum of an equilibrium ϕ_o and a perturbed ϕ potential, where $\phi_o = \vec{v}_{\perp 0}(x-x_0)^2/2$ It is important to note that the equilibrium potential profile only applies to linearly varying perpendicular flows. This is assuming KB and KT are selected, and without any scalar nonlinearity. Eq. (39) can be rewritten as follows:

$$J\left[\phi + \left(\vec{v}'_{\perp 0} + \frac{5}{3}\alpha K_{B1}\right)\frac{(x-x_1)^2}{2}, T + K_{T1}\,\tau\frac{(x-x_2)^2}{2}\right] = 0, \tag{40}$$

where $x_1 = \frac{\left[\mu_0 - \vec{v}_{\perp o} + \vec{v}'_{\perp o}x_0 - \left(\frac{5}{3}\right)\alpha K_{Bo}\right]}{\left[\vec{v}_{\perp o} = \left(\frac{5}{3}\right)\alpha K_{B1}\right]}$ and $x_2 = \frac{(2\mu_0 + 5 - 3\tau K_{T0})}{3\tau K_{T1}}$,

and although it is very difficult to analytically solve Eq. (40) for a localized vortex solution. However, if we set $x_1 = x_2$, then

$$T = K_{T1}\tau\frac{(x-x_2)^2}{2} = f_1\left[\Phi + \left(\vec{v}'_{\perp 0} + \frac{5}{3}\alpha K_{B1}\right)\frac{(x-x_1)^2}{2}\right], \tag{41}$$

The condition of $x_1 = x_2$ is a determinant for the phase velocity of the vortex. f_1 is an arbitrary function of its argument. If perturbations at infinity are minimized, we obtain

$$T = \frac{K_{T1}\tau}{\left[\vec{v}'_{\perp 0} + \left(\frac{5}{3}\right)\alpha K_{B1}\right]}\Phi = a_7\Phi. \tag{42}$$

Inserting T in Eq. (33), this is satisfied by

$$v = f_2(\Phi + \vec{v}'_{\perp 0}\frac{(x-x_3)^2}{2} + \eta_0\alpha\tau^{-1}(1 + a_7 + \alpha^{-1}\tau)(x - x_4), \tag{43}$$

Where

$$x_3 = \left(\mu_0 - \vec{v}_{\perp 0} + \vec{v}'_{\perp 0}x_0\right)/\vec{v}_{\perp 0}, x_4 = \frac{\vec{v}_{io}}{\eta_0\alpha\tau^{-1}}(1 + a_7 + \alpha^{-1}\tau), \tag{44}$$

and f_2 is an arbitrary function and we take it in the linear form such that $f_2(\chi) = F\text{–}2.\chi$, we obtain

$$\partial_\xi \vec{v} = F_2 \partial_\xi \Phi, \tag{45}$$

Equations (44), (45) and (39) can be combined to give the Jacobean

$$J = \left(\Phi + \frac{\left\{\alpha K_{T1}[1+\alpha\tau^{-1}(1+a_7)]^{-1} + \vec{v}'_{\perp 0}\right\}(x-x_5)^2}{2}, \nabla_\perp^2\Phi + \frac{\tau K_{B1}(x-x_6)^2}{2}\right) = 0, \tag{46}$$

where

$$x_5 = \frac{\{\mu_o + [1+\alpha\tau^{-1}(1+a_7)](\vec{v}'_{\perp 0}x_0 - \vec{v}_{\perp 0}) - \alpha K_{T0}\}}{[1+\alpha\tau^{-1}(1+a_7)]\vec{v}'_{\perp 0} + \alpha K_{T1}}, \tag{47}$$

and

$$x_6 = \frac{\{\mu_o + [1-(\alpha+\tau)K_{B0} + \alpha a_7 K_{B0} + \eta_0 F_2]\}}{\alpha K_{B1}(1 + a_7 + \alpha^{-1}\tau}, \tag{48}$$

Again setting $x_5 = x_6$, Eq. (46) can be integrated to obtain the general solution

$$\nabla_\perp^2\Phi + \frac{\tau K_{B1}(x-x_5)^2}{2} = f_3(\Phi + \frac{\left\{\alpha K_{T1}[1+\alpha\tau^{-1}(1+a_7)]^{-1} + \vec{v}'_{\perp 0}\right\}(x-x_5)^2}{2}, \tag{49}$$

where f_3 is an arbitrary function. Using the appropriate boundary conditions for a localized vortex solution, we obtain

$$\nabla_\perp^2\Phi = \frac{\beta^2}{\gamma^2}\Phi, \tag{50}$$

Where $\frac{\beta^2}{\gamma^2} = \tau K_{B1}/\left\{\alpha K_{T1}[1 + \alpha\tau^{-1}(1 + a_7)]^{-1} + \vec{v}'_{\perp 0}\right\}$. Using the standard procedure, Eq. (50) can be solved in cylindrical coordinates, both within and outside a circle with a radius of a. Its solution takes the form of a tripolar vortex.

5. Quadrupolar Vortex

Equation (50), can also be rewrite as

$$\nabla_\perp^2\Phi + a_8 X^2 = f_4(\Phi + a_9 X^2), \tag{51}$$

In the above expressions, $a_8 = \tau K_{B1}/2$, $a_9 = \{\alpha K_{T1}[1 + \alpha\tau^{-1}(1 + a_7)]^{-1} + \vec{v}'_{\perp 0}\}/2$, $X = (x - x_5)$, and f_4 is an arbitrary function of the given argument and we choose it as a linear one, i.e., $f_4 \approx f_{40} + (\Phi + X^2) f_{41}$. With this choice of f_4 we can rearrange the Eq. (51) defining new constants.

$$(\nabla_\perp^2 - 1)\Phi + a_{10}X^2 = f_{40} + (\Phi + X^2) f'_{41}. \quad (52)$$

where $a_{10} = 1 + a_8 + f_{41} - a_9 f_{41}$ and $f'_{41} = 1 + f_{41}$.

6. Entropy Behavior

We have augmented the drift approximation below the low-frequency ($\omega \ll \omega_{ci}$) with an entropy drift, where the ITG mode has a frequency of ω and the cyclotron frequency of ions is $\omega_{ci} = (eB_0/m_i c)$, where e is the magnitude of electron charge, c is the speed of light, and m_i is the mass of ions. Additionally, an electrostatic wave can be expressed as $E = -\nabla\phi$ (where ϕ is the wave potential) [1].

Drift approximation with entropy is communicated as

$$\vec{v}_i = \vec{v}_{EB} + \vec{v}_{Di} + \vec{v}_s + \vec{v}_\pi + \vec{v}_{pi} + \vec{v}_i \parallel \ \hat{b}\,, \quad (53)$$

Now tackle for entropy

$$\nabla.(n_i\vec{v}_s) = n_i\nabla.\vec{v}_s + \vec{v}_s\nabla n_i, \quad (54)$$

Implementing perturbation technique

$$\nabla.(n_i\vec{v}_s) = (n_o + n_i)\Big[-\frac{cm_i}{eB_o}(\hat{b}\times\nabla S_i).\nabla T_{io} - \frac{cm_i}{eB_o} T_o(\hat{b}\times\nabla S_i).\Big(-\frac{\nabla\vec{B}_0}{B_0}\Big)\Big] + \vec{v}_s\nabla n_o + \vec{v}_s.\nabla n_i, \quad (55)$$

$$\nabla.(n_i\vec{v}_s) = n_o S_o m_i\Big[\frac{cT_i}{B_o}\Big(\hat{b}\times\frac{\nabla T_{io}}{T_{io}}\Big) - \frac{cT_i}{eB_o}\Big(\hat{b}\times\frac{\nabla\vec{B}_o}{B_o}\Big) + \frac{cT_{io}}{eB_o}\Big(\hat{b}\times\frac{\nabla n_o}{n_o}\Big)\Big].\nabla S + n_o\vec{v}_s.\nabla N, \quad (56)$$

$$\nabla.(n_i\vec{v}_s) = n_o S_o m_i[\vec{v}_N + \vec{v}_T - \vec{v}_B].\nabla S + n_o\vec{v}_s\nabla N, \quad (57)$$

where

$$\vec{v}_N = \frac{cT_{io}}{eB_o}\Big(\hat{b}\times\frac{\nabla n_o}{n_o}\Big), \vec{v}_T = \frac{cT_{io}}{B_o}\Big(\hat{b}\times\frac{\nabla T_{io}}{T_{io}}\Big), \quad \vec{v}_B = -\frac{cT_{io}}{eB_o}\Big(\hat{b}\times\frac{\nabla B_o}{B_o}\Big), \eta_i = \frac{\vec{v}_T}{\vec{v}_N}, \quad (58)$$

$$\nabla.(n_i\vec{v}_s) = n_o S_o m_i[(1+\eta_i)\vec{v}_N - \vec{v}_B].\nabla S + n_o\vec{v}_s\nabla N, \quad (59)$$

After the addition of entropy, the new form of ion continuity equation is

$$\partial_t n_i + \nabla.(n_i\vec{v}_{EB}) + \nabla.(n_i\vec{v}_{Di}) + \nabla.(n_i\vec{v}_{si}) + \nabla.(n_i\vec{v}_{pi}) + \nabla.(n_i\vec{v}_i \parallel) = 0, \quad (60)$$

$$n_o\partial_t N + n_o\tau(\vec{v}_B - \vec{v}_N)\nabla\phi + n_o\vec{v}_E.\nabla N + n_o\vec{v}_B.\nabla(N+T) - n_o\delta_s^2[\partial_t + \vec{v}_E.\nabla + (1+n_i)\vec{v}_N.\nabla]\nabla_\perp^2\phi - n_o\delta_s^2[\nabla(\vec{v}_{Di}.\nabla)]\nabla_\perp\phi + n_o S_o m_i[(1+n_i)\vec{v}_N - \vec{v}_B].\nabla S + n_o\vec{v}_s.\nabla N + n_o\nabla_\parallel[(1+N)\vec{v}_{i_\parallel}]\hat{b} = 0, \quad (61)$$

Dividing by n_o, we have

$$(\partial_t\vec{v}_E.\nabla + \vec{v}_B.\nabla + \vec{v}_s.\nabla)N + \tau(\vec{v}_B - \vec{v}_N).\nabla\phi + \vec{v}_B.\nabla T - \delta_s^2[\partial_t + \vec{v}_E\nabla + (1+\eta_i)\vec{v}_N.\nabla]\partial_\perp^2\phi - \delta_s^2[\nabla.(\nabla_{Di}.\nabla)\nabla_\perp\phi + n_o S_o m_i[(1+\eta_i)\vec{v}_N - \vec{v}_B].\nabla S + \nabla_\parallel[(1+N)\vec{v}_i.\vec{v}_\parallel] = 0, \quad (62)$$

7. Entropy Drift and Non-linear Solution

We have studied how density, temperature, velocity gradients, and magnetic fields can cause instability in modified ion-acoustic waves and electrostatic drift waves with entropy drift. At a particular point in the development of the instability, non-linear effects appear to be significant for the perturbed values at certain points. To assess this, we used the same method of calculation and introduced normalized parameters to incorporate entropy into eqs. (25) and (26).

$$\Big(D_t + \frac{5}{3}\alpha k_B\partial_y\Big)T - \frac{2}{3}\partial_x(\ln N_o)\,\partial_y\Phi + \Big(\frac{5}{3} - \tau k_T\Big)\partial_y\Phi - \frac{2}{3}D_t\Phi + \partial_t\{n_o S_0 m_i[(1+\eta_i)\vec{v}_N - \vec{v}_B].\nabla S + n_o\vec{v}_s\nabla N = 0, \quad (63)$$

$$\Big(D_t + \frac{5}{3}\alpha k_B\partial_y\Big)T - \frac{2}{3}\partial_x(\ln N_o)\,\partial_y\Phi + \Big(\frac{5}{3} - \tau k_T\Big)\partial_y\Phi - \frac{2}{3}D_t\Phi + \partial_t\{n_0 S_0 m_i[(1+\eta_i)\vec{v}_N].\nabla S + n_o\vec{v}_s\nabla N = 0\,, \quad (64)$$

and

$$\partial_t(1-\nabla_\perp^2)\Phi - \alpha_o\Phi\partial_y\Phi + \alpha K_B\partial_y T - [1 - (\alpha+\tau)K_B + \alpha K_T\nabla_\perp^2]\partial_y\Phi - (1+\alpha\tau^{-1})\,J(\Phi,\nabla_\perp^2\Phi) - \alpha\tau^{-1}\nabla.J(T,\nabla_\perp\Phi) + \partial_z\vec{v} + \partial_t\{n_o S_o m_i[(1+\eta_i)\vec{v}_N - \vec{v}_B].\nabla S + n_0\vec{v}_s\nabla N = 0, \quad (65)$$

Invoking all these in Eq. (24), one has

$$-\mu_o D_\xi\vec{v} = \Big[d_x\vec{v}_{io}\frac{\partial}{\partial_\xi} - (1+\alpha\tau^{-1})\,\eta_o\frac{\partial}{\partial_\xi}\Big]\Phi - \alpha\tau^{-1}\,\eta_o\frac{\partial}{\partial_\xi}T - \mu_0 D_\xi[n_o S_0 m_i\{(1+\eta_i)\vec{v}_N.\nabla S - \alpha K_B\partial_y T\}] + n_o\vec{v}_s\nabla N = 0, \quad (66)$$

$$D_\xi\vec{v} = -\frac{1}{\mu_o}\Big[d_x\vec{v}_{io}\frac{\partial}{\partial_\xi} - (1+\alpha\tau^{-1})\,\eta_o\frac{\partial}{\partial_\xi}\Big]\Phi + \Big(\frac{\alpha\tau^{-1}}{\mu_0}\eta_0\frac{\partial}{\partial_\xi} + D_\xi n_o S_0 m_i\alpha K_B\partial_y\Big)T - \frac{n_o\vec{v}_s}{\mu_o}\nabla N + D_\xi[n_0 S_0 m_i(1+n_i)\vec{v}_N.\nabla S], \quad (67)$$

From eq. (63), we have

$$\Big(-\mu_o D_\xi + \frac{5}{3}\,\alpha k_B\partial_\xi - \mu_0 D_\xi n_0 S_0 m_i\alpha K_B\partial_\xi\Big)T - \frac{2}{3}\,(-\mu_o D_\xi)\Phi - \frac{2}{3}\,\partial_x(\ln N_o)\Phi\partial_\xi\Phi + \Big(\frac{5}{3} - \tau k_T\Big)\partial_\xi\Phi + \partial_t\{n_o S_o m_i[(1+\eta_i)\vec{v}_N - \vec{v}_B].\nabla S + n_0\vec{v}_s\nabla N = 0, \quad (68)$$

$$\left(-\mu_o D_\xi\{1+n_0 S_0 m_i \alpha K_B \partial_\xi\}+\frac{5}{3}\,\alpha k_B \partial_\xi\right)T+\frac{2}{3}\,(\mu_o D_\xi)\Phi$$
$$-\frac{2}{3}\,\partial_x(\ln N_o)\Phi\partial_\xi\Phi+\left(\frac{5}{3}-\tau k_T\right)\partial_\xi\Phi+ \tag{69}$$
$$\partial_t\{n_o S_o m_i[(1+\eta_i)\vec{v}_N-\vec{v}_B].\nabla S+n_0\vec{v}_s\nabla N=0,$$

Diving by $-\mu_0$, we arrive

$$\left(D_\xi\{1+n_0 S_0 m_i \alpha K_B \partial_\xi\}-\frac{5}{3\mu_0}\,\alpha k_B \partial_\xi\right)T-\frac{2}{3}\,(D_\xi)\Phi$$
$$+\frac{2}{3\mu_0}\,\partial_x(\ln N_o)\Phi\partial_\xi\Phi-\left(\frac{5}{3}-\tau k_T\right)\partial_\xi\Phi/ \tag{70}$$
$$\mu_0-\frac{\partial_t}{\mu_0}\{n_o S_o m_i[(1+\eta_i)\vec{v}_N-\vec{v}_B].\nabla S-\frac{n_0}{\mu_0}\vec{v}_s\nabla N=0,$$

8. Entropy Drift and Tri-polar Vortex

We are now searching for a stationary solution of eqs. (34)-(39), with an entropy drift, known as the tripolar vortex solution. To find this type of stationary vortex solution in plasma with ion sheared flow and entropy drift, we assume Gaussian profiles for density, temperature, and magnetic field. This allows us to approximate $K_B = K_{BO} + K_{B1}x$ and $K_T = K_{TO} + K_{T1}x$. The perpendicular ion flow velocity equilibrium is affected by entropy drift, which is driven by a radial electric field $-\nabla\phi_o$. This effect is incorporated in the $E \times B$ drift when entropy drift is present, by expressing the total electrostatic potential as a sum of the equilibrium ϕ_o and perturbed ϕ potentials, where $\phi_o = \vec{v}_{\perp 0}(x - x_0)^2/2$.

The effect of the equilibrium perpendicular ion flow velocity with entropy drift, which is caused by a radial electric field, is incorporated in the drift in the present of entropy drift, by writing the total electrostatic potential as a sum of the equilibrium ϕ_0 and perturbed ϕ potential, where $\phi_o = \vec{v}_{\perp 0}(x - x_0)^2/2$. It is important to remember that the equilibrium potential profile only describes linear, perpendicular flows. By selecting KB and KT, and with no scalar nonlinearity present.

Eq. (39) can be rewritten as follows:

$$J\left[\phi+\left(\vec{v}'_{\perp 0}+D_\xi\{1+n_0 S_0 m_i \alpha K_B \partial_\xi\}-\frac{5}{3\mu_0}\,\alpha k_B \partial_\xi\right)\right.$$
$$\left.\frac{(x-x_1)^2}{2},T+K_{T1}\,\tau\frac{(x-x_2)^2}{2}\right]=0, \tag{71}$$

where $x_1=\frac{\left[\mu_0-\vec{v}_{\perp o}+\vec{v}'_{\perp o}x_0-D_\xi\{1+n_0 S_0 m_i \alpha K_B \partial_\xi\}-\frac{5}{3\mu_0}\alpha k_B \partial_\xi\right]}{\left[\vec{v}_{\perp o}=\left(\frac{5}{3}\right)\alpha K_{B1}\right]}$

and $x_2=\frac{(2\mu_0+5-3\tau K_{T0})}{3\tau K_{T1}}$.. Although it is very difficult to solve Eq. (24) analytically for a localized vortex solution, setting $x_1 = x_2$ allows one to integrate the equation and obtain a solution.

$$T=K_{T1}\tau\frac{(x-x_2)^2}{2}=$$
$$f_1\left[\Phi+\left(\vec{v}'_{\perp 0}+D_\xi\{1+n_0 S_0 m_i \alpha K_B \partial_\xi\}-\frac{5}{3\mu_0}\,\alpha k_B \partial_\xi\right)\frac{(x-x_1)^2}{2}\right], \tag{72}$$

When x_1 equals x_2, it determines the phase velocity of the vortex. f_1 is a function of its argument. To ensure that perturbations become zero at infinity, we must adhere to this condition.

$$T=\frac{K_{T1}\tau}{\left[\vec{v}'_{\perp 0}+D_\xi\{1+n_0 S_0 m_i \alpha K_B \partial_\xi\}-\frac{5}{3\mu_0}\,\alpha k_B \partial_\xi\right]}\Phi=a'_7\Phi, \tag{73}$$

Inserting T in below equation this is satisfied by

$$D_\xi\vec{v}=-\frac{1}{\mu_o}\left[d_x\vec{v}_{io}\frac{\partial}{\partial_\xi}-(1+\alpha\tau^{-1})\eta_o\frac{\partial}{\partial_\xi}\right]\Phi$$
$$+\left(\frac{\alpha\tau^{-1}}{\mu_0}\eta_0\frac{\partial}{\partial_\xi}+D_\xi n_o S_0 m_i \alpha K_B \partial_y\right)a'_7\Phi-\frac{n_0\vec{v}_s}{\mu_o}\nabla N+ \tag{74}$$
$$D_\xi[n_0 S_0 m_i(1+n_i)\vec{v}_N.\nabla S],$$

$$\mathrm{v}=\mathrm{f}_2(\Phi+\vec{\mathrm{v}}'_{\perp 0}\frac{(\mathrm{x}-\mathrm{x}_3)^2}{2}+\eta_0\alpha\tau^{-1}(1+\mathrm{a}'_7+\alpha^{-1}\tau)(\mathrm{x}-\mathrm{x}'_4), \tag{75}$$

where $x_3=\left(\mu_0-\vec{v}_{\perp 0}+\vec{v}'_{\perp 0}x_0\right)/\vec{v}_{\perp 0}, x'_4=\frac{\vec{v}_{io}}{\eta_0\alpha\tau^{-1}}(1+a'_7+\alpha^{-1}\tau)$,

and f_2 is an arbitrary function and we take it in the linear form such that $f_2(\chi) = F_2.\chi$, we obtain

$$\partial_\xi\vec{v}=F_2\partial_\xi\Phi, \tag{76}$$

The above equations can be can be combined to give the Jacobean

$$J=\left(\Phi+\frac{\left\{\alpha K_{T1}[1+\alpha\tau^{-1}(1+a'_7)]^{-1}+\vec{v}'_{\perp 0}\right\}(x-x'_5)^2}{2},\right.$$
$$\left.\nabla_\perp^2\Phi+\frac{\tau K_{B1}(x-x'_6)^2}{2}\right)=0, \tag{77}$$

where

$$x'_5=\frac{\{\mu_o+[1+\alpha\tau^{-1}(1+a'_7)](\vec{v}'_{\perp 0}x_0-\vec{v}_{\perp 0})-\alpha K_{T0}\}}{[1+\alpha\tau^{-1}(1+a'_7)]\vec{v}'_{\perp 0}+\alpha K_{T1}}, \tag{78}$$

and

$$x'_6=\frac{\{\mu_o+[1-(\alpha+\tau)K_{B0}+\alpha a'_7 K_{B0}+\eta_0 F_2]\}}{\alpha K_{B1}(1+a'_7+\alpha^{-1}\tau}, \tag{79}$$

again setting $x_5' = x_6'$, Eq. (28) can be integrated to obtain the general solution:

$$\nabla_\perp^2\Phi+\frac{\tau K_{B1}(x-x'_5)^2}{2}=$$
$$f_3(\Phi+\frac{\left\{\alpha K_{T1}\left[1+\alpha\tau^{-1}(1+a'_7)\right]^{-1}+\vec{v}'_{\perp 0}\right\}(x-x'_5)^2}{2}, \tag{80}$$

where f_3 is an arbitrary function. Using the appropriate boundary conditions for a localized vortex solution, we obtain

$$\nabla_{\perp}^{2}\Phi = \frac{\beta'^2}{\gamma'^2}\Phi\,, \tag{81}$$

Where $\frac{\beta'^2}{\gamma'^2} = \tau K_{B1}/\{\alpha K_{T1}[1+\alpha\tau^{-1}(1+a'_7)]^{-1} + \vec{v}'_{\perp 0}\}$..

Eq. (81) can be solved using standard procedures in cylindrical coordinates, both inside and outside a circle with radius $r = a$. The solution is presented in the form of a tripolar vortex.

9. Entropy Drift and Quadrupolar Vortex

Eq. (80) can a1so be rewrite as

$$\nabla_{\perp}^{2}\Phi + a_8 X^2 = f_4(\Phi + a'_9 X^2), \tag{82}$$

Where $a_8 = \tau K_{B1}/2$, $a'_9 = \{\alpha K_{T1}[1+\alpha\tau^{-1}(1+a'_7)]^{-1} + \vec{v}'_{\perp 0}\}/2$, $X = (x - x_5)$, and f_4 is an arbitrary function of the given argument and we choose it as a 1inear one, i.e., $f_4 \approx f_{40} + (\Phi + X^2)f_{41}$. With this choice of f_4, we can rearrange the eq. (4.22) defining new constants.

$$(\nabla_{\perp}^{2} - 1)\Phi + a'_{10}X^2 = f_{40} + (\Phi + X^2)f'_{41}. \tag{83}$$

Where $a'_{10} = 1 + a_8 + f_{41} - a'_9 f_{41}$ and $f'_{41} = 1 + f_{41}$. A localized general solution of quadrupolar vortex due to entropy drift is shown in the above equation.

10. Conclusion

The study of entropy drift is essential to the transport of mass and energy. Initially, we examined a numerical model of tri-polar and quadrupolar vortexes in the absence of entropy drift. Subsequently, a stationary solution was derived to include entropy effects in both the tri-polar and quadrupolar vortexes. This research opens up new possibilities for researchers to investigate the entropy effect in different plasma models. The findings from this work will be used to further understand the role of entropy drift in numerical models.

Acknowledgments

This research work was funded by institutional Fund Projects under grant no. (IFPIP: 1225-130-1443). The authors gratefully acknowledge technical and financial support provided by the Ministry of Education and King Abdulaziz University, DSR, Jeddah, Saudi Arabia.

References

[1] M. Y. Khan and J. Iqbal, Phys. Plasmas **24**, 082514 (2017).
[2] J. Wesson and D. J. Campbell, Tokamaks, Oxford University Press **49**, (2011).
[3] Y. Q. Zhang, A. W. DeSilva, and A. N. Mostovych, Phys. Rev. Lett. **62**, 1848 (1989).
[4] J. Iqbal and M. Y. Khan, Phys. Plasmas **25**, 042111 (2018).
[5] B. Coppi, M. N. Rosenbluth, and R. Z. Sagdeev, The Physics of Fluids **10**, 582 (1967).
[6] A. Jarmén, P. Andersson, and J. Weiland, Nuclear Fusion **27**, 941 (1987).
[7] K. Bol, V. Arunasalam, and M. Bitter, Plasma Physics and Controlled Nuclear Fusion Research, (1978).
[8] A. B. Hassam, T. M. Antonsen, J. F. Drake, and P. N. Guzdar, Physics of Fluids B: Plasma Physics **2**, 1822 (1990).
[9] G. Murtaza, M. Nadeem, and P. K. Shukla, Physica Scripta **48**, 359 (1993).
[10] F. Romanelli, Plasma Physics and Controlled Fusion **31**, (1989).
[11] F. Romanelli, Physics of Fluids B: Plasma Physics **1**, 1018 (1989).
[12] P. K. Shukla, Physics of Fluids B: Plasma Physics **2**, 848 (1990).
[13] M. Azeem and A. M. Mirza, Phys. Plasmas **11**, 4727 (2004).
[14] P. K. Shukla, Physica Scripta **42**, 725 (1990).
[15] A. M. Mirza and M. Azeem, Phys. Plasmas **11**, 4341 (2004).
[16] M. Azeem and A. M. Mirza, Phys. Plasmas **12**, 052306 (2005).
[17] M. Azeem and A. M. Mirza, Phys. Plasmas **11**, 4727 (2004).
[18] C. Liu and P. Ying, Chinese Physics B **31**, 26201 (2022).
[19] Z. Li, M. Peng, X. Zhou, K. Shin, S. Tunmee, X. Zhang, C. Xie, H. Saitoh, Y. Zheng, Z. Zhou, and Y. Tang, Adv. Mater. **33**, 2100793 (2021).
[20] X. He, Q. Yan, H. He, J. Shang, X. Zhou, N. Chanlek, S. Tunmee, P. Kidkhunthod, W. Yao, and Y. Tang, Adv. Sustainable Syst. **6**, 2200122 (2022).
[21] S. Mu, Q. Liu, P. Kidkhunthod, X. Zhou, W. Wang, and Y. Tang, Natl. Sci. Rev. **8**, 1 (2021).
[22] Q. Zhao, J. Liu, H. Yang, H. Liu, G. Zeng, B. Huang, and J. Jia, Appl. Opt. **61**, 7225 (2022).
[23] H. Yu, Y. Chen, W. Wei, X. Ji, and L. Chen, ACS Nano **16**, 9736 (2022).
[24] H. Wang, Y. Chen, H. Yu, W. Liu, G. Kuang, L. Mei, Z. Wu, W. Wei, X. Ji, B. Qu, and L. Chen, Adv. Funct. Mater. **32**, 2205600 (2022).
[25] Y. Liang, J. Li, Y. Xue, T. Tan, Z. Jiang, Y. He, W. Shangguan, J. Yang, and Y. Pan, J. Hazard, Mater. **420**, 126584 (2021).
[26] L. Yang, H. Huang, Z. Xi, L. Zheng, S. Xu, G. Tian, Y. Zhai, F. Guo, L. Kong, Y. Wang, W. Lü, L. Yuan, M. Zhao, H. Zheng, and Gang Liu, Nat. Commun. **13**, 2444 (2022).
[27] S.-T. Yang, X.-Y. Li, T.-L. Yu, J. Wang, H. Fang, F. Nie, B. He, L. Zhao, W.-M. Lü, S.-S. Yan, A. Nogaret, G. Liu, and L.-M. Zheng, Adv. Funct. Mater. **32**, 2202366 (2022).