Diffusive Species in MHD Squeezed Fluid Flow Through non-Darcy Porous Medium with Viscous Dissipation and Joule Heating

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This article focuses the flow through non-Darcy porous medium. The flow is due to the squeezing phenomenon. The magneto viscous fluid is accounted. Formulation of the flow problem is interpreted the salient features of Ohmic heating (Joule heating), viscous dissipation and auto-catalyst and reactants (i.e. homogeneous-heterogeneous reactions). A whole analysis is carried out with different diffusion coefficients for both auto-catalyst and reactants. It is also desired to observe the dependence of convective surface condition on flow regime in heat transport process. The resulting non-linear partial differential equations are found to be governing by dimensionless ordinary differential equations with the implementation of similarity solutions. A homotopic procedure based on an iterative scheme is utilized for the solutions of the flow problem. Flow velocity, fluid temperature and concentration are addressed via graphs for different values of geometrical and rheological parameters of considered flow problem. Moreover, skin friction co-efficient and Nusselt number are sketched and discussed graphically. The analysis reveals that higher values of mass diffusion ratio parameter result reduction in concentration of species B whereas concentration of species A enhances for higher mass diffusion ratio parameter.

Keywords: squeezing flow, non-Darcy porous medium, convective boundary condition, joule heating, homogeneous-heterogeneous reactions, viscous dissipation

1. Introduction

Dissipation effects as an energy source are played a vital role in the heat transport phenomenon. Joule heating (Ohmic heating) and viscous dissipation get more significance when plates are heating or cooling. The perceptible fact of the heat transport phenomenon occurs in the processes of power generation systems, cooling of metallic sheets or electronic chips, liquid metal fluids, cooling of nuclear reactors. Moreover, the Joule heating (Ohmic heating) process generates heat due to the resistance arises by passage of electric current through the material. The Joule heating has valuable as well as adverse influence on the system. A few systems that exploit the Joule heating effects include thermistors and soldering irons, di-electrophoretic trapping, hot plate, PCR reactors, micro-valves for fluid control, electric heaters and stoves, bio-particles manipulation in dilute medium, electric fuses etc. On the other hand, inadmissible heat produces in some processes which can melts or debase the machinery parts, may create denaturation of biological samples (proteins, DNA etc.), bubble formation, malfunctioning of chip systems etc. Hayat et al. [1] portray the impact of heat transfer characterized by Newtonian and Joule heating effects on Williamson fluid flow over shrinking surface. The properties of the magneto-hydrodynamics in Sisko nano-fluid flow over stretched cylinder with Ohmic heating and viscous dissipation are examined by Hussain et al. [2]. Sulochana et al. [3] disclosed the influence of the Joule heating on MHD radiative flow of nano-fluid along the continual moving needle. The characteristics of the Joule heating in the convectively heated MHD Maxwell fluid flow through the wall jet are portrayed by Zaidi and Mohyuddin [4]. The behavior of the Joule heating in the radiative peristalsis flow in a curved channel is constructed by Hayat et al. [5]. Shagaiy et al.

Chemically reactive processes featured via homogeneous-heterogeneous reactions which behave differently in the presence or absence of the catalyst. In industrial processes, catalysts are usually observed to increase the effectiveness of the chemical reactions. The constitutive relationship among the homogeneous-heterogeneous reactions is specifically elaborated. Chemical reaction is effectively important in the manufacturing of ceramics, food processing, polymer production, metallurgy and hydrometallurgical industry, crops damage via freezing and chemical processing equipment design etc. Markin [10] explored the features of the auto-catalyst and reactants in the concept of boundary layer flow theory for isothermal model. Hayat et al. [11] disclosed the melting phenomenon in flow of stagnant Jeffrey fluid with homogeneous and heterogeneous reactions. Khan et al. [12] explained the variation of the heat and mass transport featured via heat generation (or absorption) on Maxwell fluid flow with homogeneous-heterogeneous reactions. Significance of dissipation effects on hydro-magnetic flow of stagnant Casson liquid for the isothermal model is demonstrated by Khan et al. [13]. Xu [14] presented the isothermal model for stagnation flow of the heated fluid through a plane surface. Hayat et al. [15] exhibited the effect of homogeneous-heterogeneous model on a convectively heated nano-fluid flow via porous medium. Farooq et al. [16] explored the properties of the homogeneous-heterogeneous model in the flow over a Riga plate of variable thickness with melting condition. Raju et al. [17] elaborated the behavior of induced magnetic effects on the flow of stagnant Casson fluid with homogeneous-heterogeneous reactions. Few recent contributions in the area of the homogeneous-heterogeneous reactions are made in the refs. [18, 19].

A close scrutiny of the scientific literature aimed that no studies have been appeared in the communications where the squeezing flow analysis is accounted under the Darcy Forchheimer theory for the hydro-magnetic fluid with the homogeneous-heterogeneous reactions. The objective of this analysis is to fill such void. Therefore the present attempt is described the MHD squeezing flow through a Forchheimer porous media. The heat transfer process involves convective boundary condition and dissipation effects (Joule heating and viscous dissipation). The homogeneous-heterogeneous reactions are utilized to explore the mass transport phenomenon. The convergent series solutions of the problem are evaluated by homotopic technique [20-28]. The contributions of different embedding parameters are plotted and addressed. Skin friction co-efficient and Nusselt number are exhibited through graphical data.

2. Mathematical Modeling

Consider the unsteady squeezed viscous fluid flow between parallel two plates. The incompressible fluid saturates the porous medium featuring the Darcy-Forchheimer model. The fluid is the electrically conducting and strength $B_0/\sqrt{1-\gamma r}$ of the magnetic field is applied along the $y$-direction. The lower plate is fixed at $y = 0$ and the upper plate at $h(t) = \sqrt{v(1-\gamma r)/a}$. Here $h(t)$ represents the width between the plates and $\gamma$ denotes the dimensional constant. Further, upper plate is squeezed towards the immovable lower stretching plate satisfying convective heat condition. The flow phenomenon is studied in the Cartesian co-ordinate system $(x, y)$. The direction of $x$ and $y$ co-ordinates are as shown in Fig. 1.

The heat transfer phenomenon is explored with the Joule heating and viscous dissipation. $T, a$ and $b$ represent the temperature and the concentration respectively. The convective heating process provides the temperature $T_f$ and $T_s$ denotes the upper plate temperature. Flow analysis is also carried out through a homogeneous-heterogeneous reactions of species $A$ and $B$. Basic model of the homogeneous and heterogeneous reactions are initiated by Merkin [10] i.e.

$$A + B \rightarrow 3B, \text{ rate } = k_{ab}^2,$$  

which represents the isothermal cubic autocatalysis reaction while first-order and isothermal reaction on the catalyst surface is given by

$$A \rightarrow B, \text{ rate } = k_a.$$

![Fig. 1. (Color online) Schematic of flow situation.](image)
where $k_c$ and $k_s$ denote the constant rate while $a$ and $b$ indicate the concentration of chemical species $A$ and $B$ respectively.

The conservative flow laws under consideration take the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$  
(3)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2}{\rho (1 - \gamma T)} - \frac{v^2 \phi'}{k} - \frac{1}{k^2} \frac{\partial^2 u}{\partial y^2},$$  
(4)

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{v \phi'}{k} - \frac{1}{k^2} \frac{\partial^2 v}{\partial y^2},$$  
(5)

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p (1 - \gamma)} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \frac{\alpha B_0^2}{\rho C_p (1 - \gamma)} u^2,$$  
(6)

and

$$\frac{\partial a}{\partial t} + u \frac{\partial a}{\partial x} + v \frac{\partial a}{\partial y} = D_a \left( \frac{\partial^2 a}{\partial x^2} + \frac{\partial^2 a}{\partial y^2} \right) - k_a a^2,$$  
(7)

$$\frac{\partial b}{\partial t} + u \frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y} = D_b \left( \frac{\partial^2 b}{\partial x^2} + \frac{\partial^2 b}{\partial y^2} \right) + k_b b^2.$$  
(8)

Here $u$ and $v$ represent the velocity components along $x$ and $y$ direction respectively, $p$ is the pressure, $v$ denotes the kinematics viscosity, $\sigma$ denotes the electric conductivity, $\mu$ denotes the absolute viscosity, $B_0$ is the magnetic field, $\rho$ represents the fluid density, $\phi'$ and $k'$ denote the porosity and the permeability of the porous medium respectively, $C_p = (C_s/x)$ denotes the drag co-efficient, $T$ is the fluid temperature, $k$ is the co-efficient of thermal conductivity, $C_p$ represents the specific heat capacity, $D_a$ and $D_b$ are the diffusion co-efficient of species $A$ and $B$ respectively.

The boundary conditions are as follow

$$u = U_w = \frac{dx}{1 - \gamma}, \quad v = 0, \quad \frac{\partial T}{\partial y} = -h[T_f - T],$$

$$D_a \frac{\partial a}{\partial y} = k_a a, \quad D_b \frac{\partial b}{\partial y} = -k_a a \quad \text{at} \quad y = 0,$$

$$u = 0, \quad v = v_t = \frac{dh}{dt} = \frac{\gamma}{2N d(1 - \gamma)}, \quad T = T_{in}, \quad a \rightarrow a_0, \quad b \rightarrow 0 \quad \text{at} \quad y = h(t).$$  
(9)

Here $U_w$ represents the stretching velocity, $d$ represents the dimensional constant, $h_t$ represents the convective heat transfer coefficient, $T_f$ represents the convective fluid temperature, $T_{in}$ represents the temperature of the upper plate and $a_0$ represents the constant concentration at the upper wall.

Selecting the suitable similarity variables of the form

$$\eta = \frac{v}{h(t)}, \quad \Psi' = \frac{dv}{\eta^2 - \gamma} f' \eta, \quad u = U_w f' \eta,$$

$$v = -\frac{dv}{\eta^2 - \gamma} f' \eta, \quad \theta(\eta) = \frac{T - T_{in}}{T_{in} - T_h}, \quad \phi(\eta) = a / a_0,$$

$$g(\eta) = \frac{b}{a_0}.$$  
(10)

Here $\Psi'$ represents the stream function, $\eta$ represents the similarity variable, $f' \eta$ represents the non-dimensional variable, $\theta(\eta)$, $\phi(\eta)$ and $g(\eta)$ represent the dimensionless temperature and dimensionless concentration of species $A$ and $B$ respectively.

Continuity equation is verified identically. Eliminating pressure term from equations (4)-(5) and in view of equation (10), we obtain the constitutive flow equations as below

$$f'''' + f'' f' - \frac{S}{2} (3f'' + \eta f'') - M^2 f''$$

$$- Da^{-1} f'' - 2 a f' f'' = 0,$$  
(11)

$$\theta'' + Pr \left( f \theta' - \frac{1}{2} S \eta \theta' \right)$$

$$+ Pr Ec \left( f'' \gamma \right) + 4 \delta f'' \gamma + M f'' \gamma = 0,$$  
(12)

$$\phi'' + Sc \left( f \phi' - \frac{1}{2} S \eta \phi' \right) - Sc k_1 \phi g^2 = 0,$$  
(13)

$$\delta g'' + Sc \left( f g' - \frac{1}{2} S \eta g' \right) + Sc k_1 \phi g^2 = 0,$$  
(14)

with subjected boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \quad f(1) = 0, \quad f'(1) = \frac{S}{2},$$  
$$\theta'(0) = -B_1(1 - \theta(0)), \quad \theta(0) = 0, \quad \phi'(0) = k_2 \phi(0), \quad \phi(1) = 1, \quad \delta g'(0) = -k_c \phi(0), \quad g(1) = 0.$$  
(15)

where squeezing parameter $S_y$, magnetic parameter $M^2$, inverse Darcy number $D_{in}^{-1}$, local inertia coefficient parameter $\alpha_i$, Prandtl number $Pr$, Eckert number $Ec$, length parameter $\delta$, Biot number $B_i$, ratio of mass diffusion coefficient $\delta_i$, Schmidt number $Sc$, strength of homogeneous reaction parameter $k_1$ and strength of heterogeneous reaction parameter $k_2$ are given by
In view of the equation (10), the equation (17) takes the form

\[ S_q = \frac{\dot{z}}{d}, \quad \Delta a^{-1} = -\frac{\phi'(1-\eta)}{k'd}, \quad M' = \frac{\sigma \beta'^2}{\rho d}, \quad \alpha_i = \frac{C_i}{j^2}, \]

\[ P_c = \frac{\mu C_e}{k}, \quad E_c = \frac{d^2 x^2}{C_p(1-\eta)^2 (T_f - T_h)}, \quad \delta = \frac{\sqrt{v(1-\eta)}}{x^2}, \]

\[ B_i = \frac{h_{1/2}(1-\eta)}{\sqrt{d}}, \quad \delta_i = \frac{D_i}{D_a}, \quad S_c = \frac{V}{D_a}, \]

\[ k_i = \frac{k_{d} d(1-\eta)}{d}, \quad k_2 = \frac{k_{d} \sqrt{1-\eta}}{d}. \]  

(16)

It is observed that for \( S_q < 0 \), the plates are moving away and for \( S_q > 0 \), the plates are moving towards each other respectively.

Defining local skin friction co-efficient \( C_f \) and local Nusselt number \( N_u \) as

\[ C_f = \frac{\mu(\tau_{xy})}{\rho U_w}, \quad N_u = \frac{k(1-\eta)}{k(T_f - T_h)}. \]  

(17)

In view of the equation (10), the equation (17) takes the form

\[ \left( Re \right) C_f = f''(1), \quad \left( Re \right) \frac{1}{\nu} N_u = -\theta'(1). \]  

(18)

Where \( Re = U_w/\nu \) represents the local Reynolds number.

3. Homotopy Solutions

Consider the initial guesses \( f_0, \theta_0, \phi_0, g_0 \) and linear operators \( L_f, L_{\theta}, L_{\phi}, L_g \) for homotopic procedure as

\[ f_0(\eta) = \frac{1}{2}(2\eta - 4\eta^2 + 3S_q \eta^2 + 2\eta^3 - 32\eta^3), \]

(19)

\[ \theta_0(\eta) = \frac{B_1}{1+B_1}(1-\eta), \]

(20)

\[ \phi_0(\eta) = \frac{1}{1+k_2}(1+k_2 \eta), \quad g_0(\eta) = \frac{k_2}{\delta_i(1+k_2)}(1-\eta), \]

(21)

\[ L_f = f'''', \quad L_{\theta} = \theta''', \quad L_{\phi} = \phi''', \quad L_g = g'''. \]  

(22)

with

\[ L_f(C_x + C_2 \eta + C_3 \eta^2 + C_4 \eta^3) = 0, \quad L_{\theta}(C_x + C_8 \eta) = 0, \]

\[ L_{\phi}(C_7 + C_8 \eta) = 0, \quad L_g(C_6 + C_{10} \eta) = 0. \]

(23)

where \( C_i (i = 1 - 10) \) are arbitrary constants.

3.1. Zeroth-order problems

Here,
where \( q \in [0,1] \) is embedding parameter and auxiliary non-zero parameters are \( \eta \), \( \eta \), \( \eta \), and \( \eta \).

### 3.2. nth-order problems

Here,

\[
\mathcal{L}_0[\theta_m(\eta) - \chi_m f_m(\eta)] = \eta \partial^2 \theta_m(\eta),
\]

\[
f_m(0) = 0, \quad f_m'(0) = 0, \quad f_m'(1) = 0, \quad f_m''(1) = 0,
\]

\[
\mathcal{L}_0[\theta_m(\eta) - \chi_m \theta_m(\eta)] = \eta \partial^2 \theta_m(\eta),
\]

\[
\theta_m''(0) - B\theta(0) = 0, \quad \theta_m'(0) = 0,
\]

\[
\mathcal{L}_0[\phi_m(\eta) - \chi_m \phi_m(\eta)] = \eta \partial^2 \phi_m(\eta),
\]

\[
\phi_m'(0) - k_0 \phi_m(0) = 0, \quad \phi_m(1) = 0,
\]

\[
\mathcal{L}_0[g_m(\eta) - \chi_m g_m(\eta)] = \eta \partial^2 g_m(\eta),
\]

\[
g_m'(0) + k_0 \phi_m(0) = 0, \quad g_m(1) = 0,
\]

Defining non-linear operators as follows

\[
R^0_m(\eta) = f_m''(\eta) + \sum_{k=0}^{m-1} f_m''(\eta) f_m''(\eta) - \sum_{k=0}^{m-1} f_m''(\eta) f_m''(\eta)
\]

\[
- \frac{S^m_2 (2f_m''(\eta) + \eta f_m''(\eta))}{2}
\]

\[
- M f_m''(\eta) - D a f_m''(\eta) - 2 \alpha \sum_{k=0}^{m-1} f_m''(\eta) f_k''(\eta)
\]

\[
R^0_m(\eta) = \partial_m^m + P \sum_{k=0}^{m-1} f_m''(\eta) \partial_k + \frac{1}{2} S_m \eta \partial_m^m
\]

\[
+ P m \sum_{k=0}^{m-1} f_m''(\eta) \partial_k^m + 4 \delta \sum_{k=0}^{m-1} f_m''(\eta) \partial_k^m
\]

\[
+ M \sum_{k=0}^{m-1} f_m''(\eta) \partial_k^m
\]

\[
R^0_m(\eta) = \phi_m^m + S \sum_{k=0}^{m-1} f_m''(\eta) \phi_k + \frac{1}{2} S_m \eta \phi_m^m
\]

\[
- S c k \sum_{k=0}^{m-1} \phi_m''(\eta) \sum_{p=0}^k g_k \eta \phi_p^m
\]

\[
R^0_m(\eta) = g_m^m + S \sum_{k=0}^{m-1} f_m''(\eta) g_k + \frac{1}{2} S_m \eta \phi_m^m
\]

\[
- S c k \sum_{k=0}^{m-1} \phi_m''(\eta) \sum_{p=0}^k g_k \eta \phi_p^m
\]

\[
\chi_m = \begin{cases} 
0, & m \leq 1, \\
1, & m > 1
\end{cases}
\]

for \( q = 0 \) and \( q = 1 \), we can write

\[
\tilde{f}(\eta; 0) = f_0(\eta), \quad \tilde{f}(\eta; 1) = f(\eta),
\]

\[
\tilde{\theta}(\eta; 0) = \theta_0(\eta), \quad \tilde{\theta}(\eta; 1) = \theta(\eta),
\]

\[
\tilde{\phi}(\eta; 0) = \phi_0(\eta), \quad \tilde{\phi}(\eta; 1) = \phi(\eta),
\]

\[
\tilde{g}(\eta; 0) = g_0(\eta), \quad \tilde{g}(\eta; 1) = g(\eta),
\]

and with the variation of \( q \) from 0 to 1, \( \tilde{f}(\eta; q) \), \( \tilde{\theta}(\eta; q) \), \( \tilde{\phi}(\eta; q) \) and \( \tilde{g}(\eta; q) \) vary from the initial solutions \( f_0(\eta), \theta_0(\eta), \phi_0(\eta) \) and \( g_0(\eta) \) to the final solutions \( f(\eta), \theta(\eta), \phi(\eta) \) and \( g(\eta) \) respectively. By Taylor series, we have

\[
\tilde{f}(\eta; q) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) q^m, \quad f_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \tilde{f}(\eta; q)}{\partial q^m} \right|_{q=0}
\]

\[
\tilde{\theta}(\eta; q) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) q^m, \quad \theta_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \tilde{\theta}(\eta; q)}{\partial q^m} \right|_{q=0}
\]

\[
\tilde{\phi}(\eta; q) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta) q^m, \quad \phi_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \tilde{\phi}(\eta; q)}{\partial q^m} \right|_{q=0}
\]

\[
\tilde{g}(\eta; q) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta) q^m, \quad g_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \tilde{g}(\eta; q)}{\partial q^m} \right|_{q=0}
\]

choose suitable value for auxiliary parameter that the series (42) converge at \( q = 1 \) i.e.

\[
f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta),
\]

\[
\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta),
\]

\[
\phi(\eta) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta),
\]

\[
g(\eta) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta),
\]

\[
f_m, \theta_m, \phi_m, \text{and } g_m \text{ represent general solutions for equations (32-35) in the form of special solutions } \left( f_m', \theta_m', \phi_m', \text{and } g_m' \right) \text{ are given by}
\]

\[
f_m(\eta) = f_m(\eta) + C_1 + C_2 \eta + C_3 \eta^2 + C_4 \eta^3,
\]

\[
\theta_m(\eta) = \theta_m(\eta) + C_1 + C_2 \eta,
\]

\[
\phi_m(\eta) = \phi_m(\eta) + C_1 + C_2 \eta,
\]

\[
g_m(\eta) = g_m(\eta) + C_1 + C_2 \eta.
\]
3.3. Convergence of problem

In order to develop the iterative solutions of the flow problem, our purpose is to determine the convergence region to ensure the iterative solutions incorporated in equation (44) are convergent. Convergence region of the flow problem depends upon the auxiliary parameters $h_1$, $h_2$, and $h_5$ for which $h$-curves are sketched in Fig. 2. It is found that allowable ranges for the parameters $h_1$, $h_2$, $h_5$, and $h_6$ are $-1.4 \leq h_1 \leq -0.5$, $-1.5 \leq h_2 \leq -0.2$, $-1.8 \leq h_5 \leq -0.3$ and $-1.7 \leq h_6 \leq -0.6$.

4. Discussion

This segment is graphed to elaborate the behavior of the flow parameters on the velocity components, temperature and the fluid concentration. Further, the skin friction and Nusselt number are also examined via these parameters.

4.1. Dimensionless velocity distributions

The variation of the squeezing parameter $S_q$ on the velocity field is illustrated in Figs. 3, 4. It is evident that the velocity components grow up for larger values of squeezing parameter. Physically, fluid deformed rapidly due to the squeezing force exerted by walls. Hence velocity components (i.e. horizontal and vertical velocities) increase. Further the velocity profile increases for the higher values of $\eta$. The velocity field is smaller at the lower surface while maximum at the upper plate. The influence of the magnetic parameter ($M$) on the horizontal velocity field is indicated in Fig. 5. It is reflected that horizontal velocity diminishes for increasing magnetic parameter near the lower plate whereas it dominants towards the upper plate.

Fig. 2. (Color online) (a) Convergence region for $f(\eta)$ and $\theta(\eta)$. (b) Convergence region for $\phi(\eta)$ and $g(\eta)$.

Fig. 3. (Color online) Variation of $S_q$ on $f(\eta)$.

Fig. 4. (Color online) Variation of $S_q$ on $f'(\eta)$.

Fig. 5. (Color online) Variation of $M$ on $f'(\eta)$. 
In fact, the wall parallel Lorentz force (resistive force) is stronger near the wall as compared to the central region of the flow. So that decrease in the flow velocity in the region bounded by walls will balance the increase in the velocity field within the central region give rise to the cross flow behavior which is expected in MHD flow. Fig. 6 presents that an increment in the Darcy number $Da$ causes horizontal velocity to dominate immediate to the lower plate i.e. in the region $0 \leq \eta \leq 0.5$ while reduction is observed towards the upper plate $0.5 \leq \eta \leq 1.0$. Physically, parameter $Da$ represents the resistance to the flow which decreases the fluid velocity towards the upper wall. Fig. 7 exhibits the variation in horizontal velocity field corresponds to the local inertia coefficient parameter $\alpha_1$. It is seen that higher local inertia coefficient parameter decays the horizontal velocity component in the region $0 \leq \eta \leq 0.5$ due to the increase in porosity results reduction in pore velocity while opposite behavior is featured in the region $0.5 \leq \eta \leq 1$. Fig. 8 indicates the variation of $x$ and $\eta$ on horizontal velocity component $u$. It is observed that an increment in $x$ causes to enhance the stretching velocity at the left plane while it is minimum at the right plane of the channel.

### 4.2. Temperature distribution

Figure 9 examines the variation of the squeezing parameter ($S_q$) on temperature field. It describes the reduction in the temperature for the greater squeezing parameter. Here when plates get closer, the temperature is relatively large, increasing ($S_q$) decays kinematic viscosity and as a results temperature profile decreases. Fig. 10 discloses the enhancing behavior of the temperature with magnetic parameter $M$. It is noticed that temperature field dominant for growing $M$. The physics behind is that the fluid resistance increases due to the resistive forces (Lorentz forces) because of an increment in the applied magnetic field which leads to enlargement in the temperature. Fig. 11 represents the impact of Eckert number ($Ec$) on the fluid temperature. It is found that the temperature field hike for higher Eckert number. Physically, larger Eckert number increases the kinetic energy of the fluid particles which strengthen the temperature.
The features of Biot number \( (B_i) \) on the temperature field is reflected in Fig. 12. An enlargement in the Biot number leads to strengthen the temperature profile. Physically, it justifies that for larger \( (B_i) \) the convective heating at the lower plate increases due to which the transfer of the heat to the fluid increases and as a result temperature profile grows up.

4.3. Concentration distribution

The variation in the concentration distribution for Schmidt number \( Sc \) is suggested in Fig. 13. It is evident that an increment in the Schmidt number results reduction in concentration profile \( (\phi) \) of specie \( A \). As the Schmidt number increases, the mass diffusivity decays which results away movement of the particles and consequently concentration decays. Fig. 14 discloses the variation of strength of the heterogeneous reaction parameter \( k_2 \) versus concentration distribution. Enlarge \( k_2 \) diminishes the concentration. In fact, \( k_2 \) relates inversely to the diffusion coefficient so that the reaction rate grows up for weaker diffusion rate. Hence specie’s concentration decays.
15 represents the behavior of strength of the heterogeneous reaction parameter $k_1$ on the concentration distribution. Hence larger estimation of $k_1$ leads to decays the concentration of the flow field because the reactants are consumed in the homogeneous reaction. Fig. 16 portrays the features of Schmidt number $Sc$ on the concentration rate $g(\eta)$. An increment in Schmidt number causes to strengthen the concentration rate. As expected that higher $Sc$ has larger mass diffusivity of concentration $(b)$ of specie $B$ which contributing rapid diffusion of mass. Hence the concentration rate $g(\eta)$ increases. Analysis of the ratio of the mass diffusion coefficient $\delta_1$ on the concentration field of specie $A$ is demonstrated in Fig. 17. It is found that concentration increases when $\delta_1$ increases because mass diffusivity rises for increasing $\delta_1$. In fact larger values of $\delta_1$ strengthen the diffusion process inside the fluid. Hence concentration profile grows. Fig. 18 reflected the features of the ratio of mass diffusion coefficient $\delta_1$ on concentration of specie $B$. It is noted that concentration distribution reduces for larger $\delta_1$. Physically, larger vales of $\delta_1$ yields lower mass diffusivity which leads to weak concentration distribution.

4.4. Skin friction coefficient and Nusselt number

Figure 19 reflects the variation of squeezing parameter $S_q$ and magnetic parameter $M$ on the skin friction coefficient $Cf$. It is evident that skin friction co-efficient decays with increment in $S_q$ whereas it enhances with $M$. Physically it justified that larger $S_q$ provides more momentum transfer to the working fluid due to squeezing force and as the result the $Cf$ decreases. Moreover, the magnetic parameter $M$ depends on Lorentz forces (resistive

![Fig. 16. (Color online) Variation of $Sc$ on $g(\eta)$.](image1)

![Fig. 17. (Color online) Variation of $\delta_1$ on $\phi(\eta)$.](image2)

![Fig. 18. (Color online) Variation of $\delta_1$ on $g(\eta)$.](image3)

![Fig. 19. (Color online) 3D plot of $Cf$ with $S_q$ & $M$.](image4)

![Fig. 20. (Color online) 3D plot of $Nu$ with $Pr$ & $B_i$.](image5)
force) which enhance the viscous forces and consequently skin friction co-efficient increases. Influence of Prandtl number $Pr$ and Biot number $B_i$ on Nusselt number is reflected in Fig. 20. It is noted that Nusselt number enhances with an increment in Prandtl number $Pr$ and Biot number $B_i$. In fact enlargement in Prandtl results enhancement in thermal conductivity which is responsible for more heat transfer. Hence Nusselt number enhances. Further, Nusselt number is analyzed to increase for larger values of $B_i$ which results due to sturdy thermal convection.

5. Closing Remarks

MHD squeezing flow of viscous fluid incorporates with viscous dissipation, Joule heating and homogeneous-heterogeneous reactions through non-Darcy porous medium are demonstrated. The conclusions drawn as follows:

- Larger magnetic parameter, Eckert and Biot numbers strengthen the temperature field.
- Dominant values of strength of homogeneous-heterogeneous reaction parameters are responsible for lower temperature field.
- Concentration of species $A$ and $B$ show opposite flow behavior for larger ratio of mass diffusion coefficient.
- Higher Schmidt number $(Sc)$ depicts a lower concentration distribution $(a)$ for specie A whereas concentration $(b)$ of specie B shows increasing trend for larger.

References