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Impact of Cattaneo-Christov Heat Flux Model on the Flow of Maxwell Ferromagnetic Liquid Along a Cold Flat Plate Embedded with Two Equal Magnetic Dipoles

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The target of the current study is to inspect theoretically two-dimensional boundary layer flow of a Maxwell ferromagnetic fluid toward a flat plate. An external magnetic field due to two equal line dipole which are equidistant from the wall and perpendicular to the flow plane is applied. Cattaneo-Christov heat flux model is utilized in modified form of Fourier's Law to disclose the heat transfer characteristic. Governing flow problem is normalized into ordinary differential equation by adopting similarity transform procedure. The solution of resulting non-linear ODE's are solved by shooting technique based on Runge-Kutta algorithm with the help of MATLAB. Characteristic of sundry parameter like magneto-thermomechanical (ferrohydrodynamic) interaction parameter, dimensionless thermal relaxation, Prandtl number and Debora number on velocity and temperature profile are displayed via graphs and in tabular form. It is also pointed out that temperature profile suppresses by varying values of the thermal relaxation time and Prandtl number and increasing behaviour is seen against ferrohydrodynamic interaction. Present numerical results are compared with those published previously in the literature for the case of Newtonian fluid ($\alpha_1 \to 0$) and found an excellent agreement.

Keywords : Ferrofluid, two line currents, Cattaneo-Christov heat flux model, thermal relaxation, Maxwell fluid

1. Introduction

Investigation of heat transfer in boundary layer flow is phenomenon observed in the nature because of temperature difference between objects inside the same body. It has a remarkable practical application like energy fabrication, heat exchangers for the packed bed, drying technology, atomic reactor cooling, catalytic reactors, biomedical applications, such as, heat conduction in tissues and medication has been focused by many scientists [1-3]. The heat transfer characteristic was first proposed by Fourier [4] to forecast the heat transfer mechanism in various connected circumstances. However, one of the significant drawback of this model is that it produces a parabolic energy transport for temperature profile and consequently it has weakness in the sense that any unsettling influence is lost. To overcome that paradox Cattaneo [5] remodel the Fourier law by including thermal relaxation term to indicate the "thermal inertia". The expansion of thermal relaxation time converts the energy transport in form of thermal waves with finite speed. The material-invariant formula of Cattaneo's law through Oldroyd's upper-convected model has been established by Christov [6]. Ciarletta and Straughan [7] confirmed uniqueness and mechanical immovability of the solution by utilizing Cattaneo-Christov heat flux model. Han *et al.* [8] scrutinised heat transfer characteristics of viscoelastic liquid in view of Cattaneo-Christov heat flux model enclosed by stretching plate with velocity slip effect. Recently, Mustafa [9] investigated rotating viscoelastic flow over a stretchable surface by employing Cattaneo-Christov heat flux model. He establishes that temperature of the fluid is inversely proportional to thermal relaxation time.

All the works mentioned previously are restricted to clean fluids, but the purpose of the current study is to explore the two-dimensional parallel flow of a Maxwell ferromagnetic liquid flow toward a flat plate with external magnetic field due to two equally line current dipoles using Cattaneo-Christov heat flux. In fact, a ferrofluid is a

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liquid which is highly magnetized when an external magnetic field is applied. Ferrofluids were initially created and characterized in 1963 by Pappell [10] at NASA. He establishes ferrofluid as a liquid rocket fuel that could be pinched near a pump inlet in a weightless atmosphere by using external magnetic field. Ferrofluids are micron sized colloidal magnetic nanoparticles scattered and suspended consistently in a single domain non-magnetic carrier fluid [11]. These particles have an average size of about 10 nm. It has remarkable applications aimed the recent years because of its significance in micro electro mechanical system (MEMS), purification of molten metals, shock absorbers, coolers of nuclear reactor, microfluidic actuators, leak-proof seals, microfluidic valves and pumps, lithographic patterning and many others [12-15]. The application of such flow coupled with stretching sheet has been an active research field [16-21].

The above-mentioned studies inspire the present work, the goal is to examine two-dimensional Maxwell fluid saturated with ferromagnetic nanoparticles flowing due to stretched surface with the appearance of two-line dipole current, which has not been discussed so far. We used modified Cattaneo's flux model to explore the heat transfer characteristics. Modelled equations have been solved numerically by adopting Runga-kutta integration scheme. The behavior of numerous sundry parameters appearing in the problem statement are discussed in detail with the help of graphs.

2. Mathematical Formulation

2.1. Magnetic Dipole

Magnetic scalar potential $\phi(\mathrm{H} = -\nabla\phi)$ at any point (x, y) under the influence of dipole are taken as

$$\phi = -\frac{I_0}{2\pi}\left[Tan^{-1}\left(\frac{y+d}{x}\right) + Tan^{-1}\left(\frac{y+d}{x}\right)\right] \tag{1}$$

where I_o is the strength of magnetic field and the corresponding components along the coordinate axes are H_x and H_y.

$$H_x = -\frac{\partial\phi}{\partial x} = -\frac{I_0}{2\pi}\left[\frac{y+d}{x^2+(y+d)^2} + \frac{y-d}{x^2+(y-d)^2}\right] \tag{2}$$

$$H_x = -\frac{\partial\phi}{\partial y} = -\frac{I_0}{2\pi}\left[\frac{x}{x^2+(y-d)^2} + \frac{x}{x^2+(y+d)^2}\right] \tag{3}$$

The resultant magnitude H of the force field strength is taken as

$$H = \left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2\right]^{\frac{1}{2}} \tag{4}$$

The elements of ∇H in term of scalar potential are

$$(\nabla H)_x = \frac{\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial x^2} + \frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial x\partial y}}{\left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2\right]^{\frac{1}{2}}} \tag{5}$$

$$(\nabla H)_y = \frac{\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial x\partial y} + \frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial y^2}}{\left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2\right]^{\frac{1}{2}}} \tag{6}$$

Utilizing equation (1) in Eqs. (5) and (6) and expanding up to order x^2, we get

$$(\nabla H)_y = 0. \tag{7}$$

Since, at the surface of wall $\left(\frac{\partial\phi}{\partial x}\right)_{y=0} = \left(\frac{\partial^2\phi}{\partial y^2}\right)_{y=0} = 0$ and It is assumed that $x \gg d$ so from equation (5) we have

$$(\nabla H)_x = -\frac{I_0}{\pi}\frac{1}{x^2}. \tag{8}$$

Also, the change in magnetization $\boldsymbol{M}$ can be taken as a linear function of temperature T [15].

$$M = K^*(T_\theta - T), \tag{9}$$

where, K^* is a pyro magnetic constant and T_θ is Curie temperature.

2.2. Flow analysis

Consider an incompressible flow of Maxwell ferromagnetic fluid over a flat surface with line dipole, which are

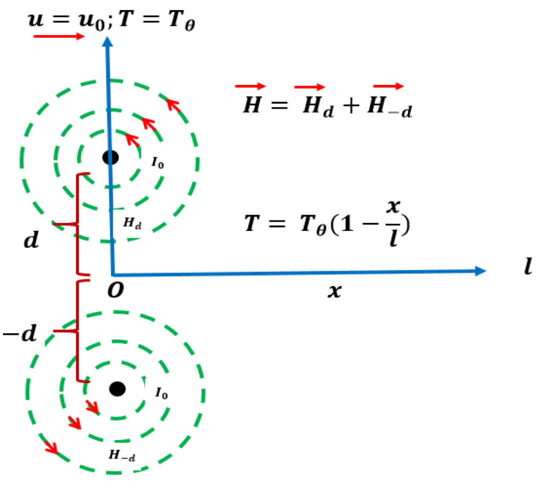

Fig. 1. (Color online) Geometry of the flow model.

equidistant d from the wall and perpendicular to the plane as shown in Fig. 1. Temperature of the wall varies linearly with length is taken as $T_w = T_\theta(1-x/l)$, where l is the length of the plate ($l >> d$). Heat transfer analysis are deliberated, because the impact of applied force field on the flow is restricted to a small region which in close to the wall where temperature T of the fluid is reasonably lower then Curie temperature ($T < T_\theta$).

2.3. Ferromagnetic flow equations

The governing equations reporting the physical situation of the flow problem in term of Maxwell ferrofluid can be expressed as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{10}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right)=\frac{\mu_0}{\rho}M\frac{\partial H}{\partial x}+v\frac{\partial^2 u}{\partial y^2} \tag{11}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=-\nabla .q, \tag{12}$$

where component of velocity along x and y direction are u and v respectively, λ_1 is relaxation time, T is fluid temperature, ν is kinematic viscosity of fluid, ρ is density of the fluid, μ_0 is magnetic permeability, μ is dynamic viscosity, c_P is specific heat, H is magnetic field strength, M is magnetization.

The heat flux model [6] for incompressible fluid taken as:

$$q+\lambda_2\left(\frac{\partial q}{\partial t}+\mathrm{V}.\nabla q-q.\nabla \mathrm{V}\right)=-k\nabla T, \tag{13}$$

where q is heat flux, k is thermal conductivity, λ_2 is the relaxation time. When $\lambda_2 = 0$, Eq. (13) is reduced to Fourier's law.

By eliminating q between Eqs (12) and (13), the transformed equation for temperature is given by:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_2\left(\begin{array}{l} u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y}+u\frac{\partial v}{\partial x}\frac{\partial T}{\partial y} \\ +v\frac{\partial u}{\partial y}\frac{\partial T}{\partial x}+2uv\frac{\partial^2 T}{\partial x\partial y}+u^2\frac{\partial^2 T}{\partial x^2}+v^2\frac{\partial^2 T}{\partial y^2}\end{array}\right) =\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}, \tag{14}$$

The corresponding boundary relations imposed to the present study are given by

$$u=v=0,\quad T=T_\theta\left(1-\frac{x}{l}\right)\quad \text{at } y=0 \tag{15}$$

$$u=u_0,\quad T=T_\theta \quad \text{at } y=\infty \tag{16}$$

3. Solution Procedure

Using the following similarity transformation [15]

$$\eta=\left(\frac{u_0}{\nu x}\right)^{\frac{1}{2}}y,\ u=u_0 f'(\eta),$$

$$v=\left(\frac{\nu u_0}{x}\right)^{\frac{1}{2}}\left[\frac{\eta f'(\eta)}{2}-\frac{f(\eta)}{2}\right],\ T=T_\theta\left[1-\frac{x}{l}\theta(\eta)\right], \tag{17}$$

By substituting Eq. (17) into the Eq's. (11) and (14), we obtained the resulting ordinary differential equations

$$f'''\left(1-\frac{\alpha_1 f^2}{2}\right)+\frac{ff''}{2}-\frac{\alpha_1}{4}(f'^2 f''\eta+2ff'f'')-\beta\theta=0, \tag{18}$$

$$\frac{1}{\mathrm{Pr}}\theta''+\frac{f\theta'}{2}-f'\theta+\frac{\alpha_2}{4}(2ff''+2ff'\theta'-f^2\theta'')=0, \tag{19}$$

The corresponding relevant boundary given in Eqs. (15)-(16) takes the form

$$\left.\begin{array}{l} f=f'=0,\quad \theta=1,\ \text{ at } \eta=0 \\ f'\to 1,\quad \theta\to 0,\ \text{ as } \eta\to\infty \end{array}\right\} \tag{20}$$

where $\beta=\frac{I_0\mu_0 KT_\theta}{\pi\rho l u_0^2}$ is the ferrohydrodynamic interaction parameter, $Pr=\frac{\rho c_p \nu}{k}$ is Prandtl number, $\alpha_1 = \lambda_1 u_0 x^{-1}$ is the dimensionless Deborah number in terms of relaxation time and $\alpha_2 = \lambda_2 u_0 x$ is the dimensionless thermal relaxation time. In view of many industrial applications, the friction factor and Nusselt number is also computed. They are defined as

$$C_{fx}=\frac{2\tau_w}{\rho u_0^2},\ Nu_x=\frac{xq_w}{k(T_\theta-T_w)}, \tag{21}$$

where shear stress is τ_w and heat flux is q_w

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\ q_w=-\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{22}$$

Non-dimisionalizing the Eqs. (21) and (22) and combining. we develop the following relations.

$$C_f Re_x^{1/2}=f''(0),\ Nu_x Re_x^{-1/2}=\theta'(0) \tag{23}$$

where $Re_x=\frac{\rho u_0 x}{\mu}$ is Reynolds number.

4. Numerical Results and Discussion

In this section, we have explored the different rheological aspects of the problem, the behavior of numerous

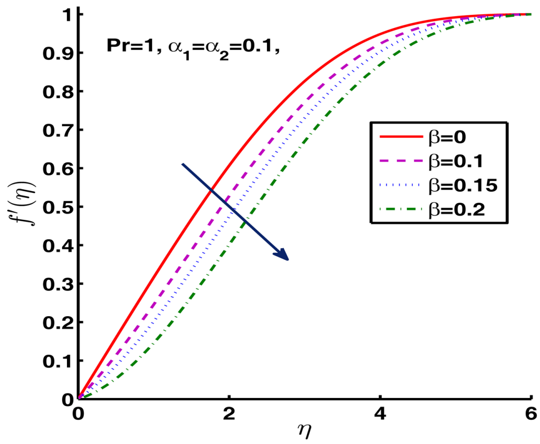

Fig. 2. (Color online) Influence of β on $f'(\eta)$.

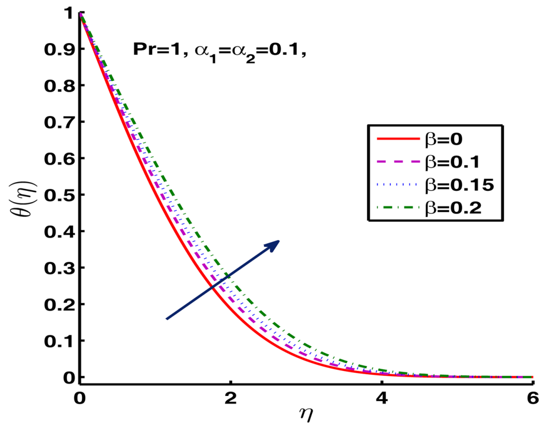

Fig. 3. (Color online) Influence of β on $\theta(\eta)$.

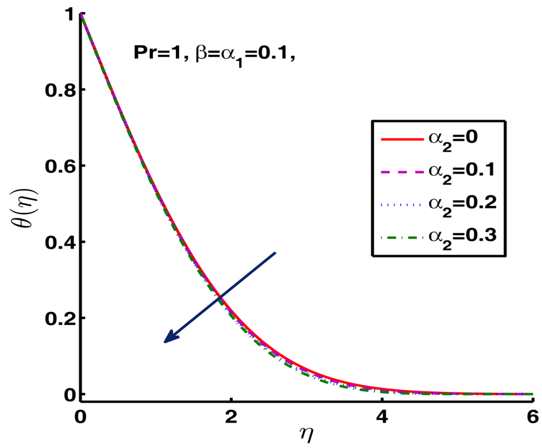

Fig. 4. (Color online) Influence of α_2 on $\theta(\eta)$.

governing parameters involved in velocity and temperature profile are illustrated in Figs. 2 to 8. For verification and efficiency of the numerical procedure, evaluation of our present results corresponding to local Nusselt number $-\theta(0)$ in case of Newtonian fluid ($\alpha_1 = 0$) are compared with those reported by [15] for several values of β are presented in Table 1 and found an excellent agreement with the available data which shows the validity of our numerical scheme.

Figure 2 is displayed to highlight impact of ferromagnetic interaction parameter $0 \leq \beta \leq 0.25$ on velocity field. It is simply perceived from the figure that in the occurrence of magnetic dipole velocity profile reduces for large values of β, this happen owing to the presence of Lorentz force, which opposes the flow and this force has tendency to develop a resistance and enhance the temperature field. Consequently, flattening the axial velocity $f'(\eta)$. It is more stimulating to observe that the occurrence of applied magnetic field reduces velocity as compared with the hydrodynamic case ($\beta = 0$) because there is an interference between the fluid motion and the stroke of magnetic field.

Table 1. Comparison of Skin friction and Nusselt number when Pr = 10, α_1 = 0, α_2 = 0.

β	$f''(0)$ [15]	$f''(0)$ Present result	$\theta'(0)$ [15]	$\theta'(0)$ Present result
0.0	0.3320	0.3321	1.1756	1.1757
0.05	0.3058	0.3058	1.1557	1.1552
0.10	0.2782	0.2777	1.1338	1.1333
0.15	0.2497	0.2491	1.1102	1.1096
0.20	0.2199	0.2193	1.0845	1.0837
0.25	0.1887	0.1880	1.0561	1.0553
0.30	0.1556	0.1548	1.0244	1.0233
0.35	0.1201	0.1191	0.9881	0.9866
0.40	0.0811	0.0798	0.9449	0.9429

This kind of interference reduces the velocity and rising the frictional heating within the fluid layers which are accountable for the increment in thermal boundary layer thickness as displayed in Fig. 3.

Figure 4 illustrates the outcome of thermal relaxation parameter $0 \leq \alpha_2 \leq 0.4$ on temperature field. It is obviously noticeable that temperature of the fluid decreases by increasing thermal relaxation parameter α_2. The physical thinking behind it is that by increasing the value of α_2, material particles need additional time to exchange energy to their neighbouring particles. Thus, for larger value of relaxation parameter α_2 causes reduction in temperature profile and opposite trend is noted for velocity profile as illustrated in Fig. 5.

Figure 6 discloses the behaviour of relaxation time α_1 on velocity profile. It is prominent that velocity profile indicates decreasing behaviour corresponding to larger values of Deborah number. This is because ratio of relaxation to observation times represent the Deborah number. So, relaxation time is upshot by increasing

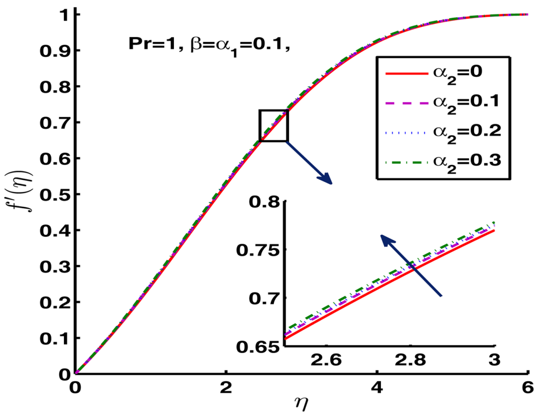

Fig. 5. (Color online) Influence of α_2 on $f'(\eta)$.

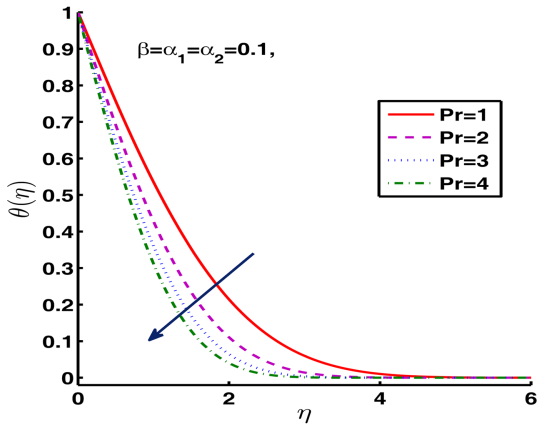

Fig. 8. (Color online) Influence of Pr on $\theta(\eta)$.

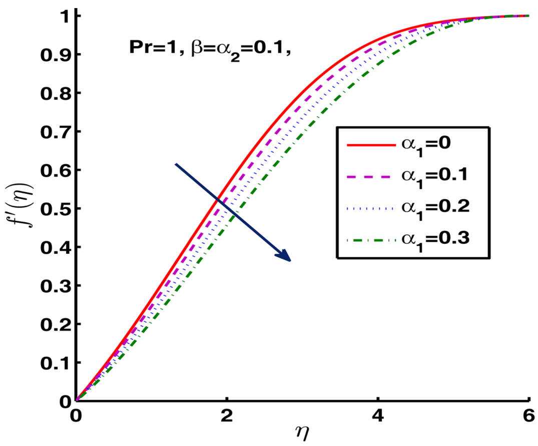

Fig. 6. (Color online) Influence of α_1 on $f'(\eta)$.

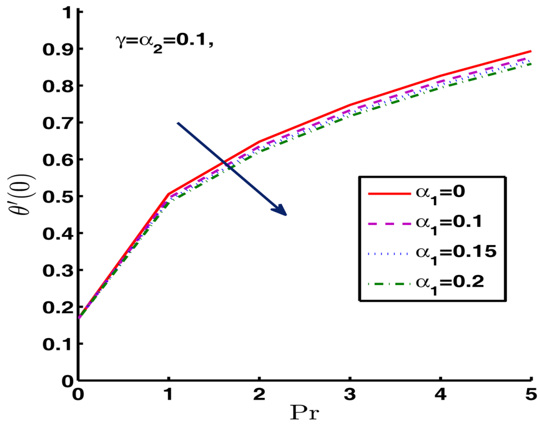

Fig. 9. (Color online) Influence of α_1 on $\theta'(\eta)$.

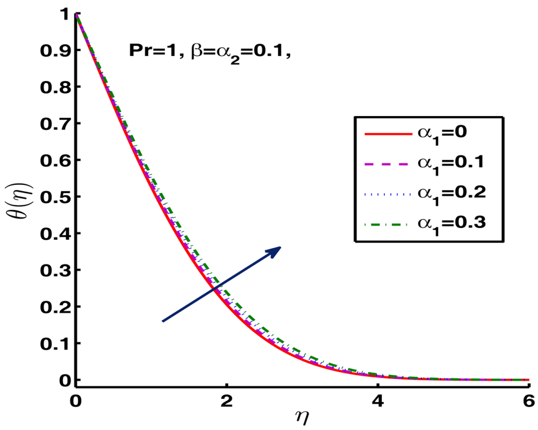

Fig. 7. (Color online) Influence of α_1 on $\theta'(\eta)$.

Deborah number which gives extra imperviousness to the motion of the fluid. Hence boundary layer thickness also decreases.

Figure 7 illustrate the physical characteristics of Deborah number α_1 on temperature field $\theta(\eta)$. One can observe that larger values of Deborah number leads to increases fluid temperature, because higher Deborah number relates to larger relaxation time which conveys opposition to the fluid motion and as a result heat is generated and therefore increases thermal boundary layer thickness. Figure 8 is sketched to discuss the behaviour of $0 \le Pr \le 5$ on temperature field. It is concluded that higher values of Prandtl number lead to decline in temperature distribution. From physical point of view Pr is the proportion of momentum diffusivity to thermal diffusivity. Reduction in thermal diffusivity consequently decreases the temperature field. In addition, it is seen that for greater Pr, heat diffuses slowly, so thinning the thermal boundary layer as contrasted to low Pr. Figure 9 shows the impact of Deborah number $0 \le \alpha_1 \le 0.5$ on Nusselt number versus Prandtl number. It is evident that rate of heat transfer increases by rising Prandtl number while reverse trend is detected for higher values of α_1.

5. Concluding Remarks

In this article, we investigate two-dimensional incompressible parallel flow of Maxwell fluid saturated with ferromagnetic liquid over a flat surface under the influence of two point line dipole. Some of the important judgements of the present problem is as follows:

- Velocity of the fluid reduces, while temperature increases by increasing the values of ferromagnetic interaction parameter β.
- Temperature profile indicate falling behavior of Cattaneo-Christov model [6] as compared with Fourier's law.
- For large values of Deborah number α_1 velocity profile decreases.
- Nusselt number enhances with the variation of Prandtl number Pr and reverse trend is examined for α_1.

References

[1] A. A. Khan, S. Muhammad, R. Ellahi, and Q. Z. Zia, J. Magn. **21**, 273 (2016).
[2] F. Selimefendigil, and H. F. Öztop, Eng. Sci. Tech. Int. J. **18**, 439 (2015).
[3] R. Ellahi, M. H. Tariq, M. Hassan, and K. Vafai, J. Mol. Liq. **229**, 339 (2017).
[4] J. B. J. Fourier, Theorie analytique De La chaleur, Paris (1822).
[5] C. Cattaneo, Atti Sem. Mat. Fis. Univ. Modena **3** (1948), 83-101.
[6] C. I. Christov, Mech. Res. Commun. **36**, 481 (2009).
[7] M. Ciarletta and B. Straughan, Mech. Res. Commun. **37**, 445 (2010).
[8] S. H. Han, L. C. Zheng, C. R. Li, and X. X. Zhang, Appl. Math. Lett. **38**, 87 (2014).
[9] M. Mustafa, AIP Advances **5**, 047109 (2015).
[10] S. S. Papell, United States Patent Office Filed Ser. 315-1963 096 (1965).
[11] R. E. Rosensweig, Ferrohydrodynamics, Courier Corporation (2013).
[12] A. Jafari, T. Tynjala, S. Mousavi, and P. Sarkomaa, Comput. Fluids **37**, 1344 (2008).
[13] C. Vales-Pinzon, R. Alvarado-Gil, Medina-Esquivel, and P. Martinez-Torres, J. Magn. Magn. Mater. **369**, 114 (2014).
[14] B. B. Yellen and G. Friedman, Nanotechnology **15**, S562 (2004).
[15] J. L. Neuringer, Int. J. Non-linear Mech. **1**, 123 (1966).
[16] M. S. Kandelousi and R. Ellahi, Zeitschrift für Naturforschung A **70**, 115 (2015).
[17] A. Majeed, A. Zeeshan, and R. Ellahi, J. Mol. Liq. **223** 528 (2016).
[18] F. Selimefendigil, H. F. Öztop, and K. Al-Salem, J. Magn. Magn. Mater. **372**, 122 (2014).
[19] M. Sheikholeslami and R. Ellahi, Appl. Sci. **5**, 294 (2015).
[20] M. M. Bhatti, A. Zeeshan, and R. Ellahi, Comp. in Bio. and Medic. **78**, 29 (2016).
[21] F. Selimefendigil and H. F. Öztop, Int. J. Ther. Sci. **86**, 258 (2014).