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Solitary Wave Solutions for MHD Flow of Viscous Fluid through Convergent or Divergent Channel

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Nonlinear mathematical problems and their solutions are of great importance in solitary waves. In soliton theory, an efficient tool to attain various type of soliton solutions is the Exp-function technique. This paper aims at finding soliton wave solutions of a viscous incompressible fluid through convergent or divergent channel in the existence of a magnetic field. By an appropriate use of local similarity transformation we obtained an ordinary differential equation. It is solved by Exp-function technique. It is observed that under discussion technique is user friendly that requires minimum computational work. Also we can extend it for physical problems of different nature as well.

Keywords : Magnetic field, non-linear differential equations, Exp-function method, viscous fluid, Maple 13.

1. Introduction

The most important problem in the fluid mechanic is the flow between converging or diverging channels with magnetic field effects, which is, due to its many applications arising in chemical, aerospace, biomechanical, civil and environmental engineering. Motion in channels and rivers are also the examples of such type of flow. Hamel and Jeffery were the first to explore such type of flow problems mathematically [1, 2]. Since then, many other researchers have studied Jeffery-Hamel flow problem [3-8].

Since the recent few years, we have observed an extraordinary progress in the soliton theory. Solitons have been studied by many mathematicians, physicists and engineers for their applications in physical phenomenon. First of all, soliton waves were observed by an engineer John Scott Russell. Soliton is a self-reinforcing solitary wave that maintains its shape while it moves at a constant velocity. It is a localized wave which preserves its identity upon its interaction with other solitons. Solitons are caused by a cancellation of nonlinear and dispersive effects in the medium. In non-dispersive media, nonlinearity may be compensated by viscous dissipation, which results in generation of dissipative solitary waves.

Wide range of phenomenon are modeled by differential equations in mathematics and physics. In nonlinear sciences, it is of great importance and interest to explain physical models and attain analytical solutions. At present, valuable progress has been made in the field of physical sciences. The great achievement is the development of various techniques to hunt for exact solitary wave solution of nonlinear differential equations. In nonlinear physical sciences, an essential contribution is the exact solutions. It can lead us to study physical behaviours and to discuss more features of the problem which gives direction to more applications. The search for solution of nonlinear evolution equations, in the form of traveling wave solution is becoming more imperative task for the scientists who are anxious in nonlinear sciences. Obviously, nonlinear evolution equations are too difficult to be solved. In addition, there is no general procedure that is applicable for all such equations. So, each equation has to be studied being considered, as an individual problem. As a result, a lot of work has been done in formulation of several convincing and significant mathematical techniques for the exact soliton solutions.

Recently, He and Wu [9] have organized a very effective method named as Exp-function technique which provides solution to many physical non-linear problems. In-depth study of the literature exhibits that Exp-function

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method is highly reliable and applicable on a vast range of differential equations. The successive works have shown the complete efficiency and reliability of the scheme under study. Mohyud-Din [12-15] enlarged the same method for nonlinear physical problems including higher-order BVPs and Zhang [20] for higher-dimensional nonlinear evolution equations. He and Wu *et al*. [10, 11] find periodic solutions of evolution equations by using this technique. Yusufoglu [19] solved MBBN equations. Oziz [16] tried this approach for Fisher's equation. Wu *et al.* [17, 18] extended compaction-like and periodic solutions of differential equations. Zhu [21, 22] solved the discrete mKdV lattice and Hybrid-Lattice system. Momani [24] found numerical and explicit solutions of the fractional KdV equation. Kudryashov [23] found exact soliton solutions of the wave dynamics equation. For recent results on exact soliton solutions and fluid flow models, see the references [26-34]. It is to be emphasized that Ebaid [25] has proved that by applying Exp-function technique to any nonlinear ordinary differential equation, $t = i$ and $l = j$ are the unique relations that can be obtained.

The elementary inspiration is that we use Exp-function method to study solitary wave phenomena of the MHD flow of incompressible viscous fluid. In this method, a trial solution is assumed for solitary wave's formation. It is noticed that this technique is highly efficient, totally compatible and extremely dependable for nonlinear differential equations and can be extended to other physical models arising in mathematical engineering, plasma physics, fluid mechanics and applied sciences.

2. Mathematical Formulation

Consider a two-dimensional steady incompressible viscous fluid flow in cylindrical polar coordinates (r, θ, z), where a source or sink at channel walls intersects in z-axis and lies in plane. Now, we assume that $v_\theta = 0$. It means that with respect to z-direction there is no change. Thus, the flow is purely depending on r and θ. There is no magnetic field along z-axis and is completely in radial direction. The polar form of Maxwell's equation of continuity and Navier-Stokes equation in lowest form is given as:

$$\rho\frac{\partial}{r\partial r}\left[rv(r,\theta)\right] = 0, \tag{1}$$

$$v(r,\theta)\frac{\partial v(r,\theta)}{\partial r} = -\frac{1}{\rho}\frac{\partial F_p}{\partial r} + \mu\left[\begin{array}{c}\frac{\partial^2 v(r,\theta)}{\partial r^2}+\frac{1}{r}\frac{\partial v(r,\theta)}{\partial r}\\ +\frac{1}{r^2}\frac{\partial^2 v(r,\theta)}{\partial\theta^2}-\frac{v(r,\theta)}{r^2}\end{array}\right] - \frac{\eta E^2}{\rho r^2}v(r,\theta), \tag{2}$$

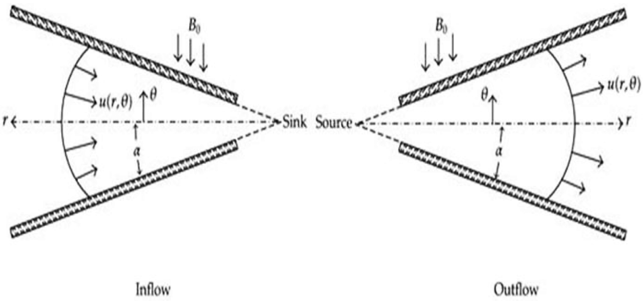

Fig. 1. MHD flow through convergent or divergent channels.

$$\frac{1}{\rho r}\frac{\partial F_P}{\partial\theta} - \frac{2\mu}{r^2}\frac{\partial v(r,\theta)}{\partial\theta} = 0. \tag{3}$$

Where μ is the coefficient of kinematic viscosity, v is velocity along radial direction, ρ is fluid density, F_p is fluid pressure, E is the electromagnetic induction strength and η represents conductivity of the fluid (Fig. 1):

From equation (1), we get

$$h(\theta) = rv(r,\theta). \tag{4}$$

Using dimensionless parameter $\xi = \frac{\theta}{\beta'}$, where β is the semi-angle joining the inclined walls.

$$h(\xi) = \frac{h(\theta)}{h_{\max}}. \tag{5}$$

Using equation (5) into equation (3) and (2), we get

$$h'''(\xi) + 2\beta.\mathrm{Re}.h(\xi).h'(\xi) + (4 - H).\beta^2.h'(\xi) = 0. \tag{6}$$

Where $H = \sqrt{\frac{\eta E^2}{\rho\mu}}$ denotes Hartman number and Re denotes the Reynolds number are defined as:

$$\mathrm{Re} = \frac{h_{\max}\beta}{\mu}\{divergent-channel: \beta > 0, h_{\max} > 0\}$$

$$\mathrm{Re} = \frac{h_{\max}\beta}{\mu}\{convergent-channel: \beta < 0, h_{\max} < 0\}$$

Subjected to boundary conditions

$$h(0) = 1, h'(0) = 0, h(1) = 0. \tag{7}$$

3. Analysis of Technique

Now we consider the general non-linear ordinary differential equation of the form:

$$P(v, v', v'', v''', v^{(iv)}, \cdots) = 0. \tag{8}$$

Where prime shows derivative of v w.r.t. ξ.

Corresponding to Exp-function technique, the assumed soliton solution is taken in the form given as:

$$h(\xi) = \frac{\sum_{b=-i}^{t} p_b \exp(b\xi)}{\sum_{a=-j}^{l} q_a \exp(a\xi)}. \tag{9}$$

Where l, j, t and i are the positive integers which are to be further find out, p_b and q_a are unknown constants which are to be further determined. We can write equation (9) again in the following equivalent form:

$$h(\xi) = \frac{p_t \exp(t\xi) + \cdots + p_{-i} \exp(-i\xi)}{q_l \exp(l\xi) + \cdots + q_{-j} \exp(-j\xi)}. \tag{10}$$

This identical formulation plays a fundamental and important rule for determining the soliton solution of problems. To find the value of t and l by using [25], we have

$$l = t, j = i. \tag{11}$$

4. Solution Procedure

The resulting non-linear differential equation is given as:

$$h'''(\xi) + 2\beta.\mathrm{Re}.h(\xi).h'(\xi) + (4 - H).\beta^2.h'(\xi) = 0, \tag{12}$$

With the boundary conditions

$$h(0) = 1, h'(0) = 0, h(1) = 0. \tag{13}$$

Where the prime represents the derivative of h with respect to ξ.

The solution of the equation (12) can be written in the form of equation (10).

We can frequently select the values of t and i, but we will understand that the final solution does not depend upon the selection of values of l and j.

Case I. For ease, we set $l = t = 1$ and $j = i = 1$ equation (9) reduces to:

$$h(\xi) = \frac{p_1 \exp(\xi) + p_0 + p_{-1} \exp(-\xi)}{q_1 \exp(\xi) + q_0 + q_{-1} \exp(-\xi)}. \tag{14}$$

Using equation (14) into equation (12), we get

$$\frac{1}{A}\begin{bmatrix} t_4 \exp(4\xi) + t_3 \exp(3\xi) + t_2 \exp(2\xi) \\ + t_1 \exp(\xi) + t_0 + t_{-1} \exp(-\xi) + t_{-2} \exp(-2\xi) \\ + t_{-3} \exp(-3\xi) + t_{-4} \exp(-4\xi) \end{bmatrix} = 0. \tag{15}$$

Where $A = (q_1 \exp(\xi) + q_0 + q_{-1} \exp(-\xi))^4$, t_i are constants that we get by Maple 13. By equating the coefficients of $\exp(b\xi)$ to zero, we get

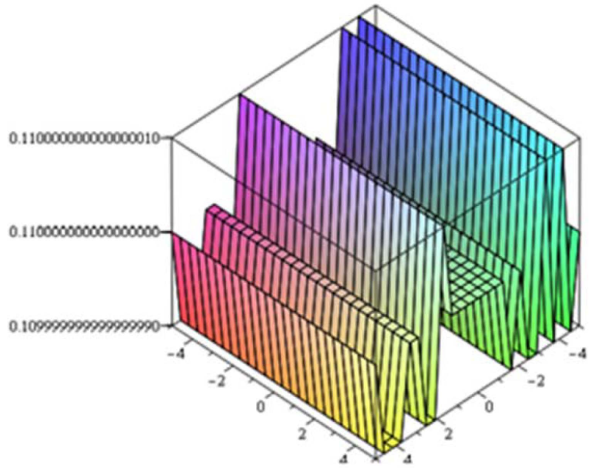

Fig. 2. (Color online) Soliton solutions of (12) for 1st solution set.

$$\begin{Bmatrix} t_{-4} = 0, t_{-3} = 0, t_{-2} = 0, t_{-1} = 0, t_0 = 0, \\ t_1 = 0, t_2 = 0, t_3 = 0, t_4 = 0 \end{Bmatrix}. \tag{16}$$

We get following solution sets which satisfy equation (12).

1st solution set:

Consider

$$p_{-1} = 0, p_0 = p_0, p_1 = p_1, q_{-1} = 0, q_0 = q_0, q_1 = \frac{q_0 p_1}{p_0}.$$

Therefore, we get the following generalized solution $h(\xi)$ of equation (12). (Fig. 2):

$$h(\xi) = \frac{q_0 + p_1 e^{\xi}}{q_0 + \frac{q_0 p_1}{p_0} e^{\xi}}.$$

2nd solution set:

Consider

$$p_{-1} = \frac{p_0 q_{-1}}{q_0}, p_0 = p_0, q_{-1} = q_{-1}, q_0 = q_0, q_1 = q_1,$$

$$p_1 = \frac{1}{-500\xi q_{-1}^2 p_0 + q_{-1}^2 q_0 (5H\xi^2 - 20\xi^2 - 5)} \left(\begin{array}{l} \dfrac{300 p_0^2 q_{-1}^2 q_1 \xi}{q_0} + \dfrac{1}{q_0} \begin{pmatrix} (200\xi q_{-1} q_1 + 100\xi q_0^2) p_0 \\ + q_{-1} q_0 q_1 (-6H\xi^2 + 24\xi^2 - 18) \\ + (q_0^3 (-H\xi^2 + 4\xi^2 + 1)) p_0 q_{-1} \end{pmatrix} \\ -100\xi p_0^2 q_{-1} q_0 + \begin{pmatrix} q_{-1}^2 q_1 (H\xi^2 - 4\xi^2 + 23) \\ + q_0^2 q_{-1} (H\xi^2 - 4\xi^2 - 1) p_0 \end{pmatrix} \end{array} \right).$$

Therefore, we get the following generalized solution $h(\xi)$ of equation (12). (Fig. 3):

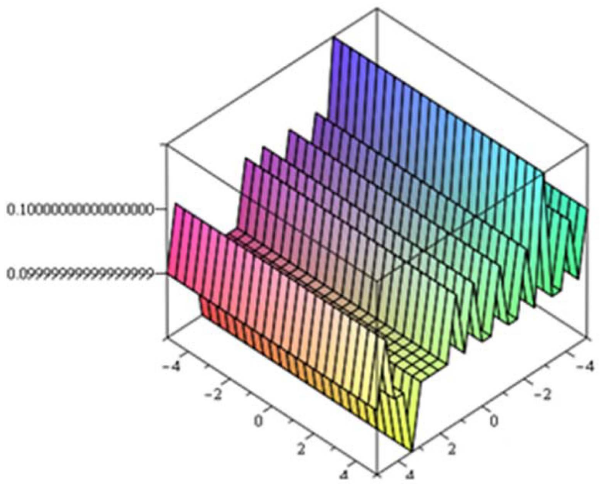

Fig. 3. (Color online) Soliton solutions of (12) for 2nd solution set.

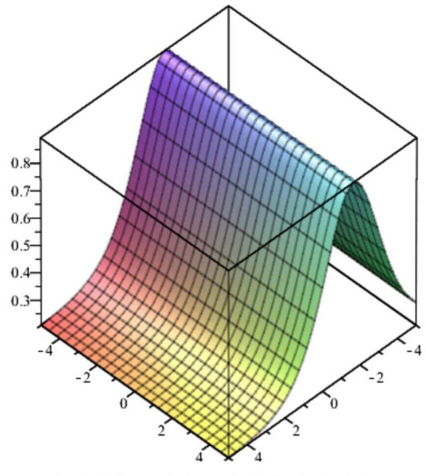

Fig. 4. (Color online) Soliton solutions of (12) for 3rd solution set.

$$h(\xi) = \frac{1}{q_{-1}e^{-\xi} + q_0 + q_1 e^{\xi}} \left(\frac{p_0 q_{-1}}{q_0} e^{-\xi} + p_0 - \frac{1}{-500\xi q_{-1}^2 p_0 + q_{-1}^2 q_0 (5H\xi^2 - 20\xi^2 - 5)} \left(\begin{array}{l} \dfrac{300 p_0^2 q_{-1}^2 q_1 \xi}{q_0} + \dfrac{1}{q_0} \left(\begin{array}{l} (200\xi q_{-1} q_1 + 100\xi q_0^2) p_0 \\ + q_{-1} q_0 q_1 (-6H\xi^2 + 24\xi^2 - 18) \\ + (q_0^3 (-H\xi^2 + 4\xi^2 + 1)) p_0 q_{-1} \end{array} \right) \\ -100\xi p_0^2 q_{-1} q_0 + \left(\begin{array}{l} q_{-1}^2 q_1 (H\xi^2 - 4\xi^2 + 23) \\ + q_0^2 q_{-1} (H\xi^2 - 4\xi^2 - 1) p_0 \end{array} \right) \end{array} \right) e^{\xi} \right).$$

3rd solution set:
Consider

$$p_{-1} = \frac{-q_{-1}(-H\xi^2 + 4\xi^2 + 1)}{100\xi^2}, p_0 = \frac{-6q_0^2 + q_0^2(-H\xi^2 + 4\xi^2 + 1)}{100\xi^2 q_0},$$

$$q_{-1} = q_{-1}, q_0 = q_0, q_1 = \frac{q_0^2}{4q_{-1}}, p_1 = \frac{3q_0^3 q_{-1}(-H + 4\xi^2 + 1)}{400\xi^2}$$

$$-\frac{1}{100\xi} \left(\begin{array}{l} \dfrac{-3q_0}{2}(-6q_0^2 + q_0^2(-H\xi^2 + 4\xi^2 + 1)) - \dfrac{q_0^3(-6H\xi^2 + 24\xi^2 + 18)}{4} \\ + (q_0^3(-H\xi^2 + 4\xi^2 + 1))((-H\xi^2 + 4\xi^2 + 1)q_{-1}) \\ - \dfrac{(-6q_0^2 + q_0^2(-H\xi^2 + 4\xi^2 + 1))^2 q_{-1}}{\xi^2 q_0} - \\ \dfrac{\frac{1}{4}(q_0^2 q_{-1}(-H\xi^2 + 4\xi^2 + 1)) + q_0^2 q_{-1}(-H\xi^2 + 4\xi^2 + 1) + (-6q_0^2 + q_0^2(-H\xi^2 + 4\xi^2 + 1))}{100\left(\begin{array}{l} 2q_0 q_{-1}^2(-H\xi^2 + 4\xi^2 + 1) + \dfrac{3qq_{-1}(-6q_0^2 + q_0^2(-H\xi^2 + 4\xi^2 + 1))}{\xi^2 q_0} \\ + q_0 q_{-1}(-5H\xi^2 + 20\xi^2 + 5) \end{array} \right)} \end{array} \right).$$

Therefore, we get the following generalized solution $h(\xi)$ of the given equation (12). (Fig. 4):

$$h(\xi) = \frac{\dfrac{-q_{-1}(-H\xi^2 + 4\xi^2 + 1)}{100\xi^2} e^{-\xi} - \dfrac{-6q_0^2 + q_0^2(-H\xi^2 + 4\xi^2 + 1)}{100\xi^2 q_0} - \mathrm{K}e^{\xi}}{q_{-1}e^{-\xi} + q_0 + \dfrac{q_0^2}{4q_{-1}} e^{\xi}}.$$

Where

$$\mathrm{K} = \frac{3q_0^3 q_{-1}(-H + 4\xi^2 + 1)}{400\xi^2} - \frac{1}{100\xi} \left(\begin{array}{l} \dfrac{-3q_0}{2}(-6q_0^2 + q_0^2(-H\xi^2 + 4\xi^2 + 1)) - \dfrac{q_0^3(-6H\xi^2 + 24\xi^2 + 18)}{4} + \\ (q_0^3(-H\xi^2 + 4\xi^2 + 1))((-H\xi^2 + 4\xi^2 + 1)q_{-1}) \\ - \dfrac{(-6q_0^2 + q_0^2(-H\xi^2 + 4\xi^2 + 1))^2 q_{-1}}{\xi^2 q_0} - \\ \dfrac{\frac{1}{4}(q_0^2 q_{-1}(-H\xi^2 + 4\xi^2 + 1)) + q_0^2 q_{-1}(-H\xi^2 + 4\xi^2 + 1) + (-6q_0^2 + q_0^2(-H\xi^2 + 4\xi^2 + 1))}{100\left(\begin{array}{l} 2q_0 q_{-1}^2(-H\xi^2 + 4\xi^2 + 1) + \dfrac{3qq_{-1}(-6q_0^2 + q_0^2(-H\xi^2 + 4\xi^2 + 1))}{\xi^2 q_0} \\ + q_0 q_{-1}(-5H\xi^2 + 20\xi^2 + 5) \end{array} \right)} \end{array} \right).$$

Case II. If $l = t = 2$ and $j = i = 1$ then equation (9) becomes

$$h(\xi) = \frac{p_2 \exp(2\xi) + p_1 \exp(\xi) + p_0 + p_{-1}\exp(-\xi)}{q_2 \exp(2\xi) + q_1 \exp(\xi) + q_0 + q_{-1}\exp(-\xi)}. \quad (17)$$

4th solution set:
Consider

$$p_{-1} = \frac{p_{-2}q_{-1}}{q_{-2}}, p_0 = \frac{p_{-2}q_0}{q_{-2}}, p_1 = \frac{p_{-2}q_1}{q_{-2}}, p_2 = p_2,$$
$$q_{-1} = q_{-1}, q_0 = q_0, q_1 = q_1, q_2 = q_2.$$

Therefore we obtain the generalized wave solution $h(\xi)$ of equation (12). (Fig. 5):

$$h(\xi) = \frac{p_2 e^{2\xi} + \dfrac{p_{-2}q_1}{q_{-2}} e^{\xi} + \dfrac{p_{-2}q_0}{q_{-2}} + \dfrac{p_{-2}q_{-1}}{q_{-2}} e^{-\xi}}{q_2 e^{2\xi} + q_1 e^{\xi} + q_0 + q_{-1}e^{-\xi}}.$$

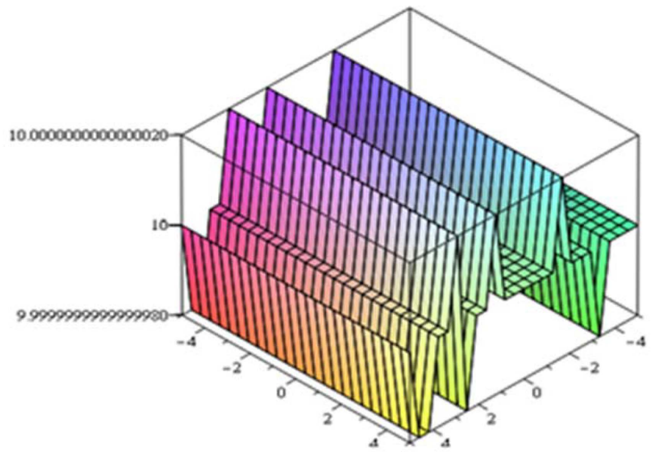

Fig. 5. (Color online) Soliton solutions of (12) for 4th solution set.

5. Results and Discussion

From graphical representations, we note that soliton is a wave which preserves its shape after it has collided with another wave of the same kind. By solving nonlinear differential equation, representing MHD flow of viscous fluid, we attain desired solitary wave solutions for different values of random parameters. The solitary wave moves right if the velocity is positive. It turns in left directions if the velocity is negative. The amplitudes and velocities are controlled by various parameters. Figures signify graphical representation for different values of parameters. Figure 2 represents traveling wave solution for different values of parameters as $q_0 = 1$, $p_0 = 0.1$, $p_1 = 1$, $\xi = 0.436$. The soliton solution is shown in Fig. 3 for values of parameters as $q_0 = 1$, $p_0 = 0.1$, $q_{-1} = 0.1$, $q_1 = 1$, $H = 10$, $\xi = 0.436$. Figure 4 shows graphical representation by using different values of parameters as $q_0 = 1$, $q_{-1} = 0.1$, $H = 10$, $\xi = 0.436$. In Fig. 5 the values of various parameters for graphical representation are $q_0 = 1$, $q_{-2} = 0.1$, $q_{-1} = 1$, $q_{-2} = 0.1$, $q_1 = 1$, $\xi = 0.436$. In both cases, for various values of parameters, we attain identical solitary wave solutions which obviously show that the final solution is not effectively based upon these parameters. So, we can choose arbitrary values of such parameters as input to our simulations.

6. Conclusion

In this paper, our main focus remained to find and analyze the novel soliton wave solutions and physical properties of nonlinear fluid model. For this, magneto-hydrodynamic flow of a viscous incompressible fluid between divergent or convergent channel is considered and we apply Exp-function method. We attain desired soliton solutions through exponential functions. The accuracy of the attained results is guaranteed by backward substitution into the original equation with Maple 13. The scheming procedure of this method is simple, straight-forward and productive. We have concluded that the under study technique is more reliable and requires minimum computational task. It is widely applicable as well. In precise, we can say that this method is quite competent and much operative for evaluating exact solution of nonlinear evolution equations. Results obtained by this method are very encouraging and reliable for solving any other type of nonlinear evolution equations. The graphical representations clearly indicate the solitary solutions.

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