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Stability Conditions for Cylindrical Domains in Ferromagnetic Elements: A Mathematical Approach to Microscopic Magnetometry

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Thin magnetic films displaying perpendicular magnetic anisotropy, with an out-of-plane easy axis, often show the presence of stripe domains that can break into cylindrical domains, producing in the end the formation of bubbles or skyrmions. Perpendicular magnetic anisotropy is considered to provide spin texture stabilization in topological materials. This paper reports a theoretical study of the stability of cylindrical domains and the interaction of cylindrical domains with each other and with the magnetic host material. In particular, the case of cylindrical domains of circular cross section has been deepened and a method for experimental determination of the saturation magnetization in a thin ferromagnetic film has been proposed.

Keywords : spintronics, cylindrical domains, skyrmions, magnetic bubbles, thin magnetic films, saturation magnetization measurements

1. Introduction

Cylindrical domains have been mainly observed in magnetic films such that the easy axis of magnetization is perpendicular to the film surface. When the anisotropy field of the material is higher than the material moment, then the magnetization will lie along the easy axis. The ratio of the anisotropy field to the moment of the magnetic film is named quality factor (Q) and is a dimensionless quantity determining many of the material's characteristics. The ratio of the minimum domain size to the width of a domain wall is proportional to Q; therefore, high Q give high domain stability. On the other hand, small values of Q allow to obtain smaller domains with higher mobility, which are desirable features in view of the realization of memory and spintronic devices [1-5]. The optimum value of Q is necessarily the result of a compromise.

In addition to the above-mentioned Q factor another parameter which determines the material's properties is the ratio of the wall energy density per unit area of wall to the magnetostatic energy density per unit volume, named material length λ. In order to have small cylindrical domains (e.g., bubbles) the film must have a thickness of the order of λ, so that the bubbles will have a diameter of the order of λ [6].

Let's consider a saturated film consisting of a single domain, magnetized normally to the surface. In order to diminish the net magnetization, a new domain (with opposite magnetization) and a corresponding domain wall must be introduced. The wall tends to remain parallel to the easy axis so that magnetic poles are avoided. In this way the domain walls are all normal to the film's surface and the domains are truncated cylinders. In this context the geometrical definition of a cylinder is: a surface generated by a line, perpendicular to a plane, tracing an arbitrary closed curve on the plane. Therefore, a cylinder has arbitrary cross section while a circular cylinder has a circular cross-section. All the domain configurations lying on a cross-section parallel to the surface of the film are identical. Under particular conditions the domains can be circular in cross-section (bubbles), that is a special case of cylindrical domain [7]. A common domain structure in the demagnetized state is that of stripe domains [8]. Upon the application of an appropriate field such a domain can contract in a bubble or skyrmion [9, 10].

An isolated bubble is not stable unless an applied field is present. Let's imagine applying a magnetic field to a demagnetized specimen. First the area of the domains magnetized parallel to the field will increase at the expense of the others. At critical field free wall domains will contract to a bubble. For a certain range of fields,

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free wall domains and bubbles will coexist. A further increase of the magnetic field above the stripe-bubble transition field, makes the bubbles to diminish in diameter until a second critical field is reached at which bubbles collapsing occurs. The bubbles can be 3 or 4 times as big at the stripe-bubble transition than they are just before collapsing. When the field reaches the value at which all the bubbles have collapsed the material becomes saturated. Then the magnetization of the sample cannot be changed even when a reverse field is applied.

A model dealing with the stability of cylindrical domains and the interaction of cylindrical domains with each other and with the magnetic host material will be proposed in the following. In particular, the case of cylindrical domains of circular cross section will receive considerable attention. An interesting consequence of the model is that a method for measuring M_s in a film supporting bubbles, through in-field magnetic force microscopy [11], can be imagined and tested. As a matter of fact, the results obtained by this method have been compared with traditional magnetometry measurements.

2. Results and Discussion

The energy E of a cylindrical domain is a function of the shape of the domain, the applied field H and the magnetic properties of the material, expressed by the p_1, …, p_n parameters characterizing it. For a given H, the possible geometries for the domain are obtained by searching the local minima of E. At local minima

$$\frac{\partial E}{\partial p_k} = 0 \qquad \text{for } k = 1, 2, \ldots, n \tag{1}$$

The stationary points of the energy are given by the simultaneous solutions to these n equations. To differentiate between maxima, minima, saddle and turning points, it is necessary that in the vicinity of the minima

$$\frac{\partial^2 E}{\partial p_k^2} > 0 \qquad \text{for } k = 1, 2, \ldots, n \tag{2}$$

We consider as the limit of stability the point at which a certain structure stops being the most likely energetically (e.g., stripe-bubble or bubble-collapse transition). The critical points can be found imposing the equality in eq. (2)

$$\frac{\partial^2 E}{\partial p_k^2} = 0 \qquad \text{for some } k \tag{3}$$

It must be underlined that we are looking for local minima. Several configurations are plausible for a given field.

The total energy of a cylindrical domain is the sum of three terms: the wall energy E_W, the energy due to the field E_H, and the internal magnetostatic energy E_M

$$E = E_W + E_H + E_M \tag{4}$$

The stability of a bubble domain will be determined studying the change in energy as a function of small perturbations. The shape of the cylindrical domain can be described by a set of quasi-orthogonal coordinates r_n and φ_n so that

$$r(\varphi) = \sum_{n=0}^{\infty} r_n \cos[n(\varphi + \varphi_n)] \tag{5}$$

In this expansion, r_0 is the radius of the bubble. Variations in r_0 are allowed to find the stable value of the radius; r_1 corresponds to a translation of the bubble in the direction in which $\varphi=\varphi_1$

The derivatives of the total energy can be written as

$$\frac{\partial E}{\partial r_0}|_0 = 2\pi h^2 \mu_0 M_S^2 \left[\frac{\lambda}{h} + a\frac{H}{M_s} - T(a)\right] \tag{6}$$

where the subscript zero refers to the point of expansion, namely the stable circular domain ($r = r_0$ and $r_n = 0$ for $n \neq 0$); h is the film thickness and a the aspect ratio of the bubble ($a = 2r/h$); $T(a)$ is

$$T(a) = \frac{2a^2}{\pi}\left[\frac{\sqrt{1+a^2}}{a} W\left(\frac{a^2}{1+a^2}\right) - 1\right] \tag{7}$$

with $W(k)$ the complete elliptic integral of the second kind.

From Eq. (1) a condition for equilibrium is found by equating Eq. (6) to zero. This leads to

$$\frac{\lambda}{h} + a\frac{H}{M_s} - T(a) = 0 \tag{8}$$

In order for a bubble to have an aspect ratio a, the application of a field given by

$$\frac{H}{M_s} = \frac{T(a) - \lambda/h}{a} \tag{9}$$

is necessary. From eq. (2) a stable bubble is obtained when

$$\frac{H}{M_s} - \frac{\mathrm{d}T}{\mathrm{d}a} > 0 \tag{10}$$

If one defines

$$S_0(a) = T(a) - a\frac{\mathrm{d}T}{\mathrm{d}a} \tag{11}$$

Table 1. Column one: film thickness. **Column two:** results of RT standard magnetometry measurements. **Column three:** error bar for the values reported in column two. **Column four:** results obtained from the bubble collapsing method by RT-MFM measurements, based on the proposed mathematical model.

Film thickness (nm)	Magnetometry - $4\pi M_s$ (kG)	Error bar	Bubble collapsing - $4\pi M_s$ (kG)
5	6,8	0,136	6,7
10	8,5	0,17	8,3
15	9,1	0,182	9,3

using eq. (9), eq. (10) becomes

$$\frac{\lambda}{h} < S_0(a) \tag{12}$$

Moreover, if one defines

$$S_n(a) = -\frac{1}{n^2-1}\left\{S_0 + \frac{a^2}{2\pi}[D_n(a) - D_n(\infty)]\right\} \tag{13}$$

with

$$D_n(a) = a\int_0^{\pi} \frac{1-\cos n\varphi}{\sqrt{\frac{1}{2}a^2(1-\cos\varphi)}}\mathrm{d}\varphi$$

it must be

$$\frac{\lambda}{h} > S_n(a) \qquad \text{for } n \geq 2 \tag{14}$$

The bubble will be stable if relations (8), (12) and (14) are satisfied. The functions T and S can be computed numerically.

The above equations allow to calculate the field necessary to obtain a stable bubble of a given aspect ratio, starting from a free wall domain. The procedure can be inverted to obtain a method for measuring λ and M_s for an unknown material. In a film of known thickness supporting bubbles, one should apply a field just large enough to make bubbles collapse and measure both the field and bubble diameter just before collapsing. The collapse aspect ratio is determined and from S the ratio λ/h can be obtained. Since h is known, λ can be calculated. Then from eq. (9) M_s can be derived since H is known.

In order to test this method thin ferromagnetic films with different thicknesses have been prepared by sputter deposition and subjected to a magnetic field generating bubbles. The bubbles collapsing field has been determined by Magnetic Force Microscopy (MFM) in-field measurements and the corresponding M_s has been calculated by the method described above. A comparison between the M_s values obtained by the bubble collapsing method and standard magnetometry measurements, both performed at room temperature (RT) on the same films, has been reported in Table 1. A good agreement between the two sets of data has been found.

In conclusion, the stability of cylindrical domains of circular cross section has been analyzed theoretically and a method for experimental determination of thin films M_s based on this analysis has been proposed.

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