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Optimal Design of Electromagnetic Devices Assisted by Black Hole and Differential Evolution Algorithms

Ziyan Ren[1]*, Siying He[2], Liping Zhang[1], Yanli Qi[1], and Chang-Seop Koh[3]

[1]*Shenyang University of Technology, Shenyang 110870, China*
[2]*CRRC Zhuzhou Electric Co., Ltd., Zhuzhou 412000, China*
[3]*Chungbuk National University, Chungbuk 28644, Republic of Korea*

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Recently, many design problems in the field of electrical engineering tend to be more complex, which are characterized by large scale in size, strong nonlinearity for performance analysis, and multi-dimensional design parameters. Therefore, it is not easy to seek for optimum effectively by traditional optimization algorithms. In order to solve optimal design of complex practical problems, in this paper, a novel hybrid optimization algorithm based on the differential evolution algorithm and the black hole theory is proposed and investigated. The differential evolution (DE) algorithm owns good diversity and flexibility, while the black-hole based optimization algorithm (BHBO) possesses faster convergence. In addition, these two algorithms have simple structures. The proposed algorithm with better merits combination may guarantee better convergence and stronger robustness than its independent counterparts of DE and BHBO. The searching performance is deeply investigated through numerical experiments on benchmark functions and practical electromagnetic applications.

Keywords : black hole, differential evolution, electromagnetic device, high-dimensional optimization problem

1. Introduction

The advanced optimization algorithm is an essential prerequisite for manufacturing electromagnetic products with better performance and lower cost. The meta-heuristic optimization algorithms such as differential evolution [1] and particle swarm optimization algorithms [2] have been proved to be effective in solving low-dimensional problems [3]. However, as the number of design variables becomes bigger or the complexity of performance analysis increases, the aforementioned algorithms may fall into local optimum easily and require more expensive searching cost. Furthermore, due to implicit relationship between objective functions and design variables in practical problems, optimal design problems tend to be more complex, strong nonlinear, and multimodal. It is significative to develop simple and effective algorithm solving complex high dimensional engineering problems.

The black-hole based optimization (BHBO) algorithm is parameter free and only has two mathematic equations for star updating and sucking [4]. Due to its simple structure, it is easy to use and has faster convergence. Until now, for the BHBO algorithm, there is no relevant research dealing with high dimensional problems. Among existing evolutionary algorithms, the differential evolution (DE) algorithm with fewer control parameters has been proved to be simple, diverse and flexible [5]. The DE normally gets high accuracy when dealing with low dimensional problems.

Based on the above background, the main contribution of this paper is to introduce a new meta-heuristic technique based on black hole and differential evolution. The suggested algorithm is capable of seeking optimum of complex electromagnetic problems. The combined hybrid algorithm is predicted to own better performance than either of independent ones.

2. Hybrid Optimization Algorithm of Black Hole and Differential Evolution (BHDE)

In space, the stars or other objects will be absorbed by the black hole when distance between object and black hole is less than the Schwarzschild radius. Based on this phenomenon, the BHBO algorithm is developed [4]. The

*Corresponding author: Tel: +86-13504018250
Fax: +86-24-25496427, e-mail: rzyhenan@163.com

differential evolution algorithm is one of population-based algorithms [1]. The DE algorithm has many variants to further improve its searching performance [6-8]. In this paper, the standard mutation operator DE/rand/1/bin is used.

To extend application of BHBO and DE in solving multi-dimensional problems, the merit of black hole is introduced into the standard differential evolution algorithm. Furthermore, from current generation to next generation, the target vector is moved based on the current black hole. Since the black hole is the best design at current iteration, so it can speed up the convergence. For a design problem with objectives to be minimized subject to some constraint conditions, the detailed BHDE algorithm is explained in the following contents.

Step* 1: *Initialization

To guarantee randomness of initial vectors, randomly generate N_p target vectors $\mathbf{x}_i^{g'}$ in a normalized design space. Based on upper limit $\mathbf{x}_U$ and lower limit $\mathbf{x}_L$ of each variable, the target vector $\mathbf{x}_i^g$ in physical design space is transformed by

$$\mathbf{x}_i^g = \mathbf{x}_L + (\mathbf{x}_U - \mathbf{x}_L) \cdot \mathbf{x}_i^{g'} \tag{1}$$

Calculate objective values and select target vector with the best fitness value as the initial black hole $\mathbf{x}_{BH}^0$.

Set numbers of maximum generation and maximum unchanged black holes as $g_{\max}$ and $uc_{\max}$ respectively.

Step* 2: *Mutation and crossover

Generate the mutation vector $\mathbf{v}_i^{g+1}$ by

$$\mathbf{v}_i^{g+1} = \mathbf{x}_{r1}^g + \mathrm{F} \times (\mathbf{x}_{r2}^g - \mathbf{x}_{r3}^g),\ r1 \neq r2 \neq r3 \neq i \tag{2}$$

where F is a weight coefficient; $r1$, $r2$, and $r3$ are subscripts of vectors which are randomly selected from $\{1, 2, \ldots, N_p\}$. Normally, the weight coefficient F belongs to the range of [0, 2]. It is used to adjust the proportion of differential vector in the mutation vector. In this paper, the weight coefficient F is set 0.5.

Generate a group of trial vector $\mathbf{u}_i^{g+1}$ by

$$u_{i,j}^{g+1} = \begin{cases} v_{i,j}^{g+1} & \text{if}(rand_j(0,1) \le C_r \text{ or } j = j_{rand}) \\ x_{i,j}^g & \text{otherwise} \end{cases} \tag{3}$$

where C_r and $rand_j(\ )$ are crossover constant and jth dimension random number among [0,1], respectively; the dimension index j_{rand} is randomly selected from 1 to n (number of design variables).

Step* 3: *Movement

The target vectors move as Eq. (4) to generate another group of trial vector $\mathbf{b}_i^{g+1}$.

$$\mathbf{b}_i^{g+1} = \mathbf{x}_i^g + rand(0,1) \times (\mathbf{x}_{BH}^g - \mathbf{x}_i^g) \tag{4}$$

where $\mathbf{x}_i^g \in R^n$ is ith star at gth generation.

Step* 4: *Survival criterion

At (g+1)th generation, trial vector $\mathbf{u}_i^{g+1}$ or $\mathbf{b}_i^{g+1}$, which gives better objective value will be selected as target vectors $\mathbf{x}_i^{g+1}$ surviving for next generation. Check the objective value of each star; if a star owns a better objective value than the current black hole $\mathbf{x}_{BH}^g$, it will become the new black hole $\mathbf{x}_{BH}^{g+1}$ for next generation.

To deal with constraint functions, there are some criterions as follows:

-If both trial vectors are feasible, select the survivor as:

$$\mathbf{x}_i^{g+1} = \begin{cases} \mathbf{u}_i^{g+1} & f(\mathbf{u}_i^{g+1}) \le f(\mathbf{b}_i^{g+1}) \\ \mathbf{b}_i^{g+1} & f(\mathbf{u}_i^{g+1}) > f(\mathbf{b}_i^{g+1}) \end{cases} \tag{5}$$

- If one of them is feasible, select the feasible one;
- If both are infeasible, select that with a lower summation of constraint violations.

Step* 5: *Vector correction

If (6a) is satisfied or the target vector is out of design space, the target vector $\mathbf{x}_i^{g+1}$ is sucked by the black hole. In order to keep a constant number of target vectors N_p, for all the sucked vectors, the corresponding new vectors will be generated randomly.

$$\left\| \mathbf{x}_i^{g+1} - \mathbf{x}_{BH}^{g+1} \right\| < R \tag{6a}$$

$$R = \left| f(\mathbf{x}_{\mathrm{BH}}^g) \right| \Big/ \sum_{i=1}^{N_p} \left| f(\mathbf{x}_i^g) \right| \tag{6b}$$

Step* 6: *Termination criterion

If the iteration number reaches a predefined maximum $g_{\max}$ or the current black hole is not changed continuously for a user-defined maximum iterations $uc_{\max}$, then terminate BHDE. Otherwise go back to step 2.

Frankly speaking, the suggested BHDE may not be superior to all meta-heuristic optimization algorithms. The proposed BHDE algorithm at least may be better than its single counterparts for solving high-dimensional problems.

3. Numerical Applications of Proposed Algorithm

In this section, the performance of the proposed BHDE algorithm is investigated by several applications. Without loss of generality, for each problem, the optimization program carries out 20 independent runs.

Table 1. Result comparisons of benchmark functions.

Function	Method	n=30		n=60		n=100	
		μ_f	σ_f	μ_f	σ_f	μ_f	σ_f
Ackley	BHBO	3.729	0.796	4.407	0.827	5.060	1.164
	DE	2.77E-11	1.63E-11	5.21E-06	1.26E-06	1.40E-03	3.00E-03
	BHDE	2.12E-09	1.71E-09	7.63E-10	4.32E-10	2.97E-09	1.38E-09
Griewank	Method	μ_f	σ_f	μ_f	σ_f	μ_f	σ_f
	BHBO	2.97E-13	5.21E-13	9.67E-12	9.91E-12	1.09E-10	8.04E-11
	DE	4.79E-23	3.15E-23	6.68E-12	3.84E-12	7.89E-07	3.10E-07
	BHDE	2.72E-16	3.70E-16	5.59E-14	7.28E-14	1.45E-11	1.18E-11
Rastrigin	Method	μ_f	σ_f	μ_f	σ_f	μ_f	σ_f
	BHBO	62.031	31.39	179.355	58.34	340.409	128.17
	DE	155.987	15.83	442.047	14.96	832.176	21.89
	BHDE	4.55E-08	1.85E-07	7.95E-09	2.45E-08	9.30E-10	3.74E-09
Schwefel	Method	μ_f	σ_f	μ_f	σ_f	μ_f	σ_f
	BHBO	2.499	1.306	14.699	40.712	10.200	3.033
	DE	2.90E-10	1.71E-10	9.73E-05	2.65E-05	0.029	9.3E-03
	BHDE	9.34E-11	8.35E-11	1.90E-11	1.34E-11	9.01E-12	5.45E-12

3.1. Benchmark Analytic Functions

Four benchmark test functions with a zero optimal objective value in (7) are selected to investigate proposed BHDE algorithm. N_p is set three times of dimensions.

$$\text{Ackley: } f(\mathbf{x}) = -20e^{-\frac{1}{5}\sqrt{\sum_{i=1}^{n} x_i^2/n}} - e^{\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)} + 20 + e, [-32, 32] \tag{7a}$$

$$\text{Griewank: } f(\mathbf{x}) = \sum_{i=1}^{n} x_i^2/4000 - \prod_{i=1}^{n}[\cos(x_i/\sqrt{i}) + 1], [-600, 600] \tag{7b}$$

$$\text{Rastrigin: } f(\mathbf{x}) = 10n + \sum_{i=1}^{n}[x_i^2 - 10\cos(2\pi x_i)], [-5.12, 5.12] \tag{7c}$$

$$\text{Schwefel: } f(\mathbf{x}) = \sum_{i=1}^{n}|x_i| + \prod_{i=1}^{n}|x_i|, \quad [-10, 10] \tag{7d}$$

For different dimensions, test results of three algorithms (BHBO, DE, and BHDE) are listed in Table 1.

Among 20 independent runs, the mean value μ_f and the standard deviation σ_f of objective function are selected as performance indices. From comparisons, the following investigations can be obtained.

(1) It is obvious that the searching ability of each algorithm becomes worse as the number of dimension n increases. Moreover, BHBO and DE fails to find true optimum of Rastrigin function under $n = 60$ and 100. By contrast, the BHDE algorithm can successfully give a better solution, which shows stronger robustness than its counterparts.

(2) Even for the low dimensional case ($n = 30$), BHBO fails to find optimal solution of all test functions except Griewank. For low-dimensional Ackley and Griewank, the DE shows outstanding performance, while the BHDE has an advantage over the other two methods for high dimensional case.

Therefore, the robustness and effectiveness of combined BHDE is better than its single counterparts especially for high dimensional problems.

3.2 Brushless DC Wheel Motor Problem

For the application of a brushless DC wheel motor design [9], the target is to maximize motor efficiency η subject to the following constraints:

$$\begin{aligned} g_1(\mathbf{x}) &= 76 - D_{int} \le 0 \\ g_2(\mathbf{x}) &= D_{ext} - 340 \le 0 \\ g_3(\mathbf{x}) &= -\text{discr} \le 0 \\ g_4(\mathbf{x}) &= 125 - I_{max} \le 0 \\ g_5(\mathbf{x}) &= T_a - 120 \le 0 \\ g_6(\mathbf{x}) &= M_{tot} - 15 \le 0 \end{aligned} \tag{8}$$

where D_{int} and D_{ext} are inner and outer diameters of motor, discr (D_s, δ, B_d, B_e) is a determinant to calculate slot height. I_{max} is peak phase current. T_a is magnet temperature. M_{tot} is the total mass of active parts including magnets, stator teeth, copper, rotor and stator yoke. Table 2 lists design variables $\mathbf{x} = (D_s, B_e, \delta, B_d, B_{cs})$ as stator diameter (D_s), maximum magnetic induction in air gap (B_e), current density in conductors (δ), average magnetic induction in teeth (B_d) and stator back iron (B_{cs}). Other details are described in [9].

Table 2. Design space of brushless DC wheel motor.

	D_s (mm)	B_e (T)	δ (A/mm^2)	B_d (T)	B_{cs} (T)
Min	150	0.5	2.0	0.9	0.6
Max	330	0.76	5.0	1.8	1.6

Table 3. Result comparisons of brushless DC wheel motor.

Items	Standard	BH-DE	BHBO	DE
D_s (mm)	201.2	201.0	204.8	203.7
B_e (T)	0.648	0.654	0.646	0.650
δ (A/mm^2)	2.044	2.067	2.048	2.037
B_d (T)	1.8	1.8	1.7	1.7
B_{cs} (T)	0.896	0.928	1.151	0.979
D_{int} (mm)	76	78	86	77
D_{ext} (mm)	239.0	239.1	242.9	242
I_{max} (mm)	125	130	125	128
M_{tot} (kg)	15	15	15	15
T_a (°C)	95.37	95.43	94.36	94.36
Discr	0.024	0.024	0.021	0.018
η (%)	95.32	95.30	95.29	95.33
Std. dev	-	1.41E-4	3.03E-4	1.07E-3

Parameters of optimizer are set as N_p = 55, $g_{\max}$ = 300, and $uc_{\max}$ = 50. The optimum searching by BHDE algorithm is compared with other two algorithms in Table 3.

In this paper, the analytic function model of brushless DC wheel motor is used for solving objective function [9]. Taking benchmark solution in [9] as a reference, the efficiency of design found by the DE is much close to standard one. However, by comparing standard deviation of 20 independent runs, the BHDE algorithm shows a better stability. Figure 1 shows convergence process of three algorithms for the best solution. After 20 independent runs searching for the optimal under different conditions, average function calls of BHDE, BHBO and DE are 10445, 14575 and 10340, respectively. Taking both

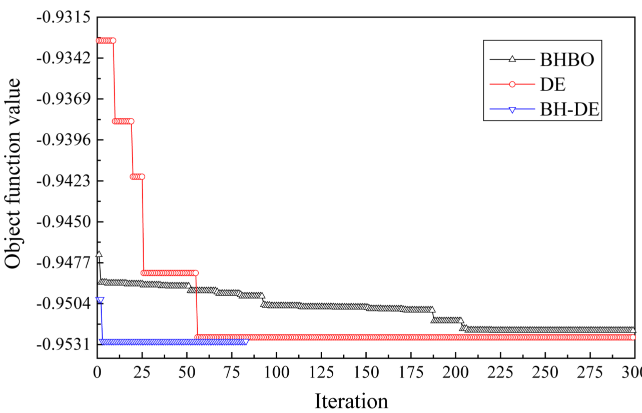

Fig. 1. (Color online) Relation between iteration number and objective function values.

searching accuracy and computing cost into account, the BHDE owns better performance than single BHBO and DE.

3.3. Superconducting Magnetic Energy Storage System

The superconducting magnetic energy storage system (SMES) is the second engineering application [10]. Design targets are to get stored energy E_0 close to180 MJ and keep a minimum stray field B_{stray}. The optimization model is defined as:

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) = B_{stray}^2 / B_{norm}^2 + \left|E(\mathbf{x}) - E_0\right| / E_0 \\ \text{subject to} \quad & g_i(\mathbf{x}) = 6.4\left|B_{m,i}\right| + \left|J_i\right| - 54 \le 0, i = 1,2 \\ & g_3(\mathbf{x}) = (R_1 + D_1/2) - (R_2 - D_2/2) \le 0 \end{aligned} \tag{9}$$

where the normal stray field B_{norm} is 200 μT. The first two constraints guarantee the quenching condition of superconductor. The last one avoids two coils being overlapped. More details about symbols definition and structure can be found in [10].

Since the SMES is a linear problem, the performance analysis is carried out by the semi-analytic method [11]. For the non-linear engineering problem, the proposed optimization method can be combined with the design of experiment method. Taking optimizer parameters of N_p = 80, $g_{\max}$ = 500, and $uc_{\max}$ = 50, optimization results are compared in Table 4. Obviously BHDE algorithm can find an optimal solution close to standard one while other two single algorithms fail to find optimal designs. Besides, through comparison of mean objection value, the BHDE shows stronger robustness. For 20 independent runs under same convergent condition, by comparing mean function values and mean number of function calls, the BHDE algorithm is proved to be better than either of other two

Table 4. Result comparisons of different algorithms for SMES.

Items	Standard	BHDE	BHBO	DE
R_1 (m)	1.5703	1.5710	1.4900	1.7044
R_2 (m)	2.0999	2.1012	2.1315	2.2521
$h_1/2$ (m)	0.7846	0.7845	0.7172	0.8191
$h_2/2$ (m)	1.4184	1.4179	1.3186	1.2886
d_1 (m)	0.5943	0.5943	0.6138	0.6245
d_2 (m)	0.2562	0.2562	0.1960	0.1797
J_1 (A/mm^2)	17.3367	17.3369	15.3367	14.0688
J_2 (A/mm^2)	-12.5738	-12.5734	-12.9892	-17.9988
f(x)	5.5203E-3	5.4741E-3	0.332329	0.556138
B_{stray}^2 (T)	2.1913E-10	2.0921E-10	4.2468E-09	1.6424E-08
Energy (MJ)	179.9924	180.0439	139.2914	153.8046
Mean f(x)	0.0055	0.0075	0.7074	0.6817
Fun. calls	-	27,330	36,480	32,310

single algorithms.

3.4. Cogging Torque Optimization of a Permanent Magnet Synchronous Machine

For further performance investigation, a permanent magnet synchronous machine (PMSM) is selected for cogging torque minimization [12]. The cogging torque is caused by the interactions among permanent magnet, stator, and rotor at no-load condition. Without consideration of skewed slots in the surface mounted PMSM, the analytic expression of the cogging torque can be summarised as follows:

$$T_{cog} = \frac{\pi z L_{Fe}}{4\mu_0}\left(R_2^2 - R_1^2\right)\sum_{n=1}^{\infty} nG_n B_{r,nz/2p}\sin(nz\alpha)$$
$$B_{r,nz/2p} = \frac{2}{n\pi}B_r^2\sin(n\pi\alpha_p) \tag{10}$$

where L_{Fe} is axial length of iron core; R_1 and R_2 are outer radius of rotor and inner radius of stator, respectively; z is number of slots; α is the intersection angle between central lines of permanent magnet and slots; α_{p} is the pole-arc coefficient; and G_{n} is coefficient of Fourier expansion.

From Eq. (10), it can be seen that the cogging torque is related with parameters of permanent magnet and armature, and different combination of slots and poles. In this paper, design variables are parameters of stator slot (Hs0, Hs1, Hs2, Bs0, Bs1, and Bs2), air-gap length δ, pole-arc coefficient α_p, and permanent magnet thickness h_M. The corresponding design spaces are shown in Table 5. The parameters required by optimizer are $N_p = 95$, $g_{\max} = 1000$, and, $uc_{\max} = 120$.

The optimum shape of stator slot type searching by the BHDE algorithm is compared with initial shape in Fig. 2 and Fig. 3. The design parameters [δ, α_{p}, $\mathrm{h_M}$] are optimized from [2.0 mm, 0.76, 6 mm] to [1.5 mm, 0.75, 7.8 mm], respectively. Other parameters are listed in Table 6.

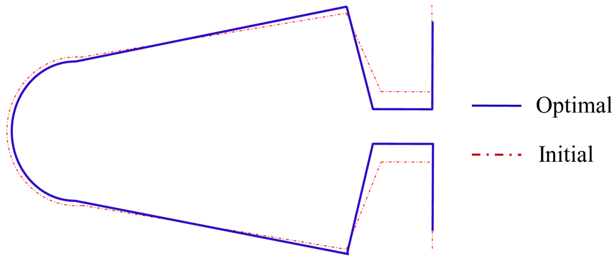

Fig. 2. (Color online) Comparision of stator slot types.

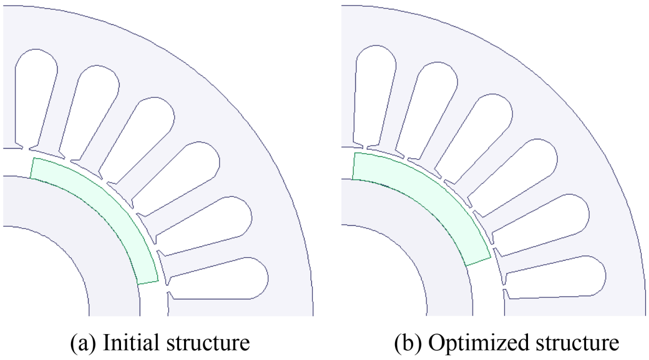

Fig. 3. (Color online) Structure comparison of electric machine before and after optimization.

From Fig. 4, it is obvious that the magnitude of cogging torque is effectively reduced from 379.5 mNm to 293.4 mNm, which decreases almost 23 %. Meanwhile, for distribution of the no-load air-gap magnetic flux density in Fig. 5, the amplitude of optimal design is slightly higher than the initial value, while shapes of waveform before and after optimization are almost same. Figure 6 shows harmonic components of the optimized flux density waveform. The corresponding harmonic distortion rate is 22.51 % while the initial design is 21.39 %. The proposed BHDE can find an optimal design, which can implement

Table 5. Design space of electrical machine.

	Hs0	Hs1	Hs2	Bs0	Bs1	Bs2	δ	α_p	h_M
Unit	mm	mm	mm	mm	mm	mm	mm	-	mm
Min	0	0.5	18.5	1	5	10	0.5	0.65	4
Max	1	1.5	23.5	3	8	14	2	0.9	8

Table 6. Design solutions of electric machine before and after optimization.

	Hs0	Hs1	Hs2	Bs0	Bs1	Bs2	δ	α_p	h_M
Unit	mm	mm	mm	mm	mm	mm	mm	-	mm
Initial	0.5	1	21.5	2	6.28	11.94	2	0.76	6
Optimal	0.8	0.7	22.7	1.4	7.39	11.71	1.5	0.75	7.8

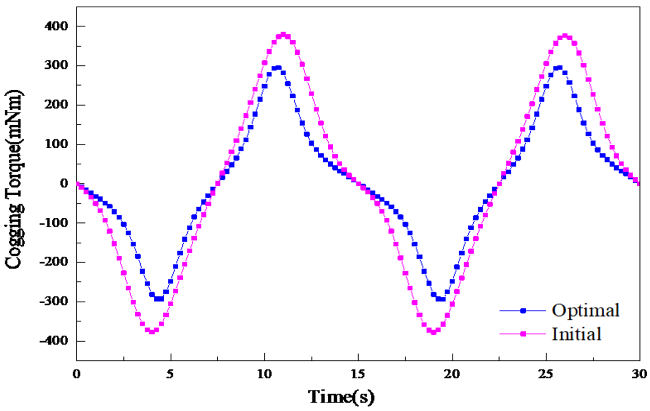

Fig. 4. (Color online) Comparison of cogging torque waveforms.

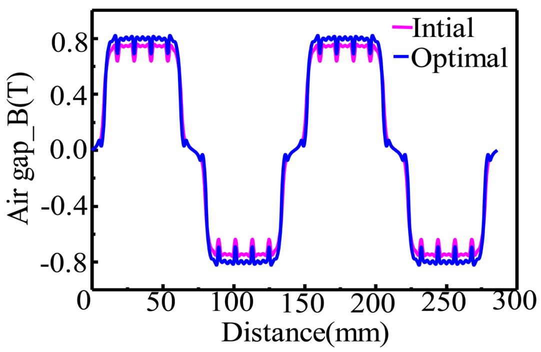

Fig. 5. (Color online) Distribution of air-gap magnetic flux density.

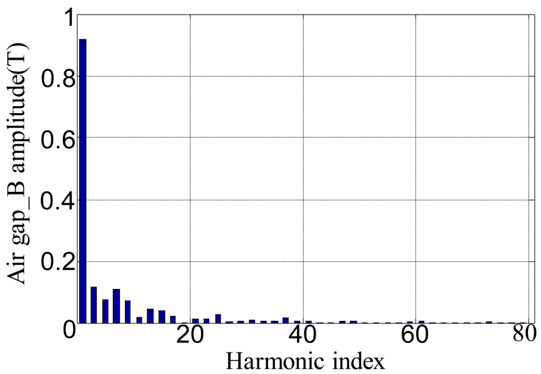

Fig. 6. (Color online) Harmonic distribution of optimal air-gap magnetic flux density.

effective control of cogging torque and not deteriorate other performances.

4. Conclusion

From the viewpoint of solving high dimensional problems in the field of electrical engineering, this paper presents a hybrid black-hole-based differential evolution (BHDE) algorithm. From several numerical experiments, the BHDE shows stronger robustness and better searching ability than independent black-hole-based algorithm and differential evolution algorithm. The BHDE with simple structure and fewer control parameters, which is easy for popularization and can be integrated with surrogate models. There is no free lunch in optimization. The BHDE may not be superior to any other optimizers. However, the BHDE laying stress on advantages combination is better than its single counterparts. In further research, the performance investigation of BHDE will be compared with other optimizers through applications on much higher-dimensional electromagnetic problems.

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References

[1] R. Storn and K. Price, J. Global Optim. 341 (1997).
[2] M.-T. Pham, M.-H. Song, and C.-S. Koh. J. Electr. Eng. Technol. **5**, 423 (2010).
[3] M.-T. Pham, Z. Ren, W. Li, and C. -S. Koh, IEEE Trans. Magn. **47**, 4163 (2011).
[4] Hotssem R. E. H. Bouchekara, IEEE Trans. Magn. **49**, 5709 (2013).
[5] G. C. Tenaglia and L. Lebensztajn, IEEE Trans. Magn. **50**, 7015404 (2014).
[6] J. Zhang and A. C. Sanderson, IEEE Trans. Evolu. Comput. **13**, 945 (2009).
[7] S. M. Islam, S. Das, and S. Ghosh, IEEE Trans. Syst. Man & Cybern B Cybern. **42**, 482 (2012).
[8] N. Baatar, D. Zhang, and C.-S. Koh, IEEE Trans. Magn. **50**, 7017504 (2014).
[9] http://l2ep.univ-lille1.fr/come/benchmark-wheel-motor/OptPb.htm
[10] G. Schoenwetter, *et al.*, IEEE Trans. Magn. **31**, 1940 (1995).
[11] D. Zhang and C.-S. Koh, IEEE Trans. Magn. **48**, 62 (2012).
[12] T. Liu, Ph.D. Thesis, Hunan University (2012).