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Effects of Gold Nanoparticles Shapes on Magnetohydrodynamic Flow and Heat Transfer in the Presence of Thermal Radiation

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The objective of present study is to examine the effects of nanoparticles shapes on magnetohydrodynamic (MHD) boundary layer flow and heat transfer of nanofluid over a flat plate in the presences of thermal radiation. Three categories of fluids such as pure water, ethylene glycol and engine oil with Prandtl number (Pr) = 7.8 containing three different shapes of Gold (Au) nanoparticles i.e. sphere, platelet and lamina have used in this study. By using unique similarity transformation, the governing partial differential equations (PDEs) are converted into a system of non-linear ordinary differential equations (ODEs) which have tackled numerically by bvp4c program. The behaviours of several pertinent parameters i.e. solid volume fraction (ϕ), magnetic field (M) and thermal radiation (R_d) for different shapes of nanoparticles also elucidated in detail. The results indicated that all selected parameters have a significant impact on the thermal boundary layer. The Nusselt number is presented in graphically form. The nanoparticles of sphere shape in Au-Ethylene glycol are more significant for temperature disturbance. The heat transfer rate has found greater for lamina shapes in Au-Engine oil.

Key words : numerical solution, nanoparticles, flat plate, thermal radiation, nanofluids, MHD boundary layer flow

1. Introduction

Nanotechnology is a widely used technology with its rapid progress in several fields, such as physics, chemistry, biotechnology, material sciences and other applications [1]. Recently, nanotechnology becomes a hot topic, which is widely discussed among the researchers due to injurious health and environmental concerns [2]. Nanoparticles can be found in numerous species as oxide, ceramics, nitride ceramics, carbines or carbon nanotubes etc. [3]. This area of study has gained plentiful prominence with the potential to enhance the heat transfer rates in many fields like a nuclear reactor, cooling application, nano-drug delivery, cryopreservation, cancer therapeutics, micro-electromechanical systems, transportation industry and some other specific industrial applications [4].

A broad range of industrial processes engaged with transfer of heat energy. Transfer of heat energy assume the main task for industrial essential because of every industrial facility heat must be removed, moved or added or from one process to next. There are numerous techniques to raise heat transfer productivity. The big heat productivity can be better by enhancing the thermal conductivity of the working fluid [5]. Nanofluids are a recent way of heat transfer fluid, which consists of small size of nanoparticles that are stably suspended in a carrier fluid. The thermal conductivity of the liquid is very low as compare to the thermal conductivity of solid nanoparticles. The suspensions of the small size of nanoparticles into dispersion with in traditional fluid energetically change its thermal conductivities. The topic of heat transfers through suspension of solid nanoparticles has got a reintroduced improvement of owed reports of the boost in thermal conductivity of suspensions carrying solid nanoparticles [6]. Many authors investigated heat

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transfer in nanofluids, for instance, a numerical study of heat transfer rate in nanofluid by radiation effect was discussed [4, 7, 8]. An excellent studied on nanofluid presented by [9-11] in review papers. The effect of thermal radiation and viscous dissipation on laminar boundary layer flow nanofluid over a vertical plate has been numerically discussed in detail by [12]. Das and Sarkar [13] ascertained the melting and thermal radiation effect on magneto hydrodynamics over a shrink sheet. Perdikis and Rapits [14], discussed numerical solution of temperature profile by considering radiation effect on non-moving plate. Cess [15], investigated heat transfer form in vertical plate by interaction of thermal radiation with laminar free convection. The heat transfer rate with the temperature distribution can be achieved by thermal radiation this technique is useful in fields i.e. nuclear reactor and power generating system. Several researchers analyzed heat transfer over moving a flat plate opposite or parallel stream velocity. The idea of flow and heat transfer by the thermal radiation effect in a flat surface by considering free stream and flat plate move in the same or opposite direction generalized by [16].

The radiation effects are delicate in the context of high temperature including space technology, for instance, developments in power plants for interplanetary flight, rocket combustion chamber, hypersonic flights, gas cold nuclear reactor, missile, re-entry as an energy mode thermal radiation have focused attention [17]. According to the above discussion and literature studied, it has concluded that heat transfer and thermal radiation are subtle aspects in astrophysical flows, electric power generation, and various industrial processes investigations.

There are many applications of MHD in Engineering such as power generators, cooling of the nuclear reactor, plasma studied, gas turbine, purification of metal form non- metal enclosures, crystal growth, and metallurgy and boundary layer control in aerodynamics. Magnetic nanofluid is a distinctive material due to both characteristics of liquid and magnetic. Physical properties of Magnetic nanofluid can be varying by turning the magnetic field [18]. Recently, several researchers have been analyzed the effects of electrically conducting nanofluid i.e. Mixture of water with a petite acid and other ingredients in the effect of magnetic field on the fluid flow and heat transfer of viscous, incompressible, stretching plate in the stationary fluid or electrical conducting fluid over a moving surface [19]. Hamad *et al.* [20] examined the effects of the magnetic field on free convection flow fluid over a sami infinite vertical flat plate. Logana and Vimala *et al.* [21] presented the effect of radiation on the laminar, steady MHD boundary layer flow of nanofluid over an exponentially stretching sheet. Nayak *et al.* [22] studied the effect of chemical reaction on MHD viscoelastic fluid over a stretching sheet through a porous medium. Khan *et al.* [23] discussed the influence of the magnetic field on the radiative fluid flow of a nanofluid past over a stretching sheet. Sheikholeslami and Ganji *et al.* [24] presented the effect of radiation on nanofluid in the presence of the magnetic field. Hayat *et al.* [25] studied MHD boundary layer fluid flow of nanofluid. Hosseinzzdeh *et al.* [26] discussed analytical solution of nanofluid flow in parallel plates in the presence of megnatic field. Gholinia *et al.* [27] numerical studied nanofluid flow over a permable circular cylinder with the effect of magnetic field. Hosseinzadeh *et al.* [28] examined Entropy generation of nanofluid flow between two rotating discs with the effect of thermal radiation and magnetohydrodynamic. Zangooee Hosseinzadeh *et al.* [29] discussed nanofluid flow between two rotating disks with heterogeneous and homogeneous reactions in the presences of magnetic field. Derakhshan *et al.* [30] discussed mass and heat process of steady nanofluid flow between two plates in the presences of uniform magnetic field.

Gold (Au) is one of the metals that have been discovered earlier. In technology fields, gold is utilized as drug delivery in nanotechnology, catalysis and as an organic photovoltaic [31]. The colloidal gold appraisal started with Michael Faraday's work in 1850s [32]. Shoyunget *et al.* [33] analyzed an experimental mechanism for the application of gold nanoparticles to cancer radiotherapy. The small size of gold nanoparticles is very important in biomedical science, it is also utilized in numerous biomedical applications [34]. Gold can actually be used to activate or prevent blood vessel growth, while some medicines may be used to increase or decrease capillary blood growth in certain illnesses. These treatments are effective for only a short time. Kanaras and his colleague disclosed that some of the issue connected with drug administration could be solve by nanoparticles. They displayed that gold particles are effective in drug carrying vehicles and drug delivery, because they can encapsulate therapeutic molecules in large amounts [35]. Gold (Au) nanoparticles are seldom utilized for studying heat transfer rate due to mixed convection. Although mixed convection happens in several technological and industrial processes i.e. thermal insulations, chemical processing, thermal insulations, electronics cooling technology and nuclear reactors the gold nanoparticles studies in this fields are inadequate. Analytical solution of MHD mixed convection fluid flow of a nanofluid by suspended gold nanoparticles into kerosene oil has been discussed by Sidra *et al.* [31].

The importance of nanoparticles shape effect on the mass and heat transfer have been shown in earlier experimental studies [36-39]. The increase in thermal conductivity of nanoparticles shape is more significant in the enhancement of heat transfer. Consequently, it is needed to find heat transfer of nanofluid exactly under the shape of nanoparticles in the nanofluid [40]. Many scientists have been performed experimental research on gold nanoparticles. In this study, we have focus on the shapes of gold nanoparticles because of its excessive thermal conductivity. The objective of this study was to investigate the effects of Gold (Au) nanoparticles shapes (i.e., sphere, platelet and lamina) on magnetohydrodynamic boundary layer flow and heat transfer over a flat plate in the presence of thermal radiation. The non-linear ODEs have been solved with the help of bcp4c program to discuss the effects of various parameters. Hamilton and crosser model was utilized in this study.

2. Mathematical Model

The present study considers the effects of nanoparticles shapes magnetohydrodynamic (MHD) boundary layer flow and heat transfer of a nanofluid in the presence of thermal radiation. The nanofluids are made of pure water, Ethylene glycol and Engine oil which contain various shapes of Gold (Au) nanoparticles. The thermos-physical characteristics of Gold (Au) nanoparticles and selected fluids have been described in Tables 1 (as [1, 31]) and Table 2. Furthermore, (x, y) Cartesian coordinate system has been considered, where x and y coordinates have measured along parallel and perpendicular to surface, respectively. The radiations and magnetic field were assumed to be applied parallel and transverse to y-axis, respectively (see in Figure 1). The basic steady state boundary layer equations of nanofluids in the Cartesian coordinate system are modeled as [41];

Table 1. Thermo-physical properties of Gold (Au), pure water, ethylene glycol and engine oil nanofluids.

Physical properties	Gold (Au)	Pure water	Ethylene glycol	Engine oil
ρ (kg/m^3)	19300	998.3	1115	884
C_p (J/kgK)	129	4182	2430	1910
k (W/m K)	318	0.60	0.253	0.144

Table 2. Values of the empirical shape factor for different nanoparticle shapes.

Shapes	Sphere	Platelet	Lamina
ϕ	1	0.526	0.186
m	3	5.7	16.2

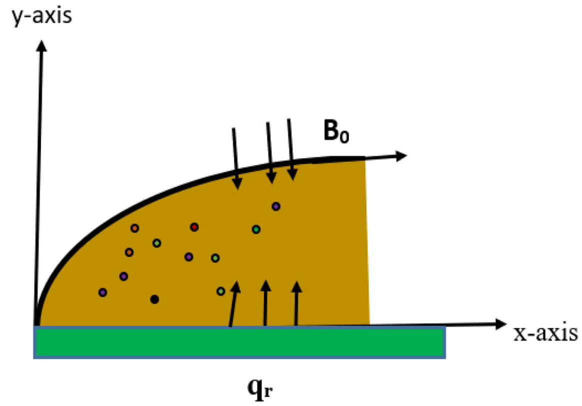


Fig. 1. (Color online) Schematic model and coordinate system.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{n_f}\partial^2 u}{\rho_{n_f}\partial y^2}-\frac{\sigma B_0^2 u}{\rho_{n_f}}\,, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{n_f}\frac{\partial^2 T}{\partial y^2}-\frac{1}{(\rho Cp)_{nf}}\frac{\partial q_r}{\partial y}\,. \tag{3}$$

The boundary conditions subjected to the problems are

$$u=U_S(x),\ v=0,\ T=T_f,\ \text{at } y=0,$$
$$u=v=0,\ T=T_\infty,\ \text{at } y\to\infty\,. \tag{4}$$

Here T_f is the temperature of fluid and T_∞ is the temperature of ambient nanofluid.

$$\alpha_{n_f}=\frac{k_{n_f}}{(\rho Cp)_{nf}},\ \rho_{n_f}=(1-\phi)\rho_f+\phi\rho_s$$
$$(\rho Cp)_{nf}=(1-\phi)(\rho Cp)_f+\phi(\rho Cp)_s\,, \tag{5}$$

In Eqs. (1-5) μ_{n_f}, ρ_{n_f}, $(\rho Cp)_{nf}$, α_{n_f} and k_{n_f} is the effective viscosity, effective density, heat capacity, thermal diffusivity, and effective thermal conductivity of nanofluid respectively. The ϕ is present the solid volume fraction. *nf*, *f* and *s* are shown the base fluid thermo-physical properties of nanofluid, base fluid and nano solid particles respectively. Furthermore, the Hamilton-crosses model is incorporated for mixture of thermal conductivity, which is effective for both spherical and nonspherical shapes of dispersed nanoparticles. According to this model mixture of thermal conductivity can be calculated as

$$\frac{k_{n_f}}{k_f}=\frac{k_s+(m-1)k_f-(m-1)\phi(k_f-k_s)}{[k_s+(m-1)k_f]+[\phi(k_f-k_s)]}\,. \tag{6}$$

The k_f presents the thermal conductivity of nanofluid, the

k_s presents the thermal conductivity of solid and m presents the shape factor of solid nanoparticles.

By using the Rosseland approximation, the radiative heat flux converted into the following form,

$$q_r = -\frac{\sigma^*}{3k^*} \cdot \frac{\partial T^4}{\partial y} \tag{7}$$

Where k^* is the mean absorption and δ^* is Stefan-Boltzmann. Moreover, it was assumed that the temperature within the flow is small such that T^4 expressed as a linear function of temperature. Hence, by expending T^4 in a Taylor series and neglecting higher order terms, we obtained

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

By using Eqs. (7), (8), Eq. (3) converted into

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{n_f}(1 + R_d)\frac{\partial^2 T}{\partial y^2} \tag{9}$$

Applying similarity transformations on Eqs. (1-3) and Eq. (4)

The Stream function ψ are defined in the following forms

$$\psi(x, y) = A(x)F(\eta),\ u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x},\ \eta = B(x)y$$

$$A(x) = \sqrt{\frac{cv_f}{2}}x,\ B(x) = \sqrt{\frac{2c}{v_f}},\ \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}. \tag{10}$$

By substituting Eq. (10) in Eqs. (1-3). The Eq. (1) identically satisfied and in the form of ordinary differential equation following results are obtained

$$f''' - \frac{1}{2}(f'^2 - ff'' + Mf')\frac{(1 + \phi)^{2.5}((1 - \phi)\rho_f + \phi\rho_s)}{\rho_f} = 0, \tag{11}$$

$$(1 + R_d)\theta'' + \frac{1}{2}Prf\theta'\frac{k_f((1 - \phi)(\rho Cp)_f + \phi(\rho Cp)_s)}{k_{n_f}(\rho Cp)_f} = 0. \tag{12}$$

The boundary value condition of Eq. (10) become

$$f(0) = 0, f'(0) = 1, f'(\infty) = 0,\ \theta(0) = 1,\ \theta(\infty) = 0. \tag{13}$$

where $M = \frac{\sigma B_0^2 x}{c(\rho_{n_f})}$, $R_d = \frac{16\sigma^* T_\infty^3}{3k_{n_f}k^*}$, and $Pr = \frac{v_f}{\alpha_f}$

The applied magnetics strength is determined by M, the effect of momentum diffusivity against thermal diffusivity is presented by Pr. The imposed nonlinear thermal radiation is measured by R_d.

The important physical quantity of interest, the Nusselt number (Nu) is defined as

$$Nu_x = \frac{xq_w(x)}{k_f[T_f - T_\infty]} \tag{14}$$

Where $q_w(x)$ is given by

$$q_w(x) = -k_{n_f}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Using Equation (10) into Equation (14), we get

$$\frac{N_{u_x}}{\sqrt{Re_x}} = -\frac{k_{n_f}}{k_f}\theta'(0)$$

3. Numerical Solution

To compute the numerical solution of non-linear ordinary differential equations (ODE$_S$), bvp4c program is very easy and conceivable. The Eqs. (11), (12) and (13) and their corresponding boundary values conditions were solved with MATLAB in bvp4c program. The first step of this method was to convert Eqs. (11), (12) and Eq. (13) into first (ODE$_S$) as following.

$$f = y_1,\ y_1' = y_2,\ y_2' = y_3,\ \theta = y_4,\ \theta' = y_5.$$

$$y_3' = 0.5 \times (y_2 \times y_2 - y_1 \times y_3 - M \times y_2) \times (1 + \phi)^{2.5} \times ((1 - \phi)\rho_f + \phi\rho_s)/\rho_f; \tag{14}$$

$$Y_5' = -0.5 \times (1 + R_d)^{-1} \times Pr \times y_1 \times y_5 \times k_f \times ((1 - \phi)(\rho Cp)_f + \phi(\rho Cp)_s)/k_{n_f}(\rho Cp)_f; \tag{15}$$

We have converted $(f, f', f'', f''', \theta, \theta')$ into $(y, y_1, y_2, y_3, y_4, y_5)$. The boundary value condition as requirement of bvp4c.

$$y_0 = 0,\ y_{01} - 1 = 0,\ y_{\infty 1} = 0,\ y_{04} - 1 = 0,\ y_{\infty 4} = 0. \tag{16}$$

The required arguments of bvp4c function are as under.
sol = bvp4c (@ex8ode, @ex8bc, solinit, options)

4. Results and Discussion

The under consideration shapes of nanoparticles are displayed in Fig. 2. Figures 3-6 Displayed variation of

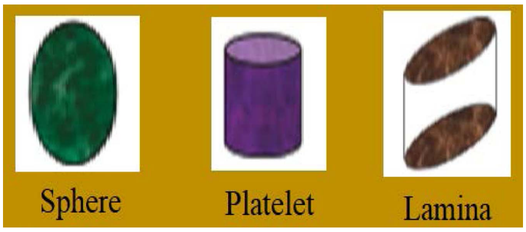
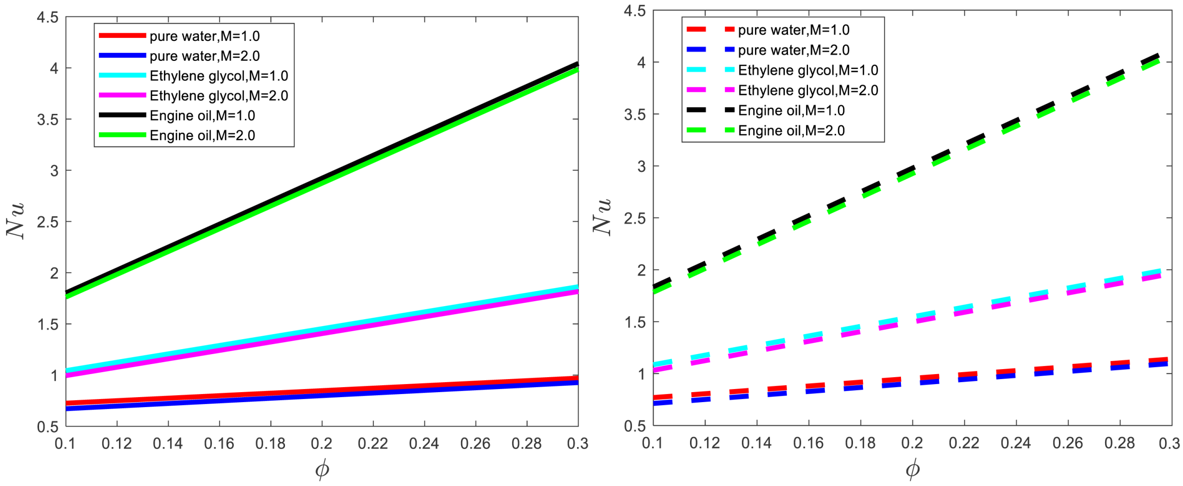
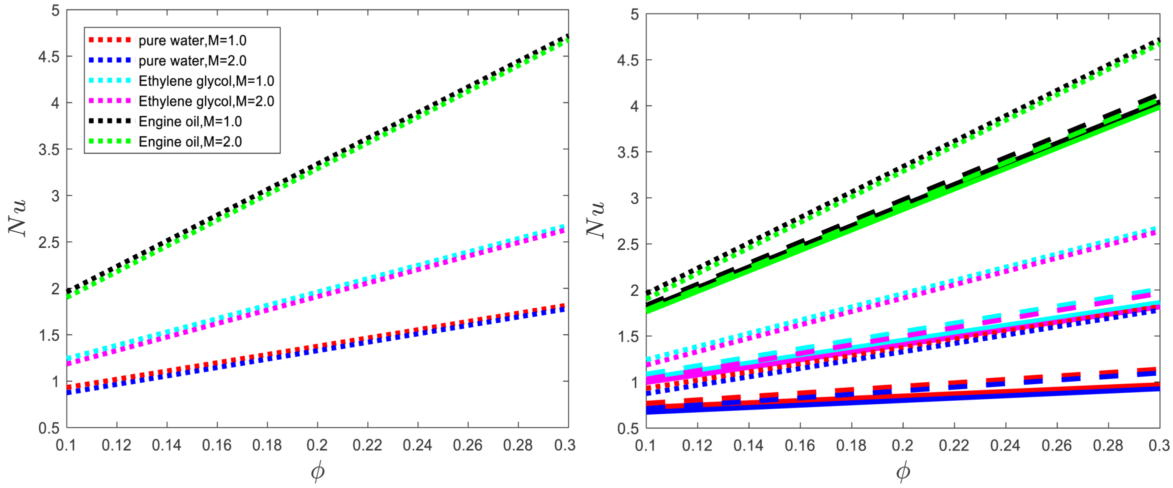


Fig. 2. (Color online) The shapes of nanoparticles under consideration.

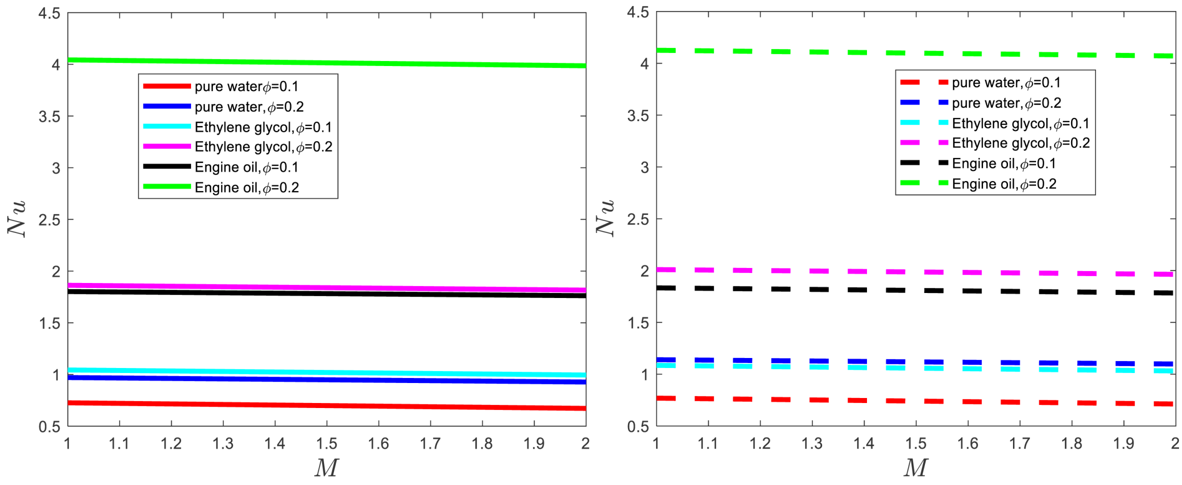


Sphere shape Platelet shape



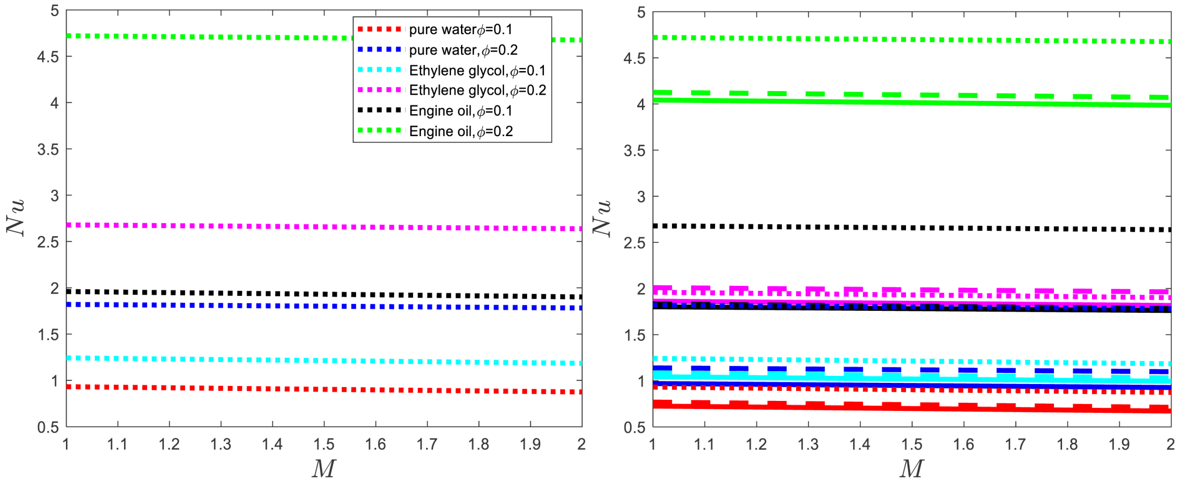
Lamina shape **Sphere, Platelet and Lamina shapes**

Fig. 3. (Color online) The effect of M on Nusselt number for $R_d = 2.0$.



Sphere shape Platelet shape

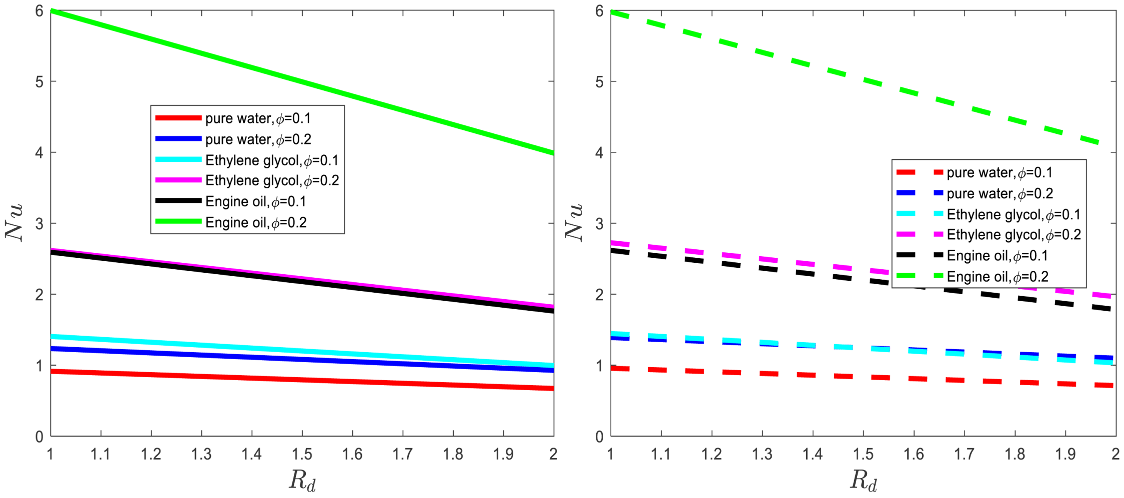
Fig. 4. (Color online) The effects of ϕ on Nusselt number for $R_d = 2.0$.



Lamina shape

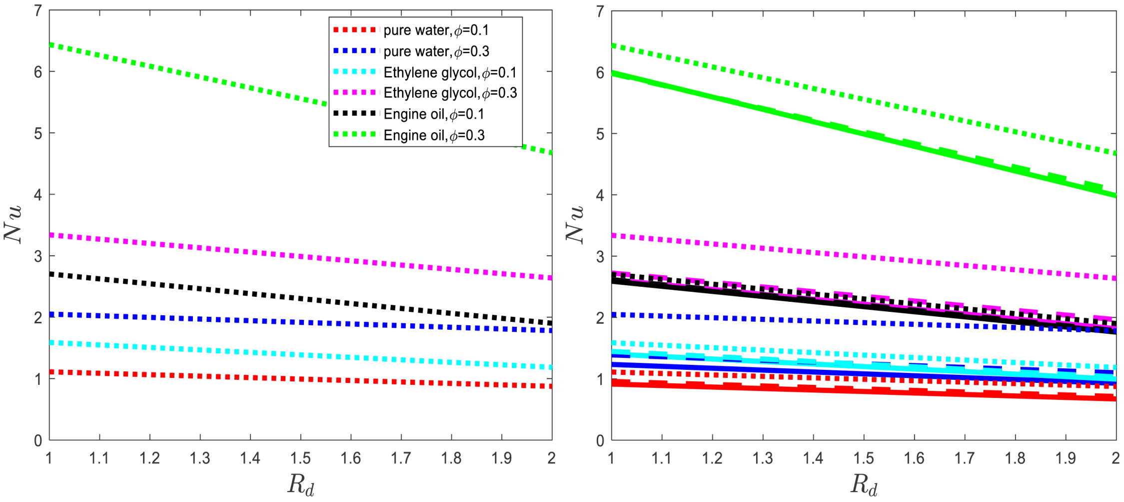
Sphere, Platelet and Lamina shapes

Fig. 4. Continued.



Sphere shape

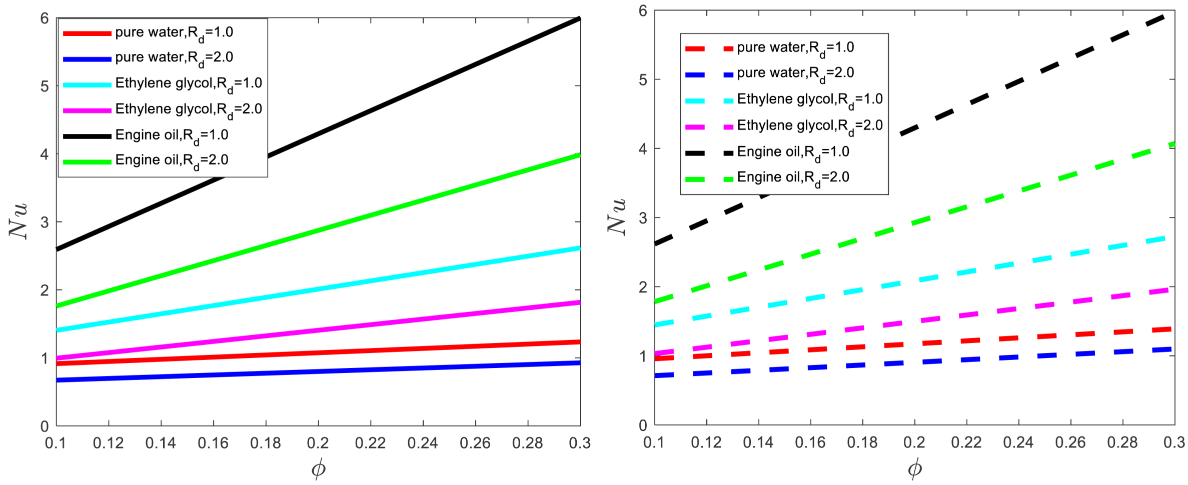
Platelet shape



Lamina shape

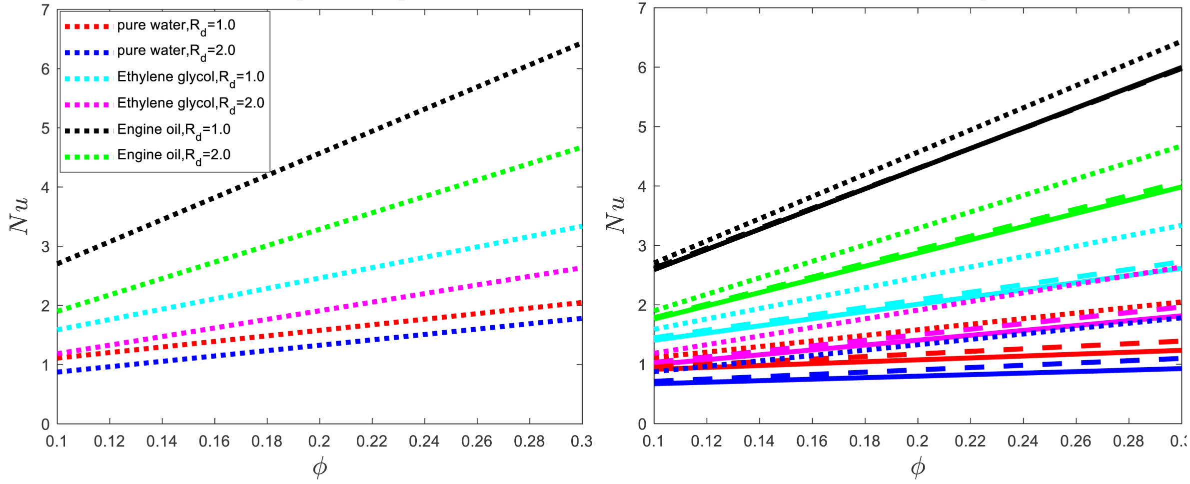
Sphere, Platelet and Lamina shapes

Fig. 5. (Color online) The effect of ϕ on Nusselt number for $M = 2.0$



Sphere shape

Platelet shape



Lamina shape

Sphere, Platelet and Laminshapes

Fig. 6. (Color online) The effect of R_d on Nusselt number for $M = 2.0$

Nusselt number in all fluids at a different value of parameters with sphere, platelet and lamina shapes nanoparticles. From Fig. 4-5, it is noted that heat transfer heat transfers for all shapes of nanoparticles increase in a various mixture of flow regime with increase the ϕ. When we increase the ϕ, the lamina shape nanoparticles played remarkable role for heat transfer in Au-Engine oil whereas, heat transfer of sphere shape nanoparticles in Au-Water is lowest, then other various mixture in the flow system. It is noted form Fig. 3 and Fig. 6, heat transfer rate of nanoparticles is inverse proportional to the M and R_d. With the influences of the M and R_d, lamina shape nanoparticles play an important role for heat transfer in Au-Engine oil, although the heat transfer rate of sphere shape nanoparticles in Au-Water is lowest than other in a various mixture in the flow regime. Figure 7 illustrates the effects of different shapes of nanoparticles on dimensionless temperature profile with the same basic fluids and selected pertinent parameters. The thermal boundary layer thickness of sphere nanoparticles in Au-Ethylene

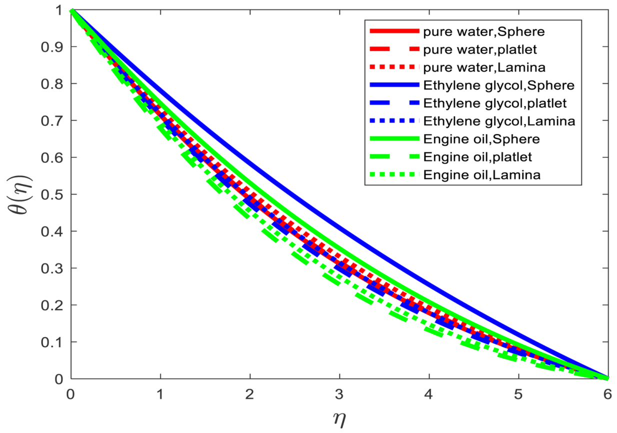


Fig. 7. (Color online) The effect of shapes on temperature profile for $M = 2.0$, $R_d = 2.0$ and $\phi = 0.02$.

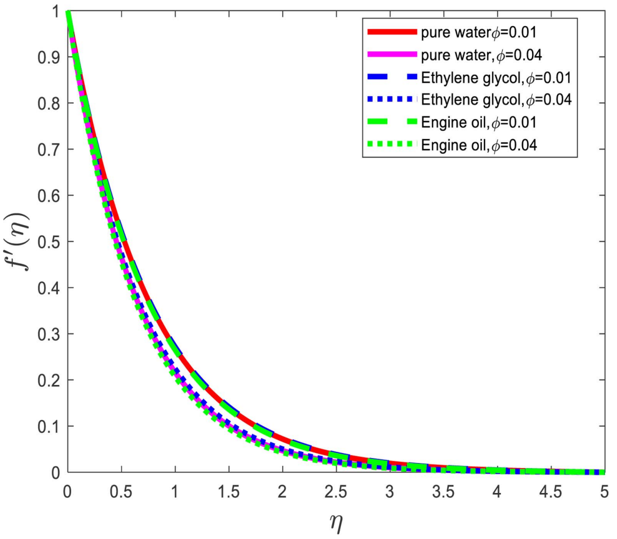
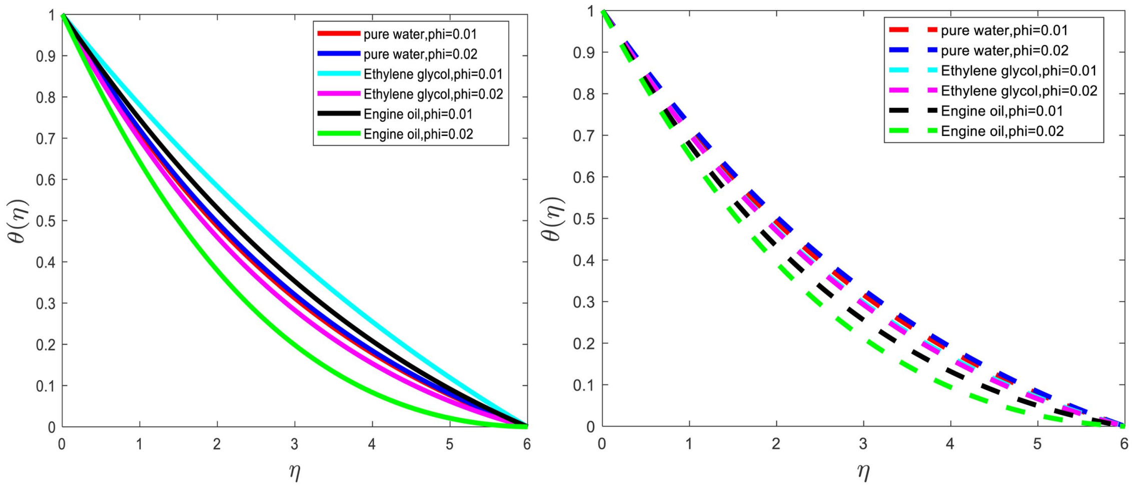
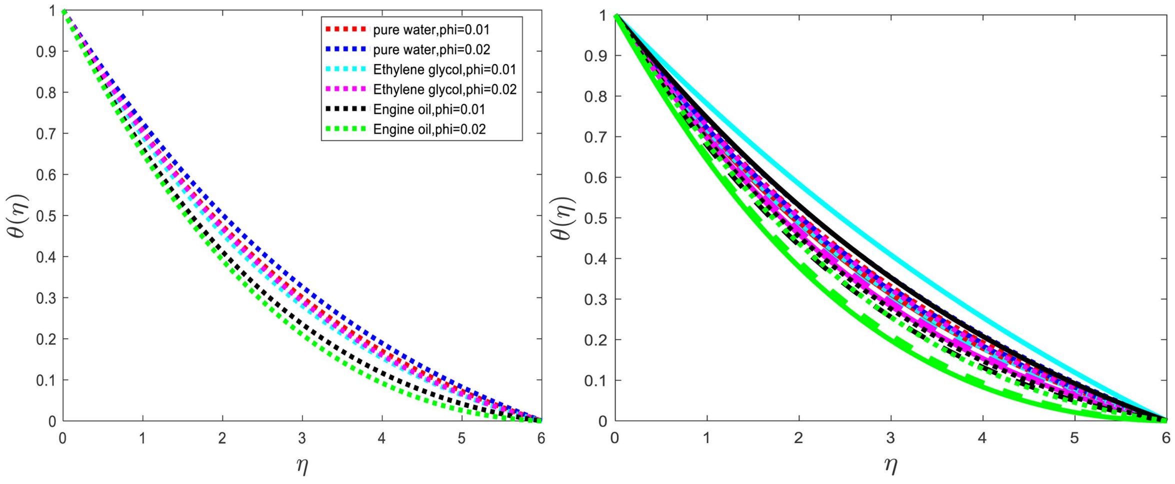


Fig. 8. (Color online) The effect of ϕ on velocity profile for $M = 2.0$.

glycol is more energetic, because of thermal conductivity and density of sphere shape nanoparticles in Au-Ethylene glycol is more significant than other mixture in regime flow. It is observed in Fig. 8, the velocity is decreasing function of ϕ. From Fig. 9, it is noted that the thermal boundary layer temperature profile (except all shapes in Au-Water and lamina nanoparticles in Au-Ethylene glycol) is decreasing function of ϕ. Physically, when ϕ intensify, the density of the nanofluid increases as a result, the fluid velocity and temperature profile decelerate. However, under the effect of ϕ, an opposite result has been found for lamina shape nanoparticles in Au-Ethylene glycol and for all shape in Au-Water. It is noted that in the presence of ϕ, sphere shape nanoparticles in Au-Ethylene glycol act an essential aspect on temperature distribution in a various mixture of flow regime. Figure 10. depicts the effects of M on velocity profiles. The velocity profile of nanoparticles decreases with an increase in the magnetic



Sphere shape   Platelet shape



Lamina shape   **Sphere, Platelet and Lamina shapes**

Fig. 9. (Color online) The effect of ϕ on temperature profile for $M = 2.0$ and $R_d = 2.0$.

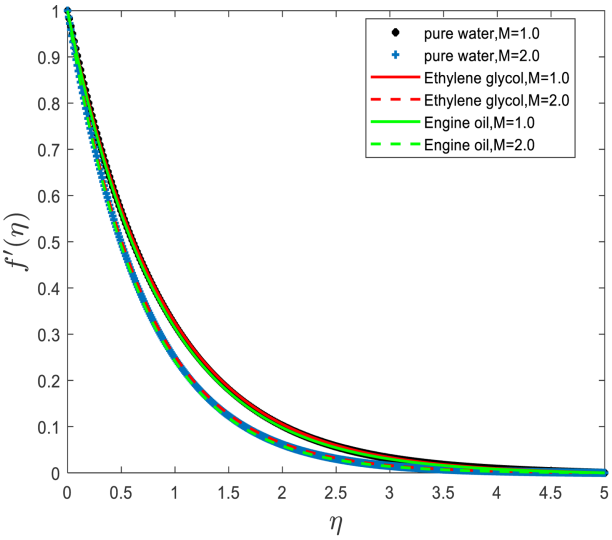
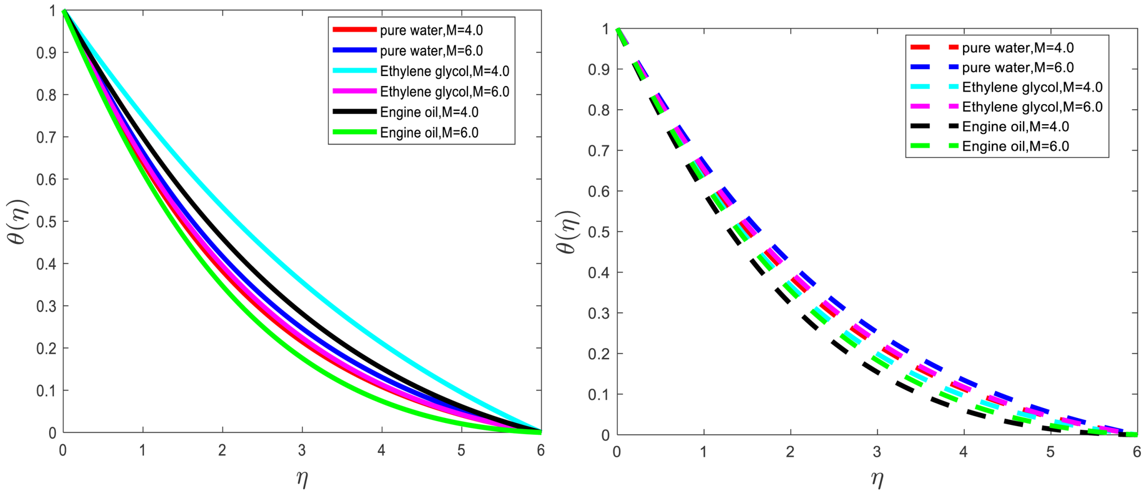


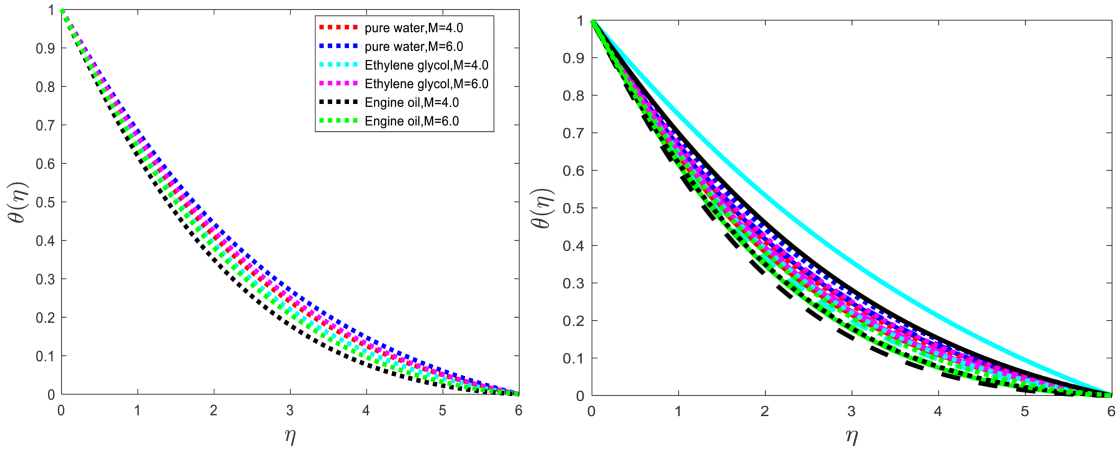
Fig. 10. (Color online) The effect of M on velocity profile for $\phi = 0.02$.

strength. Figure 11 depicts the impacts of M on temperature profile. When M increases the velocity and temperature profiles (except sphere shape nanoparticles in Au-Ethylene glycol and Au-Engine oil) decrease and increase respectively due to to induction of resistance force which opposes the motion of nanofluids. Nevertheless, an opposite trend is found in sphere shape nanoparticles in Au-Ethylene glycol and Au-Engine oil. The sphere shape Au-Ethylene glycol in the presence of M shows a prevalent role on temperature distribution. It is noted that the thermal boundary layer of engine oil is lower than ethylene glycol and pure water for all shapes of Gold (Au) nanoparticles. The Fig. 12 demonstrates the influence of R_d on temperature profiles which have direct relation (except sphere shape nanoparticles in Au-Ethylene glycol and Au-Engine oil), the reason is that for the greater value of R_d correspond to an increase in the dominance of conduction over radiation, and hence increment in the



Sphere shape

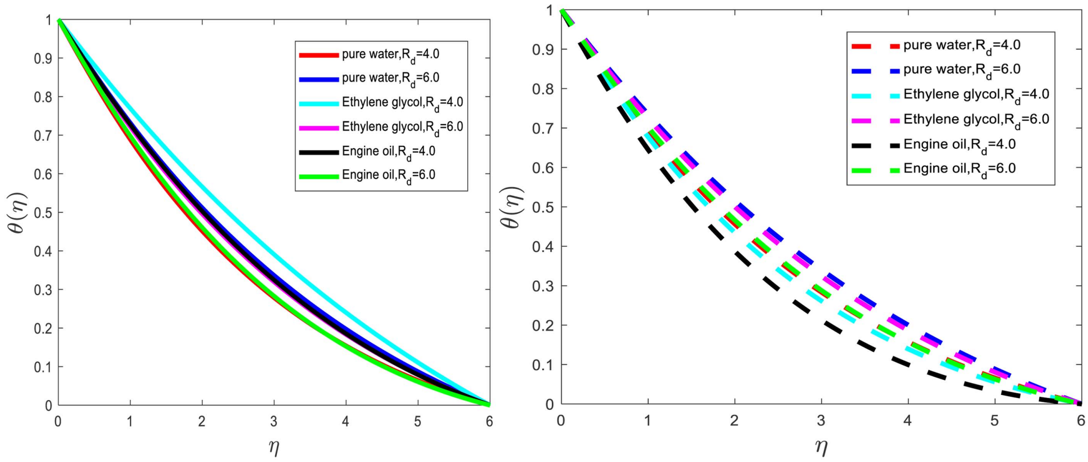
Platelet shape



Lamina shape

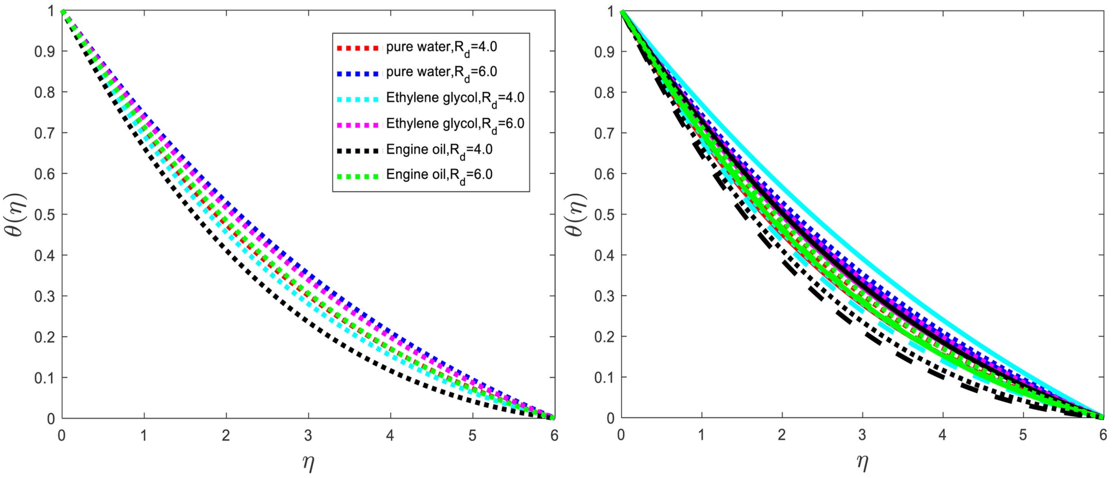
Sphere, Platelet and Lamina shapes

Fig. 11. (Color online) The effect of M on temperature profile for $R_d = 2.0$ and $\phi = 0.01$.



Sphere shape

Platelet shape



Lamina shape

Sphere, Platelet and Lamina shapes

Fig. 12. (Color online) The effect of R_d on temperature profile for $M = 2.0$ and $\phi = 0.02$.

buoyancy force and the thermal boundary layer thickness. It is interesting to note that the behavior of sphere shape nanoparticles under the effect of R_d the ethylene glycol and engine oil dissimilar to other various mixture of flow regimes. From figure, it has observed that with increasing the thermal radiation thermal boundary layer of sphere shape nanoparticles Au-Ethylene glycol is stronger than other mixture in flow regime.

To varifiaction of our results, for $R_d = 0$, $\frac{k_f((1-\phi)(\rho Cp)_f+\phi(\rho Cp)_s)}{k_{n_f}(\rho Cp)_f} = 1$. we have compaired our results with [42, 43] (see in Table 3) and a successful conclusion has been achieved.

Table 3 varifiaction of results for $\theta'(0)$.

Pr	Present Result	Anderson and Aaseth [43]	Bachok *et al.* [42]
10	1.68025	–	1.6803
1	0.44390	–	0.4437
0.7	0.34748	0.3492	0.3492

5. Conclusion

The impacts of different shapes of Gold (Au) nanoparticles on magnetohydrodynamic boundary layer and heat transfer in the presence of thermal radiation have discussed numerically. The impact of different shapes of nanoparticles namely sphere, platelet and lamina in flow system are determined. Non-linear thermal radiation has taken into account. Hamilton-Crosser model is employed. The influences of pertinent governing parameters including solid volume fraction, magnetic field and thermal radiation on velocity and temperature profiles and heat transfer rate have explored with details. The Prandtl number (Pr) of the fluids was fixed at 7.8. The deduced observations are the following:

The sphere shape nanoparticles in Au-Ethylene glycol have shown dominance for disturbance in thermal boundary layer thickness in a various mixture of flow regime. In the capability of solid volume fraction, magnetic strength and thermal radiation, lamina shape nanoparticles in Au-Engine oil showed higher heat transfer rate in a various mixture of flow regime. Heat transfer rate of platelet shape nanoparticles in a various mixture of flow system was found amid of sphere and lamina shapes nanoparticles.

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