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Non Darcy Mixed Convection Flow of Magnetic Fluid over a Permeable Stretching Sheet with Ohmic Dissipation

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This paper aims to discuss the Non Darcy boundary layer flow of non-conducting viscous fluid with magnetic ferroparticles over a permeable linearly stretching surface with ohmic dissipation and mixed convective heat transfer. A magnetic dipole is applied "a" distance below the surface of stretching sheet. The governing equations are modeled. Similarity transformation is used to convert the system of partial differential equations to a system of non-linear but ordinary differential equations. The ODEs are solved numerically. The effects of sundry parameters on the flow properties like velocity, pressure, skin-friction coefficient and Nusselt number are presented. It is deduced the frictional resistance of Lorentz force decreases with stronger electric field and the trend reverses for temperature. Skin friction coefficient increase with increase in ferromagnetic interaction parameter. Whereas, Nusselt number decrease.

Keywords : Ferromagnetic particle, Buoyancy effects, line source dipole, Non-Darcy Porous medium, ohmic dissipation, heat transfer

1. Introduction

Ferro magnetic fluid is colloidal dispersion of magnetic nano size particles and viscous carrier fluid. Usually, maghemite (γ-Fe_2O_3), magnetite (FeO. Fe_2O_3) or cobalt ferrite (CoO. Fe_2O_3) are used as nano particles in these fluids [1]. Magnetic fluids can be categorized as a type of functional fluids whose flow can be controlled by variation in external magnetic field. The practical applications of such fluids induced scientist and numerical analyst to work in this field. Neuringer [2] simulated the effects of external magnetic field on two dimensional stagnation point flow of a heated ferrofluid against a cold wall and the two dimensional parallel flow along a wall with linearly decreasing surface temperature. Ganguly *et al.* [3] investigated the flow of hot ferrofluid in a channel. He considercold wall which is under the influence of a line source dipole. It was shown that the local vortex resulted from the magnetic field alters the advection energy transport and enhances the heat transfer. More recently, Sheikholeslami and Ganji [4] investigated the effects of an outer magnetic field on flow of ferrofluid in a semi annulus along with heat transfer effects. Sheikholeslami *et al.* [5] extended the mentioned work and investigated the effects of thermal radiation on heat flow in ferrofluid. The studies related to the use of ferrofluids with MHD nanofluid on different geometries have been conducted by [6-9].

The problem on non-Darcian flow phenomena over solid stretching surfaces has received much attention in recent year. Elbashbeshy [10] analyzed the influence of non-Darcian porous medium with mixed convection along a horizontal surface. Whitaer [11] shows the derivation of Darcy's law with the Forchheimer correction for homogeneous porous medium. Pal and Mondal [12] studied boundary layer flow in non-Darcian porous media driven by mixed convection with simultaneous effects of non-uniform heat source/sink and non-uniform viscosity. Chen [13] considered the problem of Ohmic heating of MHD driven by natural convection. Pal and Mondal [14] performed flow and heat transfer analysis over a stretching surface with perpendicular magnetic field embedded in non-Darcian porous medium. He also discusses the effects of thermal radiation and Ohmic dissipation.

Motivated by applications of such studies, this present analysis deals with mixed convective flow of ferromagnetic fluid over a stretching surface embedded in Non-Darcian porous media with ohmic dissipation. The effect

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of various physical parameters like Hartmann number (Ha), Electric parameter (E_1), Drag coefficient (F^*) and mixed convection parameter (G^*) has been analyzed on velocity, temperature and pressure profile.

2. Mathematical Formulation

In this study two dimensional steady incompressible viscous base fluid with ferromagnetic particles are taken in account. It is assumed that an electrically conducting fluid is set to flow over a stretching sheet with a magnetic dipole "a" distance below origin. The wall is stretched with a linear velocity u_w while keeping the origin fixed as shown schematically in Fig. 1. A transverse magnetic fields $\vec{B} = (0, B_0, 0)$ and electric field $\vec{E} = (0, 0, -E_0)$ is applied across the sheet. The magnetic field are governed by Maxwell's equation. The ferromagnetic fluid is saturate with magnetic field due to the influence of point dipole. The temperature at the surface is maintained T_w and Curie temperature is assumed to be T_c. At this temperature fluid unable to magnetize until it starts to cool. The flow of ferrofluid are controlled using magnetic dipole. Hence, the scalar magnetic potential is given by

$$\Phi = \frac{\gamma}{2\pi}\left(\frac{x}{x^2+(y+a)^2}\right), \tag{1}$$

where γ displays the strength of magnetic field due to point source. The magnetic field intensity can be defined as

$$H_x = -\frac{\partial \Phi}{\partial x},\ H_y = -\frac{\partial \Phi}{\partial y}, \tag{2}$$

Hence, the magnitude of magnetic field intensity H can be given as

$$H = \left[\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2\right]^{\frac{1}{2}}, \tag{3}$$

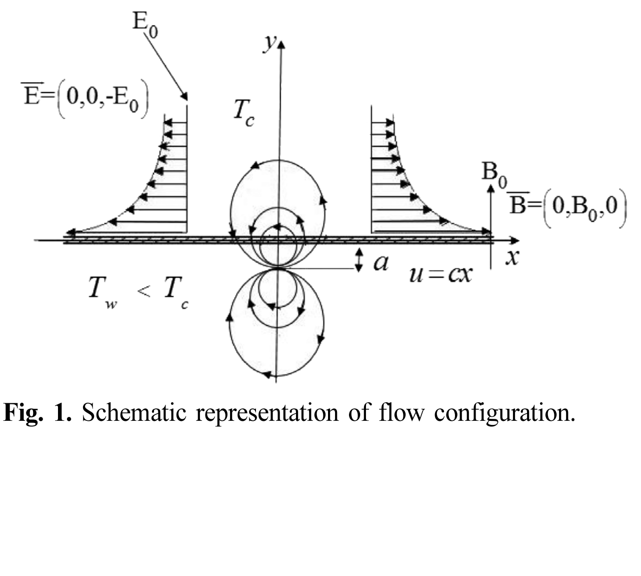

Fig. 1. Schematic representation of flow configuration.

Expanding using Taylor series for x up to $O(x^2)$.

$$\frac{\partial H}{\partial x} = -\frac{\gamma}{2\pi}\left(\frac{2x}{(y+a)^4}\right),\ \frac{\partial H}{\partial y} = \frac{\gamma}{2\pi}\left(\frac{-2}{(y+a)^3} + \frac{4x^2}{(y+a)^5}\right), \tag{4}$$

Andersson and Valnes [15] assumed magnetization M as a linear function of temperature T as $M = K^*(T_c - T)$, where K^* is called pyromagnetic coefficient.

For a porous space having permeability K of the porous medium and F is Forchheimer correction, the resistive force generated by this is $\frac{\nu}{K}u + Fu^2$. It reduces to first order Darcy resistance if $F = 0$. $\mu_0 M\frac{\partial H}{\partial x}$ and $\mu_0 M\frac{\partial H}{\partial y}$ depicts force per unit volume due magnetization and if magnetic gradient vanishes these forces will disappear. It give birth to adiabatic magnetization in thermal boundary layer. The flow behavior on the boundaries are defined with no-slip effects and variable wall temperature. The boundary conditions are

$$u = u_w = cx,\ v = 0,\ T = T_w = T_c - \frac{Dx^2}{l^2} \text{ at } y = 0 \tag{5}$$

$$u = 0,\ T = T_c,\ p + \frac{1}{2}\rho(u^2 + v^2) = \text{constant as } y \to \infty \tag{6}$$

where $c > 0$ is the stretching rate, $l = \sqrt{\frac{\nu}{c}}$ is the characteristic length. To model the flow situation continuity equation, Navier-Stoke's equation and heat equation along with body forces described in Eqs. (1)-(4) and assuming steady incompressible flow [16]. Equations reduces to

$$f''' + ff'' - f'^2 + 2P_2 + \frac{2\beta\theta_1}{(\eta+\alpha_1)^4} + Ha^2(E_1 - f')$$
$$- K_1 f' - F^* f'^2 + G^*\theta_1 = 0, \tag{7}$$

$$P_1' - f'' - ff' - \frac{2\beta\theta_1}{(\eta+\alpha_1)^3} - K_1 f + -Ha^2 f = 0, \tag{8}$$

$$P_2' - \frac{2\beta\theta_2}{(\eta+\alpha_1)^3} + \frac{4\beta\theta_1}{(\eta+\alpha_1)^5} = 0, \tag{9}$$

$$\theta_1'' + \Pr(f\theta_1' - 2f'\theta_1) + \frac{2\lambda\beta(\theta_1 - \varepsilon)}{(\eta+\alpha_1)^3} - 4\lambda(f')^2$$
$$- \Pr Ha^2 E_c(f' - E_1)^2 = 0, \tag{10}$$

$$\theta_2'' - \Pr(4f'\theta_2 - f'\theta_2) - \lambda\beta(\theta_1 - \varepsilon)$$
$$\times\left[\frac{2f'}{(\eta+\alpha_1)^4} + \frac{4f}{(\eta+\alpha_1)^5}\right] + \frac{2\lambda\beta\theta_2 f}{(\eta+\alpha_1)^3} - \lambda(f'')^2 = 0, \tag{11}$$

Also the boundary conditions (6) and (7) are transformed as

$$f = 0,\ f' = 1,\ \theta_1 = 1,\ \theta_2 = 0, \quad \text{at} \quad \eta = 0, \qquad (12)$$

$$f' \to 0,\ \theta_1 \to 0,\ \theta_2 \to 0,\ P_1 \to -P_\infty,\ P_2 \to 0 \text{ as } \eta \to \infty, \qquad (13)$$

After using transformations describe by Andersson and Valnes [15] and Zeeshan *et al.* [16]. Here, prime is the derivate w.r.t. η, which is the dimensionless coordinates defined as

$\eta = \sqrt{\frac{c\rho}{\mu}} y$. The components of velocity can be derived as

$$u = \frac{\partial \Psi}{\partial y} = cx.f'(\eta),\ v = -\frac{\partial \Psi}{\partial x} = -\sqrt{cv}.f\ (\eta), \qquad (14)$$

where β is the ferro-magnetic parameter, λ is viscous dissipation parameter, $G^* \geq 0$ is the buoyancy or mixed convection parameter, Pr is Prandtl number, K_1 is Daracy porosity parameter, Ha is Hartmann number, E_c is Eckert number, E_1 is the local electric parameter, F^* is Forchheimer correction and ε is Curie temperature ratio.

The local skin friction coefficient C_{f_x} for the shear stress on the fluid and rate of heat transfer defined as local Nusselt number Nu_x are important physical properties, can be defined in,

$$C_f \mathrm{Re}_x^{1/2} = -2f''(0),\ Nu_x/\mathrm{Re}_x^{1/2} = -(\theta_1'(0) + \xi^2 \theta_2'(0)), \qquad (15)$$

where $\mathrm{Re}_x = \frac{\rho c x^2}{\mu}$ is the local Reynold number. Ferromagnetic parameter β directly effects the flow field. The non-dimensional differential Eqs. (7)-(11) are highly nonlinear and cannot be solved analytically, hence, Runge-Kutta Fehlberg fourth-order technique is used. The step-size is taken as $\nabla \eta = 0.01$ to fulfil the convergence criteria 10^{-5}.

3. Result and Discussion

The main aim of present study is to investigate the non-Darcy mixed convection flow of ferromagnetic fluid over a stretching sheet through porous medium along with Ohmic dissipation. Effects of various physical parameters are discussed with the help of figures and tables. Throughout the problem the values of the parameters for computational work are taken as $\mathrm{Pr} = 0.72$, $\lambda = 0.01$, $Ec = 0.1$, $\varepsilon = 2.0$, $\alpha_1 = 1.0$. The effects of sundry parameters on velocity, pressure temperature, skin friction and heat transfer rate are displayed graphically in Figs. 2-12. Figs. 2 and 3 depicts the influence of Hartmann number (Ha) on velocity and temperature. Figure 2 shows that Hartmann number decreased the momentum boundary layer thickness and hence velocity decreases and increases the temperature profile as seen in Fig. 3 i.e. thermal boundary layer thicken as magnetic field grow stronger it

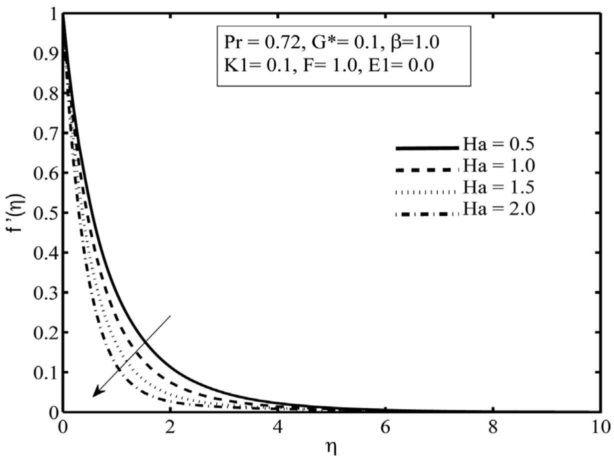

Fig. 2. Variation of f' versus η for different values of Ha.

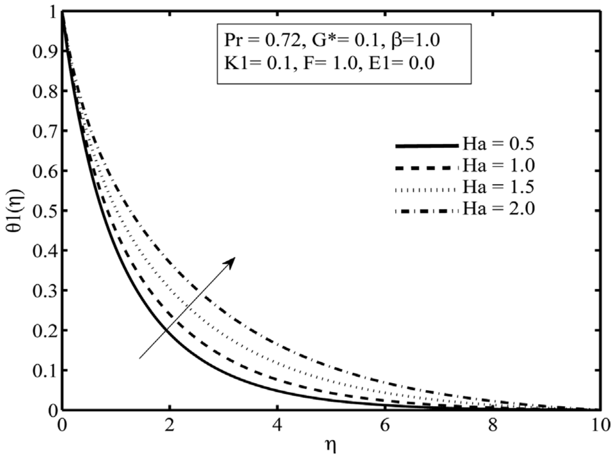

Fig. 3. Variation of θ_1 versus η for different values of Ha.

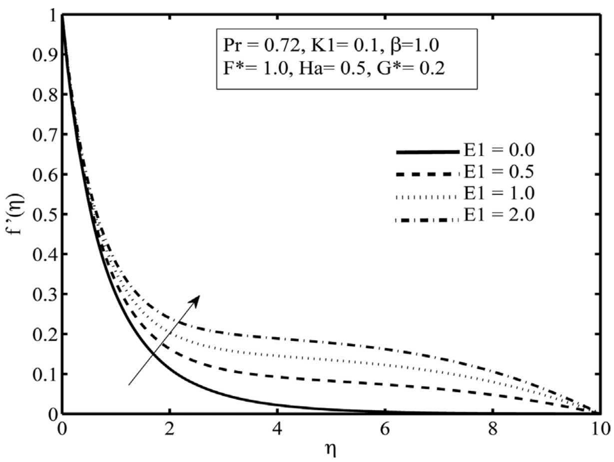

Fig. 4. Variation of f' versus η for different values of E_1.

is due to the fact that Lorentz force produces resistance in the flow and hence, increases the temperature. Figure 4 was drawn to see the effects of different values of electric

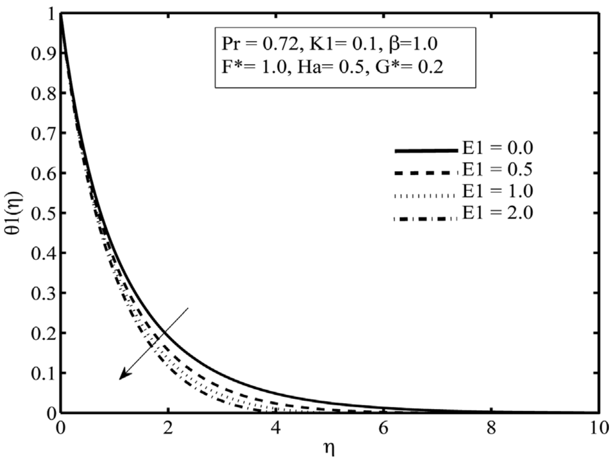

Fig. 5. Variation of θ_1 versus η for different values of E_1.

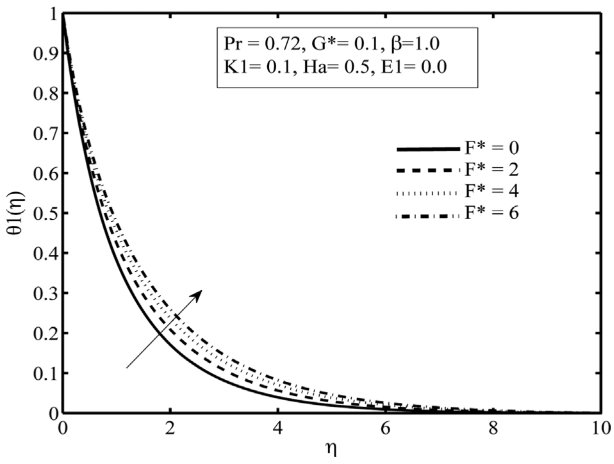

Fig. 7. Effect of porous medium of inertia coefficient F^* on temperature profile.

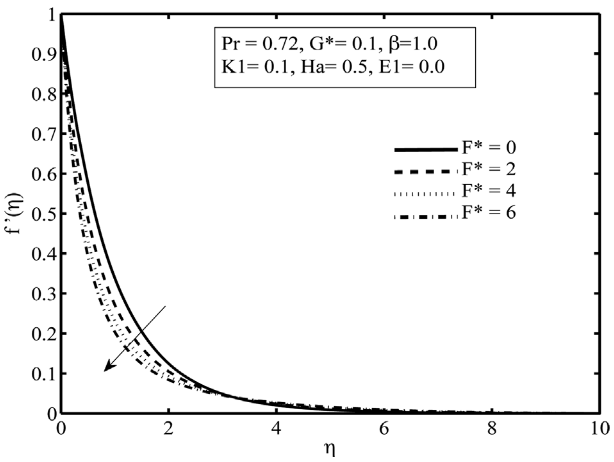

Fig. 6. Effect porous medium of inertia coefficient F^* on velocity profile.

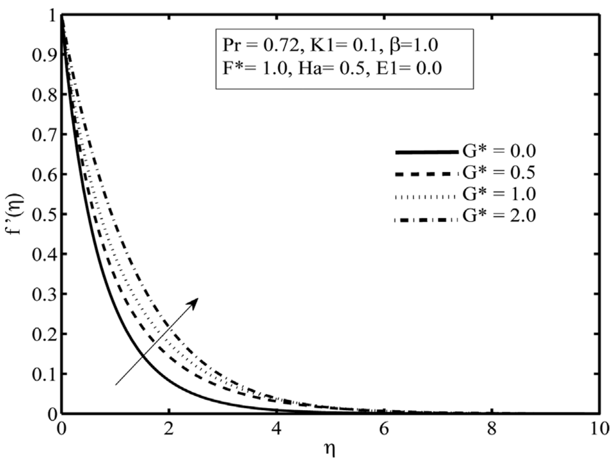

Fig. 8. Variation of f' versus η for different values of G^*.

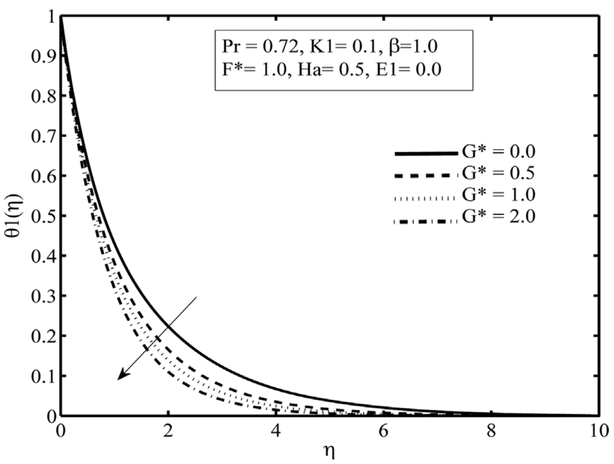

Fig. 9. Variation of θ_1 versus η for different values of G^*.

field parameter E_1 on velocity. As value of electric field parameter increases the velocity profile shows a gain in speed. The graphical results reveals that electric field assisted the flow. It is deduced the frictional resistance of Lorentz force decreases with stronger electric field, hence, the trend reverses for temperature as shown in Fig. 5.

Figures 6-7 indicates the displays the effects of the non-dimensional porous medium. For different values of Darcy-Forchheimer correction parameter F^* the graphs of velocity and temperature respectively. From Figure 6, it was depicted that drag coefficient decrease the velocity profile in the boundary layer. However, the temperature profile shows an increase in thermal boundary layer region with greater values of F^*. This is consistent with the fact that increase in resistance to the flow increases the temperature of the fluid. Figures 8 and 9 illustrate the influence of mixed convection parameter or buoyancy parameter G^* on velocity and temperature profile. It is observed from these figures that the velocity distribution increases with increase in the value buoyancy parameter G^*. Buoyancy force acts like a favorable pressure gradient

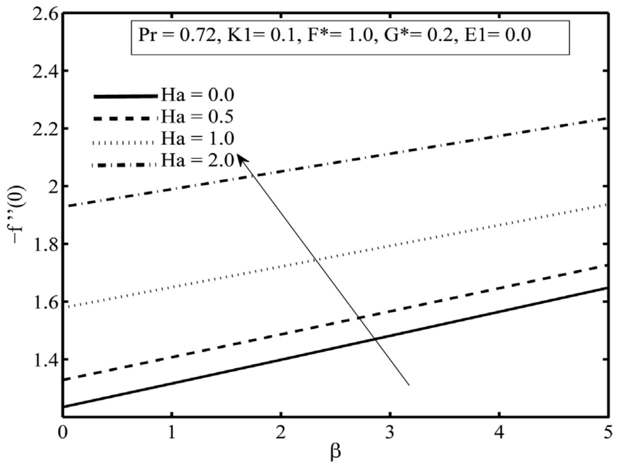

Fig. 10. Skin friction coefficient versus β for different values of *Ha*.

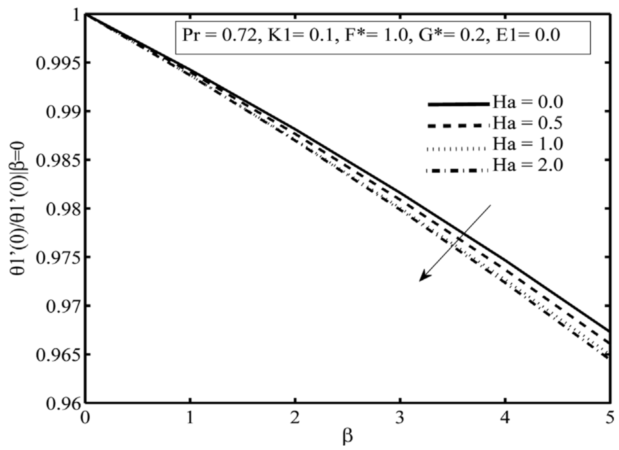

Fig. 11. Nusselt number verses β for different values of *Ha*.

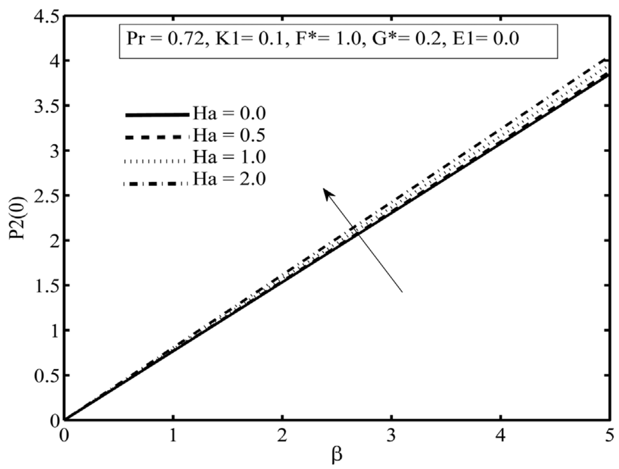

Fig. 12. Pressure profile verses β for different values of *Ha*.

which accelerates the fluid within the boundary layer, whereas the effect of buoyancy parameter G^* decrease the temperature profile in the boundary layer as shown in Fig. 9. Figures 10-12 are plotted for skin friction coefficient, Nusselt number and pressure profile against the ferromagnetic interaction parameter β for different values of Hartmann number. From the figures it observed that both skin friction coefficient and pressure increases with the variation of β as well as Hartmann number *Ha*, whereas Nusselt is decreases with the variation of β.

4. Concluding Remarks

The present paper deals with the investigation of steady, two dimensional incompressible mixed convection flow ferromagnetic fluid embedded in a non-Darcy porous space past a stretching surface. Also, the effects of Ohmic dissipation is anticipated. Fourth order Runge-Kutta Fehlberg scheme is implemented to find the solutions of nonlinear differential equations. The effects of various physical parameters are shown graphically and discussed. The following important findings of our analysis are listed below.

- Velocity decreases with increase in value of Hartmann number and local inertial coefficient whereas opposite trend is seen for temperature profile.
- The local electric parameter effects positively the velocity and decrease the temperature throughout the boundary layer.
- Skin friction coefficient and pressure profile increases with the variation of ferromagnetic interaction parameter (β) as well as Hartmann number (*Ha*).
- Nusselt number is decreases with the variation of ferromagnetic interaction parameter (β).

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