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A Robust Optimization Method Utilizing the Variance Decomposition Method for Electromagnetic Devices

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Uncertainties in loads, materials and manufacturing quality must be considered during electromagnetic devices design. This paper presents an effective methodology for robust optimization design based on the variance decomposition in order to keep higher accuracy of the robustness prediction. Sobol' theory is employed to estimate the response variance under some specific tolerance in design variables. Then, an optimal design is obtained by adding a criterion of response variance upon typical optimization problems as a constraint of the optimization. The main contribution of this paper is that the proposed method applies the variance decomposition to obtain a more accurate variance of the response, as well save the computational cost. The performance and robustness of the proposed algorithms are investigated through a numerical experiment with both an analytic function and the TEAM 22 problem.

Keywords : robust optimization, variance decomposition, electromagnetic device, TEAM 22 problem

1. Introduction

The conventional designs may not always be robust against the interference due to uncertainties that exist in different sources. In order to improve the performance of a product, robust optimization is employed to tackle the uncertainties. The target of robust design is to improve the quality of products by reducing their sensitivity to variations, thereby reducing the effects of variability [1].

A number of robust optimization methods have been proposed to be applied to electromagnetic devices [2-6]. A large part of the optimization approaches use the gradient index (GI) of parameters to represent the robustness due to its simplicity and low computational cost [7]. GI methods based on the sensitivity analysis (SA), especially the first-order SA, are extensively reported to evaluate the robustness of a design [2, 3]. In order to provide more accurate results under uncertainty, higher order (i.e., second-order) sensitivity analysis is employed in Ref. [4]. For the same reason, a second-order sensitivity assisted worst case optimization (SA-WCO) method has been proposed in Ref. [5], providing a faster calculation while keeping high accuracy. Moreover, the values obtained from SA are modified utilizing Taguchi's quality method (TM) to improve the accuracy in Ref. [6].

In practice, dimensions of electromagnetic devices may vary within the range of tolerances due to manufacturing process and cost limitation [8]. However, GI-based robust designs are only based on the local derivative at the nominal value. Although these designs are reported providing good performance, they still are grouped to the deterministic robust optimizations that essentially neglect information surrounding the nominal value. Thus, in contrast to the previous works, we develop a more accurate evaluation of robustness regarding problems having tolerances in design variables. Here, we calculate the variance of the response through a fully quantitative variance-based global analysis, which is also called Sobol' variance decomposition [9, 10]. This decomposition identifies the most important sources of the uncertainty and focus attention only on those dimensions of input space. Then the variance of the response would be briefly denoted by the main contributed terms. More details about the theory and applications can be found in the Ref. [11-14].

This paper, in the view of above, proposes a robust optimization method concerning tolerances in design variables. The response variance with respect to ranges of design variables is estimated based on Sobol' variance decomposition theory. The formulation of optimization is defined by adding a criterion of response variance upon

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the typical optimization problem as a constraint. The proposed method is applied to two examples, one an analytic function used to demonstrate the calculations and to provide a comparison with other variance estimated methods, the second a TEAM 22 problem is used to demonstrate its validity for electromagnetic devices optimization.

2. Robust Optimization Algorithm Based on Sensitivity Analysis

2.1. Nonrobust formulation

Generally, the formula of typical optimization aiming to minimize an objective function $f(x)$, subject to constraint $g(\boldsymbol{x})$, is expressed as

$$\begin{aligned} &Minimize: \ f(\boldsymbol{x}) \\ &subject\ to: \ g(\boldsymbol{x}) \le 0 \\ &\qquad\qquad \boldsymbol{x}_L \le \boldsymbol{x} \le \mathbf{x}_U \end{aligned} \tag{1}$$

where $\boldsymbol{x}_L$ and $\boldsymbol{x}_U$ are the lower and upper boundaries of the design variable vector $\boldsymbol{x}$, respectively. It is clear that there is no robustness information involved in this optimization definition. To illustrate the idea, an example function $f(x)$ is chosen with respect to one-dimensional variable x with tolerance assumed as Δ, as shown in Fig. 1. Point A will be chosen as the optimal design by using optimization algorithm (1), however it will bring sharply increase when x is perturbed from the nominal value.

2.2. Classical robust optimization using gradient index

In classical robust optimization, the sensitivity index of the objective function is consideredby estimating the maximum partial derivation, thus for a n-dimensional vector of design variables $x_1, x_2, \ldots, x_n$, $GI(\boldsymbol{x})$ defined as

$$GI(x) = \max_{1 \le i \le n} \left| \frac{\mathrm{d}f(\boldsymbol{x})}{\mathrm{d}x_i} \right| \tag{2}$$

Based on (2), the classical robust optimization algorithm is formulated as follows

$$\begin{aligned} &Minimize: \ Gi(\boldsymbol{x}) \\ &Subject\ to: \ g(\boldsymbol{x}) \le 0 \\ &\qquad\qquad f(\boldsymbol{x}) \cong F \\ &\qquad\qquad \boldsymbol{x}_L \le \boldsymbol{x} \le \boldsymbol{x}_U \end{aligned} \tag{3}$$

where F is the target value of objective function $f(\boldsymbol{x})$ assuming the design is a target-aimed design, whose objective function value is decided by the designers. Typically F is set to the optimal value obtained by solving the typical nonrobust optimization problem in (1). When this method is applied to the example function in Fig. 1, the most probable result is point B, where the GI meets

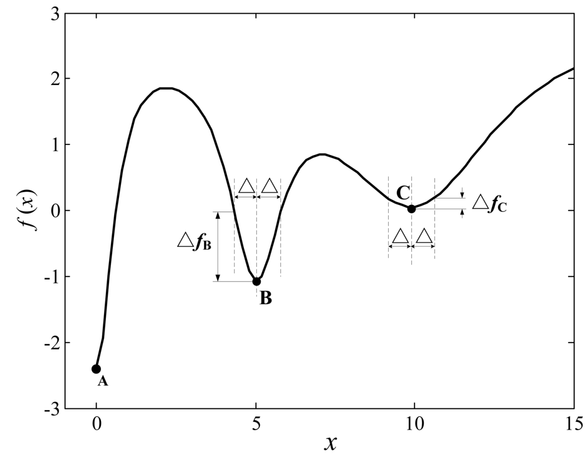

Fig. 1. Example function $\left(f(x)=3-\frac{5}{1+x^2/0.4}-\frac{3.2}{1+(x-5)^2}-\frac{2.8}{1+(x-10)^2/10}\right)$.

zero. However, the absolute deviation Δf_B over the range of variable (i.e., 2Δ) may exceed an acceptable tolerance of response. Consequently, point B may not be a robust design as desired.

2.3. Proposed optimization using variance decomposition method

A robust optimization should concern the absolute change of response introduced by the input variations. As the example in Fig. 1, another local optimum point C giving a smaller change Δf_C becomes an alternative result for the robust optimization problem. The most common index is the response variance to measure output variation due to tolerances in design variables. Therefore, we propose a robust optimization as follows

$$\begin{aligned} &Minimize: \ f(\boldsymbol{x}) \\ &Subject\ to: \ V \le V_{crit} \\ &\qquad\qquad g(\boldsymbol{x}) \le 0 \\ &\qquad\qquad \boldsymbol{x}_L \le \boldsymbol{x} \le \boldsymbol{x}_U \end{aligned} \tag{4}$$

where V is the response variance, V_{crit} is the boundary of variance, which is defined according to the specific problem.

One way to get the variance of the response, denoted as V, is a point method where, through the transmission of moments using Taylor series expanded about the mean, approximation of the variance in matrix form is given as

$$V \approx \frac{\partial f(\boldsymbol{\mu}_x)}{\partial \boldsymbol{x}^{\mathrm{T}}} \boldsymbol{V}_x \left(\frac{\partial f(\boldsymbol{\mu}_x)}{\partial \boldsymbol{x}^{\mathrm{T}}} \right)^T \tag{5}$$

where $\boldsymbol{\mu}_x$ and $\boldsymbol{V}_x$ are the means and the covariance matrix of the design variables.

Another way to get the variances uses a global-based method sometimes called analysis of variance. More particularly, in this paper we develop the Sobol' variance decomposition to provide variance. Compared with the Taylor series-based approach based on the point value, it considers the variance of response over the entire feasible region, which specifies the range of $\boldsymbol{x}$. Moreover, it assigns the response variance to each design variable and the cross terms of design variables. Generally, for electromagnetic problems, analytic Sobol' method may become difficult to be implemented since the response model is nonlinear and complex. Hence, Monte Carlo simulation is employed to estimate the Sobol' indices. Consider two random vectors $\boldsymbol{x}$ and $\boldsymbol{x}'$, and let $\boldsymbol{x} = (\boldsymbol{y}, \boldsymbol{z})$ and $\boldsymbol{x}' = (\boldsymbol{y}, \boldsymbol{z}')$, where $\boldsymbol{y}$, $\boldsymbol{z}$ and $\boldsymbol{z}'$ are the subsets of the corresponding vector. After N trials, crude Monte Carlo estimates are obtained as [10]

$$\frac{1}{N}\sum_{j=1}^{N} f(\boldsymbol{x}_j) \to^{p} f_0$$
$$\frac{1}{N}\sum_{j=1}^{N} f(\boldsymbol{x}_j)\,(\boldsymbol{x}_j') \to^{p} V_{\boldsymbol{y}}^{f} + f_0^2 \tag{6}$$

where f_0 is the mean value of the response function, V^f is the variance of response, $V_{\boldsymbol{y}}^f$ is the variance of $f(\boldsymbol{x})$ introduced by the subset of variables $\boldsymbol{y}$, and $\to^p$ implies absolute convergence. Therefore, the global sensitivity measures, called first-order Sobol' indices, are provided by

$$S_{x_i}^f = \frac{V_{x_i}^f}{V^f} \tag{7}$$

where $S_{x_i}^f$ and $V_{x_i}^f$ are the Sobol' index and the variance of function $f(\boldsymbol{x})$ with respect to the ith element of $\boldsymbol{x}$. In practice, the sum of first-order Sobol' indices, denoted as $S^{(1)}$, usually makes up a large part of the whole variance. If all or part of the first order Sobol's index satisfies the condition $1 - S^{(1)} < \varepsilon$, where ε is a small number (in this paper, $\varepsilon = 0.1$), the rest of Sobol' indices are negligible. Thus, the corresponding sum of variance $V^{(1)}$ is obtained and used to represent the response variation due to tolerances in design variables.

Latin Hypercube sampling is used in this paper to generate sample points because of its significant less computational expense. Assume that M sample sets with respect to n variables are generated as $\{x_1^{(1)}, x_2^{(1)}, \ldots, x_n^{(1)}\}$, $\{x_1^{(2)}, x_2^{(2)}, \ldots, x_n^{(2)}\}$, … , $\{x_1^{(M)}, x_2^{(M)}, \ldots, x_n^{(M)}\}$ using LH sampling. Here, the superscripts $^{(1)}$, $^{(2)}$, … , $^{(M)}$ indicate the sequential number of M sampling data for each variable x_i ($i = 1, 2, \ldots, n$). To accomplish variance estimation given in (6), extra sets of samples are needed to bring in. There is no doubt that there are many ways to rearrange the samples based on (6). The basic principal is holding the variable to be studied in the original place while changing the sequences of the other terms. For example, to calculate $V_{x_1}^f$, another M sets of samples $\{x_1^{(1)}, x_2^{(2)}, \ldots, x_n^{(2)}\}$, $\{x_1^{(2)}, x_2^{(3)}, \ldots, x_n^{(3)}\}$, … , $\{x_1^{(M)}, x_2^{(1)}, \ldots, x_n^{(1)}\}$ should be formed. Similarly, n-1 else sets of samples would be recombined to study the other n-1 variables. In summary, the general framework of the proposed robust optimization algorithm is depicted in Fig. 2, and it can be described in the following steps:

(1) According to the information on the uncertain parameters and the system response, determine the basic optimization definitions, for example, the objective function, the tolerances, upper and lower limits of the n-dimensional design variables.

(2) Generate a set of random design variables $\boldsymbol{x}$ with M samples, based on which another n sets ($M \times n$ sample points) would be regenerated.

(3) Get the studied responses from finite element analysis (FEM) and construct the approximate models of the objective function and the constraint function in (4). In this paper, radial basis function (RBF) is employed and considered as an accurate fit with the maximum approximation errors (percentage errors) equal to 10%.

(4) By using Sobol' variance decomposition method,

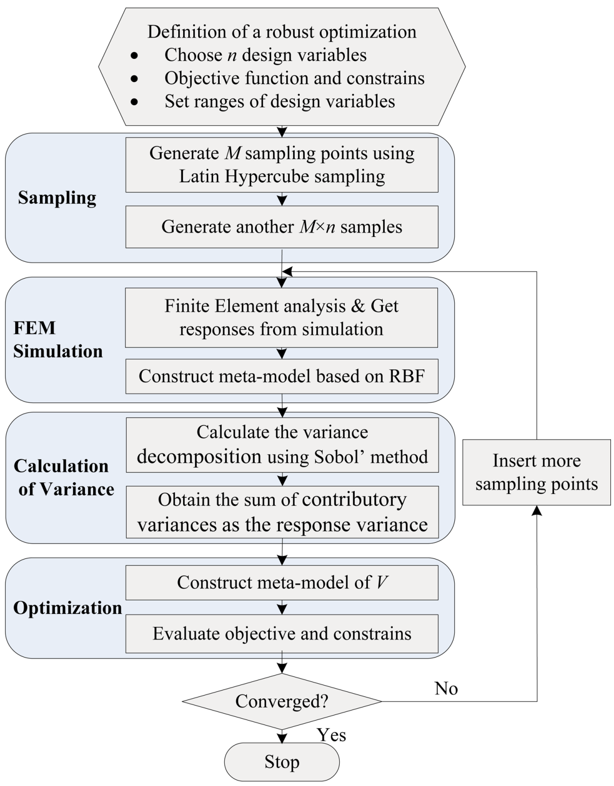

Fig. 2. (Color online) Summaries of the proposed robust optimization method.

consider the less significant variables as deterministic without variation, and at the same time obtain the sum of the main contributory variances as the response variance.

(5) Run the optimization and check convergence. If there is a feasible result, go to the 6th step; otherwise, insert more sampling points and go to the 3rd step.

(6) Obtain the optimum design and the corresponding objective function value.

3. Optimization Results

3.1. Analytic function

An analytic function is presented to verify the performance of the proposed algorithm formulated as

$$f(\boldsymbol{x}) = 2 - e^{\frac{(x_1-1)^2+(x_2-1)^2}{-1}} - 0.8e^{\frac{(x_1-1)^2+(x_2-3)^2}{-1.5}} - 1.2e^{\frac{(x_1-3)^2+(x_2-3)^2}{-0.5}} \quad (8)$$

with independent variables $\boldsymbol{x} = \{x_1, x_2\}$ uniformly distributed in the range of $0 \le x_1, x_2 \le 4$, and the response surface is shown in Fig. 3. In this case, assume the ranges are set to 1 for both x_1 and x_2 (i.e. the tolerance is ± 0.5). Therefore, the robust optimization problem using proposed method is given as

$$\begin{aligned} &Minimize: \ f(\boldsymbol{x}) \\ &subject\ to: \ V \le V_{crit} \\ &\qquad\qquad 0 \le x_1, x_2 \le 4 \end{aligned} \quad (9)$$

(7×7) points, shown as the blue points in Fig. 4, are chosen as the training set to construct a meta-model of response variance based on Radius Basis Function (RBF), who can also provide the approximate variance decomposition value over the entire range of variables [15]. Since the parameters have a tolerance of ± 0.5, the range of Monte

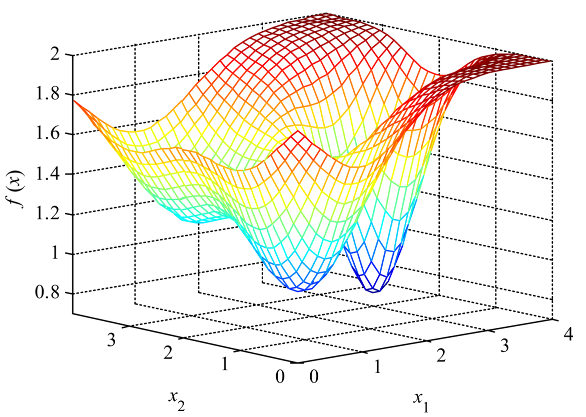

Fig. 3. (Color online) Response of the analytic function.

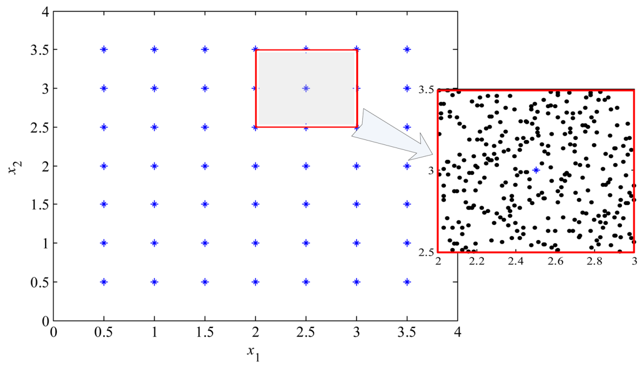

Fig. 4. (Color online) Basic information of the optimization definition.

Carlo samples for one training point is graphically represented as the shaded red box in Fig. 4. For each training point, 3000 samples are generated within the variable range, 1000 of which are generated using Latin Hypercube method and another 2000 points are obtained from resampling. The samples are indicated by black dots in Fig. 4. In this case, the percentage error of the approximating model by means of 49 points is 2.3%.

As shown in Fig. 3, the function $f(\boldsymbol{x})$ has three troughs at positions (1, 1), (1, 3) and (3, 2) with the objective function value of 0.9444, 1.1816 and 0.7647, respectively. Graphically, one of these three positions would be the solution for the optimization problem in (9) under a certain criterion of variance given as V_{crit}. Another two ways to obtain variance: Taylor series expanded based method and Monte Carlo Simulation are employed to calculate the response variances centered at positions (1, 1), (1, 3) and (3, 2), and the results are compared with Sobol'-based method in Table 1. It can be seen that results from Sobol'-based method are close to those from MCS, which are considered accurate with a relatively large sample size (i.e., 100,000). Moreover, the sobol' index for x_1 and x_2 are generally even, thus both of the variables take crucial parts of providing response variance. However, Taylor series gives quite different results that could not show the real variation around the troughs although it is slightly faster than others.

Table 2 compares the optimization results by using the proposed algorithm under different variance criterions. It can be found that the optimization results are changing along with different variance criterions. Both Sobol'-based and MCS-based optimization give reasonable results according to Table 1. As well, the results provided by Taylor Series Expanded shift from one trough to another, however, the variances obtained by Taylor Series-based method are far from the real values, which make the robust optimization design meaningless. As to the optimi-

Table 1. Comparison of response variance at three troughs.

$\boldsymbol{x} = (x_1, x_2)$	$f(\boldsymbol{x})$	Variance V		
		Taylor Series	Sobol'	MCS (100,000 samples)
(1, 3)	1.1816	4.50×10^{-4}	2.60×10^{-3} ($Sx_1=0.52$, $Sx_2=0.48$)	2.61×10^{-3}
(1, 1)	0.9444	1.84×10^{-3}	8.69×10^{-3} ($Sx_1=0.48$, $Sx_2=0.52$)	8.72×10^{-3}
(3, 2)	0.7647	9.35×10^{-4}	2.95×10^{-2} ($Sx_1=0.48$, $Sx_2=0.52$)	2.99×10^{-2}
Average time (s)	–	0.337	0.533	0.557

Table 2. Optimization results of analytic function.

Method	V_{crit}				Average time (s)
	5×10^{-4}	5×10^{-3}	2×10^{-2}	5×10^{-2}	
Taylor series	(1, 3)	(3,2)	(3,2)	(3,2)	10.21
Sobol'	N/A	(1,3)	(1,1)	(3,2)	61.02
MCS	N/A	(1,3)	(1,1)	(3,2)	206.08

zation time for all the approaches, Taylor series-based method shows an economical solution with the lowest computational cost. Besides, due to the use of meta-modeling of response variance, the proposed Sobol' method turns out with an acceptable optimization time.

3.2. Electromagnetic application

This section presents the application of the proposed method to a practical problem in electromagnetic devices design. The example (TEAM 22 problem) chosen for study here has been used extensively in previous studies, and its configuration is shown in Fig. 5. We considered the three-parameter problem presented in [2, 16, 17]. For this problem, the shape of SMES can be defined using three design variables that are modeled by a vector $x = \{R_2, H_2, D_2\}$. All random design variables are statistically independent and distributed normally with a standard deviation of 0.02. The parameters are summarized in Table 2.

The following specifications have to be satisfied in this case.

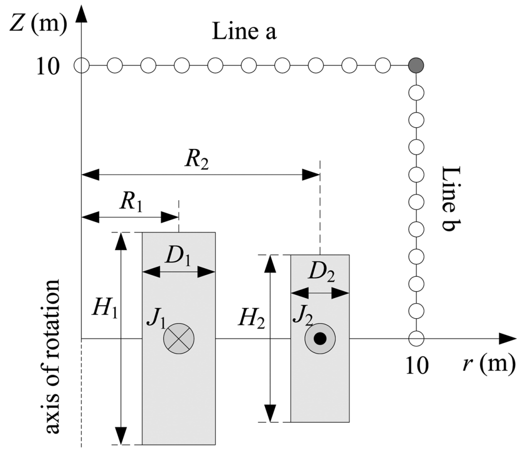

Fig. 5. Robust configuration of the TEAM 22 problem.

1. The energy stored in the device should be $E_0 = 180$ MJ.
2. The mean stray field, B_{stray}, at 22 measurement points along line a and line b in Fig. 5 at a distance of 10 m should be as small as possible.
3. The materials of the coils must not exceed the prescribed bounds established by the quench condition, approximated by the equation

$$\|J\| = -6.4\|B\| + 54 \tag{10}$$

Using the proposed method, a robust optimization is formulated as follows

Table 3. TEAM 22 parameters.

	R_1 m	R_2 m	$H_1/2$ m	$H_2/2$ m	D_1 m	D_2 m	J_1 A/mm^2	J_2 A/mm^2
Lower limit	–	2.6	–	0.204	–	0.1	–	–
Upper limit	–	3.4	–	1.1	–	0.4	–	–
Fixed	2.0	–	0.8	–	0.27	–	22.5	–22.5

$$\begin{aligned} &\text{Minimize}: f(\boldsymbol{x}) = \frac{B_{stray}^2}{B_n^2} + \frac{|E - E_0|}{E_0} \\ &\text{subject to}: \hat{V} \le V_{crit} \\ &\qquad \|J\| = +6.4\|B_{\max}\| - 54 \le 0 \\ &\qquad R_1 + \frac{D_1}{2} < R_2 + \frac{D_2}{2} \\ &\qquad \boldsymbol{x}_L \le \boldsymbol{x} \le \boldsymbol{x}_U \end{aligned} \tag{11}$$

where $\hat{V}$ is the estimated variance of objective function $f(\boldsymbol{x})$, $B_n = 3.0 \times 10^{-3}$ T, E is the stored magnetic energy in the device, $B_{\max}$ is the maximum magnetic flux density, and the stray field B_{stray} is evaluated by the magnetic flux density of each point on line a and line b.

$$B_{\text{stray}}^2 = \frac{1}{22}\sum_{i=1}^{22} B_{s,i}^2 \tag{12}$$

The Sobol' variance decomposition is performed based on the approximated model of $f(\boldsymbol{x})$ by means of the FEM simulations. Considering the design variables are at their initial values {3, 1, 0.2} to roughly examine the first-order Sobol' indices. Each index is obtained by varying one single variable, while the others stay constant and equal to the initial values. In the current case, the results of the first-order sensitivity indices for the objective function are shown in Fig. 6. Graphically, as each variable scans in the range of lower and upper limits, the sensitivity indices fluctuate all the time. As shown in Fig. 6, none of the sensitivity indices stay large or small for the whole time, thus all the design variables should be regarded as random variables with certain standard deviations. To verify whether the sum of the first-order variance corresponding to three variables (i.e., $V_{R_2}^f$, $V_{H_2}^f$ and $V_{D_2}^f$) could represent the total variance, 20 point are chosen evenly in every subfigure of Fig. 6. The mean value of the sum of first-order Sobol' indices reaches 0.934, which means that the symbol $\hat{V}$ in (11) is denoted as the sum of $V_{R_2}^f$, $V_{H_2}^f$ and $V_{D_2}^f$.

In order to save the computational cost, especially the cost of finite element computation, both the objective

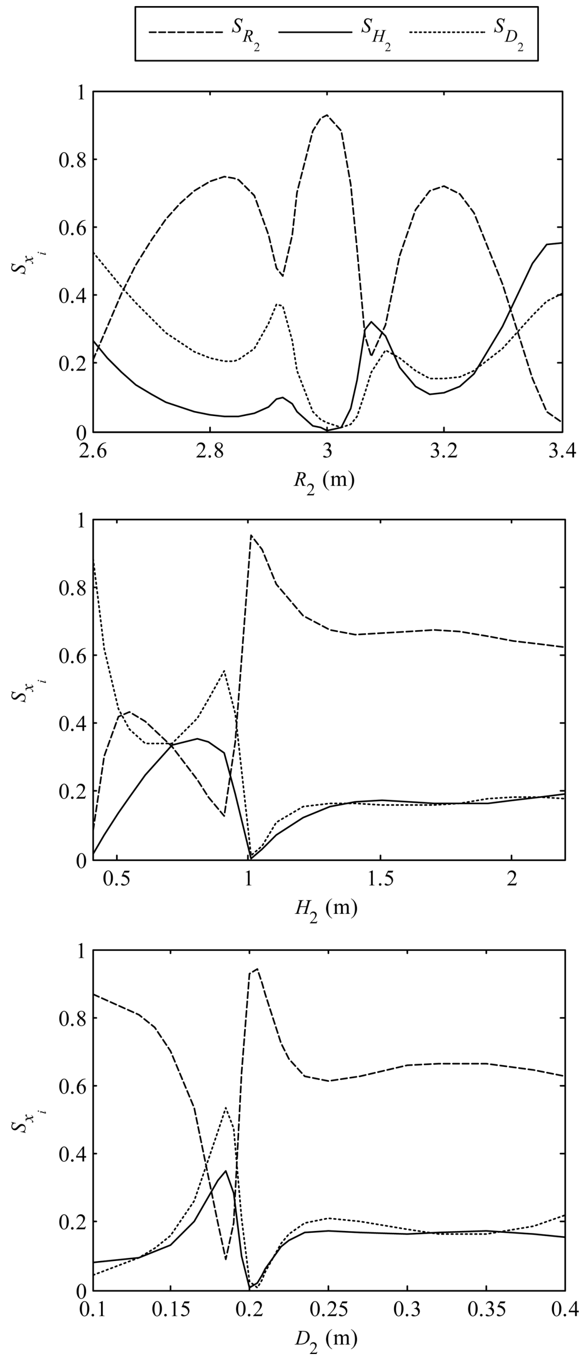

Fig. 6. Non-dimensional first-order sensitivity indices.

Table 4. Optimization results.

V_{crit} ($\times 10^{-3}$)	Method	R_2 (m)	H_2 (m)	D_2 (m)	$f(\boldsymbol{x})$	$\hat{V}$ ($\times 10^{-3}$)	V ($\times 10^{-3}$)	Numbers of function call: $f(\boldsymbol{x})$	Numbers of function call: constraint	Time (s)
2	Sobol'	3.255	0.569	0.291	0.154	1.49	2.54	2156	4213	155.3
	Taylor Series	3.143	0.580	0.312	0.109	1.55	3.98	3773	4319	124.2
	MCS	3.269	0.587	0.279	0.161	1.89	1.89	2175	4253	5882.6
3	Sobol'	3.109	0.785	0.236	0.106	2.92	4.67	3028	5796	143.6
5	Sobol'	3.08	0.478	0.394	0.088	4.23	5.97	2408	4208	163.8

function and the constraint function are approximated adopting the RBF method. 3000 samples are chosen to bulid the approximate models with the percentage error of 5.6%, 6.8% and 2.8% for the objective function, the maximum magnetic flux density and the variance, respectively. Since genetic algorithm (GA) works well in obtaining global optimum, it is chosen as the solver in the proposed robust optimization. The GA parameters are set at a population size of 50, a cross-over rate of 0.5, and a mutation rate of 0.01.

In this paper, three variance criterions are chosen to demonstrate the application of proposed method. By applying (11), the optimization results under different criterions are listed in Table 4. When V_{crit} is set to a relatively smaller value (i.e. $V_{crit} = 2 \times 10^{-3}$), optimal results from Sobol'-based method (Design 1) are compared with those using Taylor series-based method and MCS. Considering the evaluation of response variance, the proposed Sobol'-based method gives a closer approximated value (1.95×10^{-3} out of 2.54×10^{-3}) than Taylor series-based method, whose estimated value is only 1.55×10^{-3} while the real variance reaches 4.08×10^{-3}. As to the computational cost, there is no obvious difference in the numbers of function call; however, MCS takes much more time than the proposed method and Taylor series-based method. From the results, the proposed method shows an economical solution with much lower computational cost over MCS, as well provides a more accurate solution than Taylor series-based method.

As V_{crit} becomes larger to be 3×10^{-3} (Design 2) and 5×10^{-3} (Design 3), different results providing smaller objective function values are given. Comparison between three designs using proposed method is presented as histogram in Fig. 7. To get the distribution of objective function, 10,000 samples are randomly chosen within the variable ranges for each design. It is seen that a smaller variance criterion let the optimal solution turn out more robust; however, the mean value of the objective function appears larger. This means that the optimization solution with a restrictive variance criterion would compromise in performance, and designers can choose a proper design according to the importance between performance and robustness.

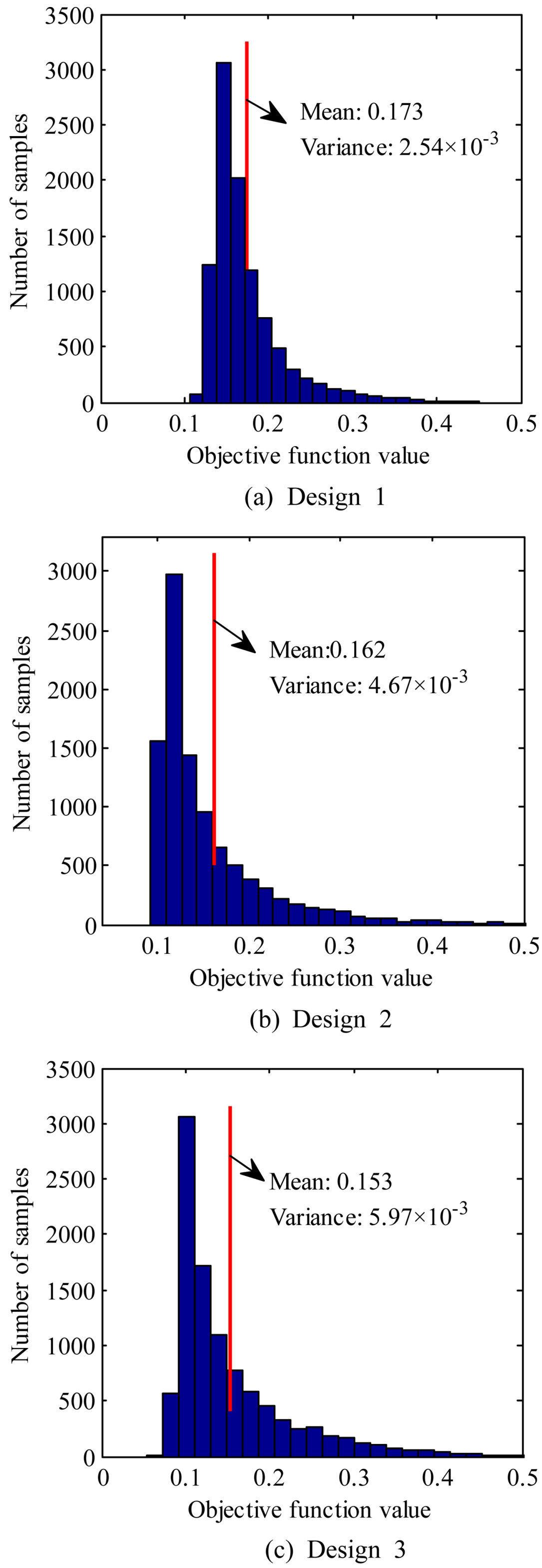

Fig. 7. (Color online) Comparison of objective function distributions.

4. Conclusion

In this paper, we proposed the usage of Sobol' variance decomposition to calculate the response variance considering tolerances in design variables. An optimal result having better robustness can be obtained by choosing a variance criterion as an optimization constraint. Our proposal was tested on both an analytic function and the TEAM 22 problem, and the results have been compared with those from other variance evaluation approaches. The accuracy and efficiency of the proposed method has been demonstrated. The designers can select their best designs through balancing good performance against higher robustness by setting different variance criterions.

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