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An Approximate Calculation Model for Electromagnetic Devices Based on a User-Defined Interpolating Function

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Optimization design and robust design are significant measures for improving the performance and reliability of electromagnetic devices (EMDs, specifically refer to relays, contactors in this paper). However, the implementation of the above-mentioned design requires substantial calculation; consequently, on the premise of guaranteeing precision, how to improve the calculation speed is a problem that needs to be solved. This paper proposes a new method for establishing an approximate model for the EMD. It builds a relationship between the input and output of the EMD with different coil voltages and air gaps, by using a user-defined interpolating function. The coefficient of the fitting function is determined based on a quantum particle swarm optimization (QPSO) method. The effectiveness of the method proposed in this paper is verified by the electromagnetic force calculation results of an electromagnetic relay with permanent magnet.

Keywords: Electromagnetic device (EMD), approximate model, user-defined interpolating function, robust design

1. Introduction

The key of optimization design or robust design lies in analysis of the influence of the input parameter and its tiny fluctuations on the output features. Here, substantial calculation needs to be conducted on the basis of the input-output relation [1]. Current calculation methods for the electromagnetic device (EMD) mainly consist of the magnetic equivalent circuit (MEC), finite element method (FEM), and approximate model.

The traditional MEC has low calculation precision. In recent years, many scholars have conducted studies on improving the calculation precision of MEC, and optimizing the EMD on the basis of the MEC. Amrhein *et al.*[2-4] established a three-dimensional magnetic circuit model for the EMD with a magnetic resistance network method, according to the distribution of the space magnetic field. Chillet *et al.*[5] constructed an analytical model of an electromagnetic system on the basis of the magnetic resistance network method, and improved the calculation precision by considering the leakage flux and magnetic saturation, and evaluating the magnetic resistance distribution of different positions. Scholars also developed corresponding software packages (such as PASCOMA) for optimization design of the EMD based on the analytical model of a magnetic circuit.[6-10] However, the MEC model usually becomes very complicated in order to get accurate results; and sometimes, it cannot establish a precise model, because of the leakage flux. In addition, the MEC also has nonlinear processing problems of ferromagnetic material. When the magnetic circuit is quite complicated, substantial nonlinear equations need to be worked out, and that might take a long time.

The opposite of the MEC is the FEM. Although the FEM has high calculation precision, it may be difficult to adapt to the robust design or optimization process, due to the long calculation time. Considering the complementary feature of the FEM and MEC, the approximate model method has gradually been introduced into the calculation of electromagnetic systems in recent years.

Choi *et al.*[11] modified the dimension parameters of the EMD according to the calculation result of the FEM, and constructed a geometric model whose calculation result matches the FEM, with the thought of space mapping. When doing optimization of the EMD, the MEC method is used, based on the modified dimension parameters. The optimization results are obtained with the mapping model. This is a good method for the optimization design of the EMD, but when the magnetic circuit is complicated, and the number of design parameters is large, the mapping

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relationship will be hard to build. Zhai *et al.*[12] also constructed a mapping relationship for MEC and the FEM with a compensation factor based on the space mapping method, and obtained a rapid calculation approximate model of the EMD. The core of this compensation factor method is to construct a magnetic circuit, and calculate the compensation factor. For a single-loop magnetic circuit with simple structure, it can achieve high-precision calculation results; but for complicated magnetic circuits, (such as a multi-loop magnetic circuit with permanent-magnet), the compensation factor near the zero point may be very large, owing to the zero passage condition of the output feature, which may result in huge error in the calculation result.

With the development of the intelligence algorithm, it was proposed by scholars that an approximate model of the EMD could be established with a mathematical approach by jumping out of the constraints of a magnetic circuit, which may realize the rapid calculation of electromagnetic features. Xia *et al.*[13] constructed the Kriging approximate model of the EMD, and optimized the parameter of a superconducting coil through the design of experiments (DOE) method. Kim *et al.*[14] applied a Kriging approximate model and Latin hypercube sampling to optimize the shape of a rotor.

The precision of the Kriging or similar methods depends on the selection of the basis function type. However, currently there is no agreed method for this kind of selection. For the EMD, because its input-output relationship has a high nonlinear feature, the normally used basis function (lower order polynomial) will no longer be applicable. Furthermore, it is very hard to find a suitable higher order basis function with sampling data under this condition.

This paper proposes a new approach for establishing an approximate model of the EMD, and the approximate model of design parameters (dimension and material properties of the EMD) and output features is determined by the multivariate function Taylor's Formula, and fitting of a polynomial. Since the output features of different coil voltages and air gaps need to be calculated by aiming at the transient process of the EMD, there is the problem of a huge calculation amount. Consequently, the relationship between the output characteristics of datum points (finite element simulation points) and the output characteristics of the rest points is established, based on the user-defined interpolation thought; and meanwhile, the coefficients of the interpolation function are worked out with a quantum particle swarm optimization (QPSO) method. According to the calculation result of an electromagnetic relay with permanent magnet, the method proposed in this paper is effective and reasonable.

2. Approximate Calculation Model of the EMD

The transient state output of the EMD is co-determined by the electromagnetic force and mechanical force, as Eq. (1) shows.

$$\begin{cases} u_0 = i(\psi, \alpha)R + \dfrac{\mathrm{d}\psi}{\mathrm{d}t} \\ J\dfrac{\mathrm{d}\omega}{\mathrm{d}t} = T(U, \alpha) - T_f(\alpha) \\ \omega = \dfrac{\mathrm{d}\alpha}{\mathrm{d}t} \end{cases} \tag{1}$$

where, u_0 is the supply voltage of the coil loop, i is the coil current, R is the coil resistance, ψ is the flux linkage, J is the rotational inertia of the armature, ω is the rotational angular speed of the armature, U is the coil voltage, α is the rotational angle of the armature, T is the electromagnetic torque, and T_f is the mechanical torque.

Normally, the coil voltage U and rotational angle α of EMD will change with time during the operational process. Therefore, the output characteristics in equation (1), for example T and T_f, have two conditions: static state (U and α are fixed), and dynamic state (U and α may change with time). The voltage U and rotational angle α are not the inherent structural parameters of the EMD, but are variables introduced by the transient process, and in this paper they are termed the process variables.

When U and α are fixed, and the design parameters fluctuate within a certain range, the output of the EMD can be achieved with the interpolation of a few FEM simulation results, and Eq. (2) shows the basic form.

$$F = F_0 + \Delta F \tag{2}$$

where, F is the output characteristic, and F_0 stands for the corresponding output result, when the design parameters are the central value within the fluctuating range. When the approximate mode is established, the result of F_0 will be obtained by the FEM; ΔF is the output characteristic variation caused by the fluctuation of design parameters.

With the multivariate function Taylor's Formula, the output characteristic change ΔF can be transformed to Eq. (3).

$$\begin{aligned} \Delta F(\Delta x_1,\dots\Delta x_n) &= \Delta F(0,\dots0) + \sum_{i=1}^{n} \Delta x_i \frac{\partial(\Delta F)}{\partial x_i} + \Delta^2(\Delta x_i) \\ &= \sum_{i=1}^{n} \Delta x_i \frac{\partial(\Delta F)}{\partial x_i} + \Delta^2(\Delta x_i) \\ &\approx \sum_{i=1}^{n} \Delta F_i + \Delta^2(\Delta x_i) \end{aligned} \tag{3}$$

where, Δx_i is the variation of the design parameter x_i, and $\Delta^2(\Delta x_i)$ is the higher order infinitesimal of Δx_i .

Since Δx_i is relatively small, $\Delta^2(\Delta x_i)$ can be neglected, and the output characteristic expression can be represented as:

$$F = F_0 + \sum_{i=1}^{n} \Delta F_i \tag{4}$$

F_0 can be obtained with the FEM. ΔF_i is the approximate function of Δx_i, U, and α, namely,

$$\Delta F_i = G_i(\Delta x_i, U, \alpha) \tag{5}$$

For steady state, the voltage U and rotational angle α are unchanged; then, $G_i(\Delta x_i, U, \alpha)$ will become a one-variable function of Δx_i. Taking $\{\Delta x_{i1}, \Delta x_{i2} \dots \Delta x_{ij}\}$ and $\{G_{i1}, G_{i2} \dots G_{ij}\}$, in which,

$$G_{ij} = G_i(\Delta x_{ij})|_{(U,\alpha)} \tag{6}$$

Since Δx_i has a small variation range, the variation trend of $G_i(\Delta x_i)$ can be reflected with the fitting of a polynomial. The order of the polynomial should not be too high. With the approximate model established as above, the output result corresponding to the multi-variable changes with fixed (U, α) can be calculated.

3. Approximate Model When the Process Variables Change

The output characteristics for the EMD should be calculated under different process variables, namely a different (U, α). Each (U, α) can be regarded as a single working point. For each working point, the model requires a group of corresponding FEM simulation results as the reference points, to get the approximate results of the output characteristics. In this way, substantial finite element simulation is still required to calculate the output characteristics of the EMD.

We adopt the user-defined interpolation function to solve the above-mentioned problem. The key points (defined as datum points) in the computed area of the output characteristics are first determined (such as the nine points in Fig. 5), then the function relation between the output characteristics of the datum points and the output characteristics of the other points is established, so as to determine the output characteristics under different (U, α).

Defining function $f = g(x_1, x_2)$, and defining $[m-\Delta x_1, m, m+\Delta x_1] \subset X_1$, $[n-\Delta x_2, n, n+\Delta x_2] \subset X_2$, $(\Delta x_1, \Delta x_2 > 0)$, if f has monotonicity for Δx_1 and Δx_2, then $g(m, n)$ is among the four points $g(m-\Delta x_1, n-\Delta x_2)$, $g(m+\Delta x_1, n-\Delta x_2)$, $g(m-\Delta x_1, n+\Delta x_2)$, and $g(m+\Delta x_1, n+\Delta x_2)$, as Fig. 1 shows.

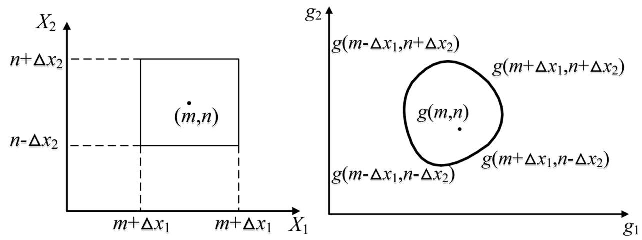

Fig. 1. Relational graph of variable and function.

$g(m, n)$ can be represented as:

$$g(m,n) = \sum_{x_1=m-\Delta x_1, x_2=n-\Delta x_2}^{x_1=m+\Delta x_1, x_2=n+\Delta x_2} g(x_1,g_2) \cdot l|_{x_1,x_2} \tag{7}$$

where l is the weight coefficient of each point, standing for the influence of each point on the target point, and it should also simultaneously satisfy the following two conditions:

(a) When (x_1, x_2) is located at one of those four nodes, the weight coefficient of this node should be 1, and the weight coefficient of the other three nodes should be 0.

(b) The value range of the weight coefficient is $0 \le l \le 1$.

The output characteristics of EMD satisfy the monotonicity for U and α, if four boundary points are selected, including (U_{m0}, α_{n0}), (U_{m1}, α_{n0}), (U_{m0}, α_{n1}), (U_{m1}, α_{n1}), $(U_{m0} \le U \le U_{m1}, \alpha_{m0} \le \alpha \le \alpha_{m1})$, and it can be obtained according to Eq. (7) that:

$$\Delta F_i = \sum_{(m,n)=(m_0,n_0)}^{(m_1,n_1)} \Delta F(\Delta x_i)|_{(U_m,\alpha_n)} \cdot h(U, \alpha)|_{(U_m,\alpha_n)} \tag{8}$$

where, $h(U, \alpha)$ refers to the user-defined interpolation function.

This user-defined interpolation function must meet the requirements of the constraint conditions (a) and (b). Meanwhile, it also needs to be able to reflect the relationship between the process variables and the variation of the output characteristics. That is, the functional form is determined by the curve shape of the relationship. For the EMD in this paper, the exponential function is selected as the user-defined interpolation function. Eq. (9) gives the specific form of the corresponding interpolation function.

$$h(U,\alpha)|_{(U_m,\alpha_n)} = e^{-k_1 \cdot \left|\frac{U-U_m}{(U_{m0}+U_{m1}-U_m)-U}\right| - k_2 \cdot \left|\frac{\alpha-\alpha_n}{(\alpha_{n0}+\alpha_{n1}-\alpha_n)-\alpha}\right|} \tag{9}$$

where, k_1 stands for the influence coefficient of U, and k_2 is the influence coefficient of α. k_1 and k_2 indicate the influence of the process variables on the output, and they have nothing to do with the design parameters.

The coefficient in Eq. (9) is determined with the QPSO method in this paper. The principle for the optimization is error minimization, that is, Eq. (10) achieves minimum value. QPSO will encode the position of particles based on the quantum bit, which effectively overcomes the premature convergence problem of particle swarm optimization (PSO).[15] N samples $X_i = [U_i, \alpha_i]$, $i = 1, 2 \ldots N$ are obtained according to the Latin hypercube sampling, and the corresponding output H_i can be obtained with the FEM. Equation (10) is the error function for stopping the iteration process.

$$Err = \sqrt{\frac{\sum_{i=1}^{n} [H_i - F(X_i)]^2}{N}} \qquad (10)$$

4. The Flowchart for EMD Analysis with the Approximate Model

The task of EMD analysis is to work out the output characteristics under different U and α; and in most cases, the design parameters fluctuation (Δx_i) should be considered at the same time.

With the approximate model in section 2 (defined as model A), the output characteristics of EMD with certain U and α can be worked out, under the condition of fluctuating design parameters. Theoretically, the output characteristics of EMD under different U and α can also be obtained with model A. But in fact, the substantial finite element simulation required will make this solution unacceptable. For example, there are 6 design parameters of the EMD in the case study, and for each parameter, 7 simulations need to be carried out. Meanwhile, there are 29 U and 21 α (a total of 609 working points) that need to be analyzed. This means 609 × 7 × 6 simulations need to be done. The approximate model in section 3 (defined as model B) can solve this problem. By using model B, there are only 609 (for F_0 in Eq. (4)), plus 7 × 6 (for ΔF_i in Eq. (4)) simulations that need to be done. The 9 datum points are among these 609 working points, as Fig. 4 shows.

Suppose the number of different U is a, and the number of different α is b; then Fig. 2 shows the flowchart for

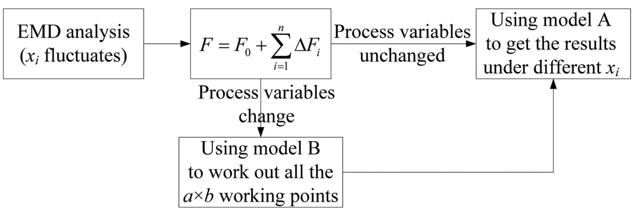

Fig. 2. The flowchart for EMD analysis.

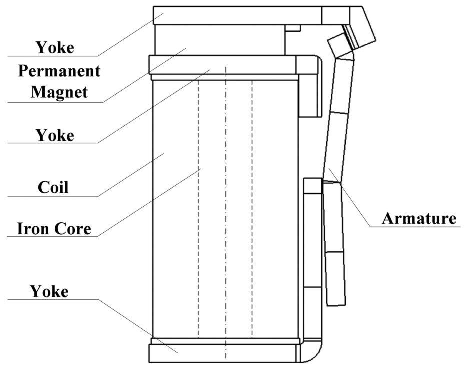

Fig. 3. Electromagnetic system structure drawing.

EMD analysis.

5. Case Study

Figure 3 is the structural diagram of an electromagnetic relay with permanent magnet, whose rotational angle range is 0-5.8 degrees, and coil voltage ranges between 0 and 28 V (D.C.). It is a small size electromagnetic relay of 26 mm × 13 mm × 13 mm dimension.

The design parameters requiring analysis mainly consist of the residual magnetism, height of the permanent magnet, thickness of the permanent magnet, radius of the iron core, width of the armature, and length of the armature. Table 1 shows the parameter fluctuation range.

Figure 4 shows the corresponding electromagnetic torques under different coil voltages (29 voltages) and rotational angles (21 angles) (there are 29 × 21 = 609 points on each electromagnetic torque curve). Certainly, for these output characteristics, the design parameters of the relay take the central values. By considering the curve shape of the electromagnetic torques, 9 points are selected as the interpolation points (namely the datum points), including (0V, 0°), (0V, 0.9°), (0V, 5.8°), (13V, 0°), (13V, 0.9°), (13V, 5.8°), (28V, 0°), (28V, 0.9°) and (28V, 5.8°), as Fig. 4 shows.

With the residual magnetism as an example, 7 nodes (the node number depends on the specific condition) are taken evenly within –0.04T and 0.04T, and the electromagnetic torque in (U, α, ΔB_r) is calculated with the

Table 1. Fluctuation range of the design parameters.

Parameters	Fluctuation range	Fixed values
residual magnetism	–0.04~0.04 (T)	0.38T
height of permanent magnet	–0.11~0.11 (mm)	8.5 mm
thickness of permanent magnet	–0.05~0.05 (mm)	2 mm
radius of iron core	–0.075~0.075 (mm)	2 mm
width of armature	–0.075~0.075 (mm)	9 mm
length of armature	–0.1~0.1 (mm)	1.3 mm

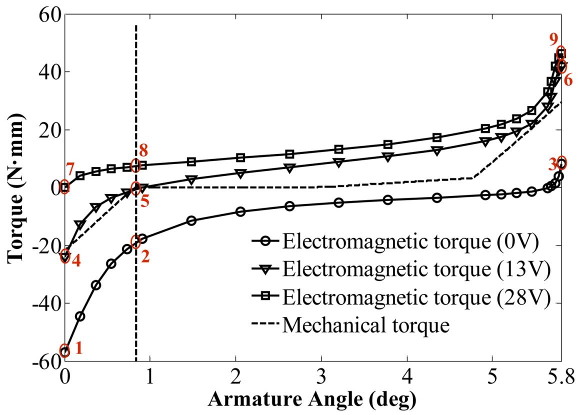

Fig. 4. (Color online) The nine datum points.

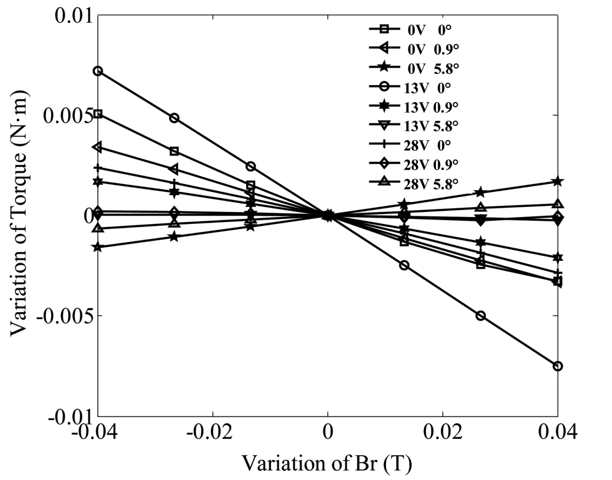

Fig. 5. Variation of the electromagnetic torque under different residual magnetism.

FEM. With the cubic spline interpolation method, the relation between the residual magnetism variation and the electromagnetic torque variation of the nine datum points is obtained, as Fig. 5 shows.

Nine datum points divide the variable space into four regions. For example, point 1, point 2, point 4 and point 5 form the region I, as Fig. 4 shows. The Latin hypercube sampling method is applied in the four regions to select N points (here N is set to 9, but it has nothing to do with the datum points number "9" in Fig. 4) as the sampling points to determine the influence coefficients. According to Eqs. (9) and (10), 9 samples $X_i = [U_i, \alpha_i]$, $i = 1, 2 \ldots 9$ are formed, and the corresponding G_i are obtained with the FEM.

As mentioned before, the influence coefficients have nothing to do with the design parameters; therefore, by taking $\Delta B_r = 0.033$T as an example (ΔB_r can be some other value), the data for determining the influence coefficients are obtained, as Table 2 shows.

Table 2. The sampling points to determine the influence coefficients.

(U, α)	Output variation ΔT (N·mm)
(3V, 5.68°)	0.000544747
(3V, 2.63°)	−0.0004083243
(3V, 0.18°)	−0.0000480795
(9V, 4.35°)	−0.00098206308
(9V, 5.68°)	−0.0004132662
(9V, 0.36°)	−0.00419130597
(16V, 5.1°)	−0.0050144419
(16V, 0.18°)	−0.0041702402
(16V, 5.78°)	0.0000336375

The influence coefficients are determined with optimization based on the QPSO method, and the basic parameters are set as follows: the solution range of the influence coefficient is (0~10), acceleration coefficients $c_1 = 5$ and $c_2 = 10$, basic step length $\theta = 0.01\pi$, variable coefficient $P_m = 0.05$, population quantity $L = 1000$, maximum generations 100, allowable error of the objective function $\varepsilon = 1e{-}6$, and results after optimizing of $k_1 = 0.758$ and $k_2 = 0.514$. Then the approximate model that can reflect the relation of electromagnetic torque and design parameters is determined according to Eq. (1).

Figures 6 and 7 show the comparison result and calculation error under the condition of several design parameters changing at the same time, and the coil voltages are 0V, 3V, 13V, and 28V, respectively. These four voltages are very typical, and can be selected to verify the calculation results of the approximate model.

When the coil voltage is 3V, the model gets the largest calculation error. Table 3 shows a comparison of the

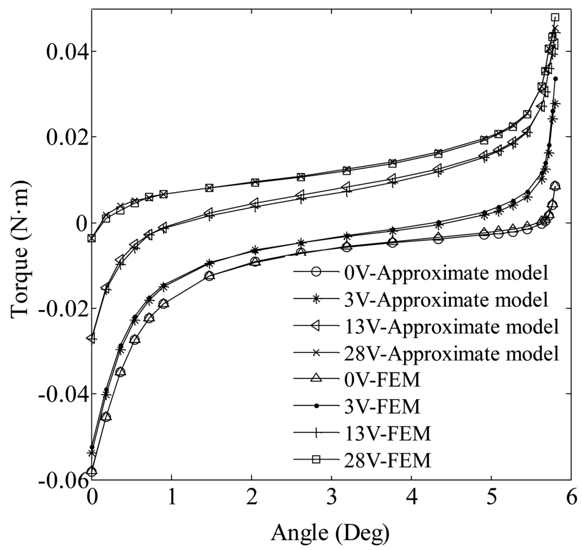

Fig. 6. Comparison of the FEM and the approximate model.

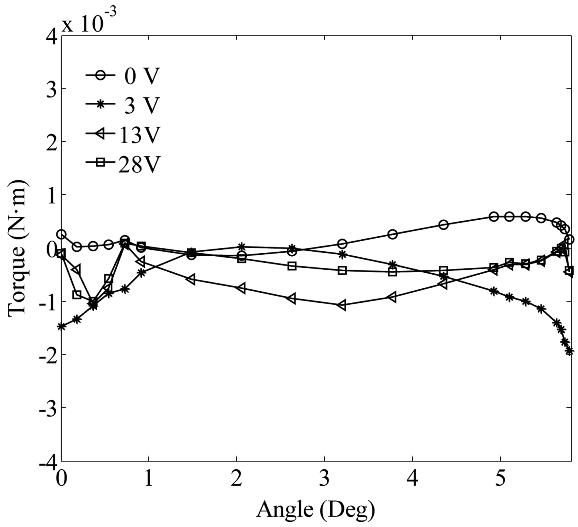

Fig. 7. Error under different voltages.

Table 3. Results of the FEM and the approximate model under 3V coil voltage.

Angle (Deg)	FEM (N·m)	Approximate model (N·m)
0	−0.05238	−0.05386
0.1816	−0.03892	−0.04025
0.3637	−0.02857	−0.02965
0.5456	−0.02186	−0.02271
0.7275	−0.01744	−0.01821
0.9093	−0.01461	−0.01507
1.482	−0.009327	−0.00941
2.055	−0.006494	−0.006468
2.628	−0.004582	−0.004595
3.201	−0.003048	−0.003172
3.774	−0.001562	−0.00187
4.347	0.0001379	−0.0004008
4.919	0.002556	0.001741
5.1	0.003656	0.002741
5.28	0.005094	0.004092
5.46	0.007265	0.006117
5.64	0.01172	0.01031
5.685	0.01407	0.01253
5.73	0.01815	0.01637
5.775	0.02627	0.02434

Table 4. Comparison of the calculation efficiency.

Computation method	Calculation number	Computation time (s)
FEM	609	604800
approximate model	609	4.8

Computer used: Core™ i5-3470 3.2 GHz PC

results of the FEM and the approximate model. The maximum error is 0.00193N·m, while the average error is 0.00088N·m.

The calculation error is within the acceptable range, and the approximate model proposed in this paper can greatly shorten the calculation time. Table 4 shows a comparison of the calculation efficiency of the FEM and this method.

6. Conclusion

The calculation of electromagnetic property, as a fundamental work, is crucial for the design, optimization, and even the manufacturing of EMDs. With the wide application of the FEM in the electromagnetic calculation field, how to achieve high-precision calculation results based on the FEM is a hot issue for scholars. In this paper, a new approach to establish an approximate model of the EMD is proposed, based on the current research achievements.

(1) With the user-defined interpolation function, the proposed method solves the substantial calculation problem caused by multi-working points.

(2) Using the method proposed in this paper, the calculation result of the case study shows that the calculation speed is substantially improved, when compared to the FEM (in this case, by 126,000 times).

(3) The Kriging method has basis-function-selection problems for EMD properties analysis, and the approximate model based on space mapping cannot solve the calculation problems caused by zero passage of the electromagnetic force. These problems are well solved in this paper.

(4) The selection of the datum points, as well as the form of the interpolation function, will impact the calculation precision of the approximate model. Owing to the time limit, specific studies on the above problems have not been carried out yet, and it should be the focus of research in the next step.

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