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Optimization of a SMES Magnet in the Presence of Uncertainty Utilizing Sampling-based Reliability Analysis

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This paper proposes an efficient reliability-based optimization method for designing a superconducting magnetic energy system in presence of uncertainty. To evaluate the probability of failure of constraints, sampling-based reliability analysis method is employed, where Monte Carlo simulation is incorporated into dynamic Kriging models. Its main feature is to drastically reduce the numbers of iterative designs and computer simulations during the optimization process without sacrificing the accuracy of reliability analysis. Through comparison with existing methods, the validity of the proposed method is examined with the TEAM Workshop Problem 22.

Keywords : electromagnetics, optimization, reliability theory, sensitivity analysis

Introduction

Due to growing demand for high-reliability electromagnetic (EM) devices, attention has recently focused on handling uncertain design parameters occurring in manufacturing process or operating condition. To incorporate such uncertainties into an early design stage, two design approaches have been used to date [1-11]: one is robust design optimization (RDO), and the other is reliability-based design optimization (RBDO).

RDO improves product quality by minimizing variation of an objective function within a feasible design space, where constraint conditions are satisfied. Several methods, such as worst-case scenario, gradient index, six sigma, etc., were used to seek a robust optimum [1-6]. They do not, however, address the quantitative probabilistic assessment on nominal designs. That is, they do not provide accurate probabilistic information as to the confidence/reliability level, at which an EM design is achieved. On the other hand, RBDO involves an objective function as deterministic optimization, and also contains probabilistic constraints where the desired probability of failure/success is imposed [7-11]. To achieve product reliability, RBDO probes a design point within a feasible space, which satisfies the target failure probability of constraints prescribed.

Recently, a few attempts have been made to perform RBDO of EM devices. Jeung *et al.* [10] applied RBDO based on the reliability index approach (RIA) to a superconducting magnetic energy storage (SMES) system, named TEAM benchmark problem 22 [12]. The same design problem was solved by another RBDO method, which adopts the performance measure approach (PMA) for reliability analysis of probabilistic constraints [11]. The two methods, RIA and PMA, use different numerical techniques to evaluate the probability of failure, but both are based on the sensitivity information of probabilistic constraints. Such sensitivity-based RBDO methods have a double-loop optimization structure in common: one is for main optimization, and the other is for reliability analysis based on the design sensitivity. This complicated scheme requires heavy computational cost mostly due to reliability analysis during the RBDO process. For the above reason, only three of the total eight design parameters of the SMES model were selected as random variables in the previous works [11, 12].

To overcome the aforementioned difficulty, this paper proposes an efficient sampling-based RBDO method for EM design, which utilizes the dynamic Kriging (DKG) method and Monte Carlo simulation (MCS). Elaborate DKG-based surrogate models are first generated in not a

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global window (whole design space) but a hyper-spherical local window (relatively small region). Then, the failure probability and sensitivity of probabilistic constraints are calculated by applying MCS to surrogate models. Unlike the sensitivity-based RBDO, the proposed optimization scheme has a simple single-loop optimization structure, so it can drastically reduce computational cost. Finally, numerical efficiency and accuracy of the proposed method is verified through three-parameter and eight-parameter designs of the benchmark problem 22.

2. Sampling-Based RBDO

In this section, the RBDO formulation is briefly summarized, and basic concepts of DKG and MCS combined with surrogate models are explained. Lastly, the program architecture, which integrates the sampling-based reliability analysis method into RBDO, is presented.

2.1. RBDO Formulation

The mathematical formulation of a RBDO model is expressed in [8-11] as:

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{d}) \\ &\text{subject to} \quad P_f(g_i(\mathbf{x}) > 0) \le P_{t,i},\ i = 1, 2, \dots nc \\ &\qquad\qquad \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U,\ \mathbf{d} \in R^n \end{aligned} \tag{1}$$

where f is an objective function, $\mathbf{d}$ is the design vector given by $\mathbf{d}=\boldsymbol{\mu}(\mathbf{x})$, $\boldsymbol{\mu}$ denotes the mean of a random vector $\mathbf{x}$, $P_{t,i}$ is the target probability of failure with respect to the ith constraint g_i, and nc is the number of probabilistic constraints. The symbols, $\mathbf{d}^L$ and $\mathbf{d}^U$, mean the lower and upper bounds of $\mathbf{d}$.

The failure probability P_f is evaluated using a joint probability density function (PDF) $f_{\mathbf{x}}(\mathbf{x})$ as:

$$P_f(g_i(\mathbf{x}) > 0) = \int I_{\Omega_{Fi}}(\mathbf{x}) f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} \tag{2}$$

where Ω_{Fi} is a failure set of $\Omega_{Fi} \equiv \{\mathbf{x}\colon g_i(\mathbf{x}) > 0\}$ for reliability analysis of the ith constraint function. The symbol $I_{\Omega_{Fi}}$ is called an indicator function, and is defined by

$$I_{\Omega_{Fi}} \equiv \begin{cases} 1, \mathbf{x} \in \Omega_{Fi} \\ 0, \text{otherwise.} \end{cases} \tag{3}$$

2.2. Surrogate Model

In the Kriging method, the outcomes are considered as a realization of a stochastic process [9, 13, 14]. The goal is to estimate a response $\mathbf{y} = [y(\mathbf{x}_1), \cdots, y(\mathbf{x}_n)]^T$ with $y(\mathbf{x}_i) \in \mathbf{R}^1$ based on n samples, $\mathbf{x} = [\mathbf{x}_1, \cdots, \mathbf{x}_n]^T$ with $\mathbf{x}_i \in \mathbf{R}^m$. The response consists of a summation of two parts: mean structure of the response $\mathbf{F}\boldsymbol{\beta}$ and realization of the stochastic process $\mathbf{e}$ as:

$$\mathbf{y} = \mathbf{F}\boldsymbol{\beta} + \mathbf{e} \tag{4}$$

where $\boldsymbol{\beta}$ is the vector of regression coefficient.

Applying fairly routine mathematical processes such as the maximum likelihood estimator and the Lagrange multiplier, the prediction $\hat{y}$ of (4) which interpolates the n samples around a prediction point $\mathbf{x}_0$ is expressed as:

$$\hat{y}(\mathbf{x}_0) = \mathbf{f}_0^T \boldsymbol{\beta} + \mathbf{r}_0^T \mathbf{R}^{-1} (\mathbf{y} - \mathbf{F}\boldsymbol{\beta}) \tag{5}$$

where $\mathbf{f}_0$ is the basis function vector at $\mathbf{x}_0$, $\mathbf{r}_0$ is the correlation vector between $\mathbf{x}_0$ and $\mathbf{x}$, and $\mathbf{R}$ is the symmetric correlation matrix. In DKG, the genetic algorithm (GA) selects the optimal basis function set at $\mathbf{x}_0$ in order that the generated surrogate model has the best accuracy. It means the best combination up to the highest-order basis function prescribed is decided by GA, which screens all the foreseeable basis function sets under certain convergence criteria.

To incorporate the predictor of (5) with MCS, random samples are generated in a hyper-spherical local window by using the truncated Gaussian sampling (TGS) technique [9]. The radius R of the window is determined as:

$$R = c\beta_t \tag{6}$$

where c is the coefficient, which is usually between 1.0 and 2.0, and β_t is the target reliability index in RBDO [8, 10]. Random samples are first produced in a standard normal space. Then, the generated samples in the hypersphere of (6) are transformed back to an original space.

2.3. Probability of Failure and Its Sensitivity Calculation

MCS is applied to the surrogate model $\hat{g}_i(\mathbf{x})$ instead of the real constraint function $g_i(\mathbf{x})$. The probability constraint of (2) is approximated by

$$P_f(g_i(\mathbf{x}) > 0) \cong \frac{1}{M} \sum_{m=1}^{M} I_{\hat{\Omega}_{Fi}}(\mathbf{x}^m),\ \hat{\Omega}_{Fi} \equiv \{\mathbf{x}\colon \hat{g}_i(\mathbf{x}) > 0\} \tag{7}$$

where M is the MCS sample size, $\mathbf{x}^m$ is the mth realization of $\mathbf{x}$, and $\hat{\Omega}_{Fi}$ is the failure set for the surrogate model. The sensitivity of the approximated probability constraint of (7) is obtained as:

$$\partial P_f / \partial \mu_i = \frac{1}{M} \sum_{m_f=1}^{M_f} s_{\mu i}^{(1)}(\mathbf{x}^{(m_f)}; \boldsymbol{\mu}) \tag{8}$$

where μ_i is the mean of the ith random variable, M_f is the number of failed samples, and $s_{\mu i}^{(1)}$ denotes the first-order score function for μ_i described in [9]. As shown in (7) and (8), the evaluation of the failure probability and its sensitivity does not require the sensitivity of the surrogate model but only uses function values of the surrogate model and derivative of the input distributions [9].

2.4. Implementation

In the sampling-based RBDO, the original MCS is combined with accurate DKG models generated in the local window to decide success/failure status of probabilistic constraints at random samples. That enables a single-loop RBDO structure as shown in Fig. 1, which leads to a significant saving in computational cost without degrading accuracy of reliability analysis. The optimization program was implemented by means of Matlab functions. The EM simulations at samples were executed with a commercial finite element analysis (FEA) code, called MagNet [15], and the sequential quadratic programming algorithm was utilized for handling the constrained optimization problem like (1).

The proposed program architecture follows as:

1) Input the number of initial sampling points, random variables and window size ($c = 1.2$ is used),

2) Scan the local window space with the center of a given design point and, if samples are founded, reuse them,

3) Generate samples in a local window by TGS,

4) Execute computer simulations at given samples,

5) Construct a surrogate model (5) based on DKG,

6) If the surrogate model satisfies a specified accuracy, go to next step. Otherwise, insert sequential samples and then go to step 4,

7) Assess the failure probability of failure (7) and its sensitivity (8) by MCS,

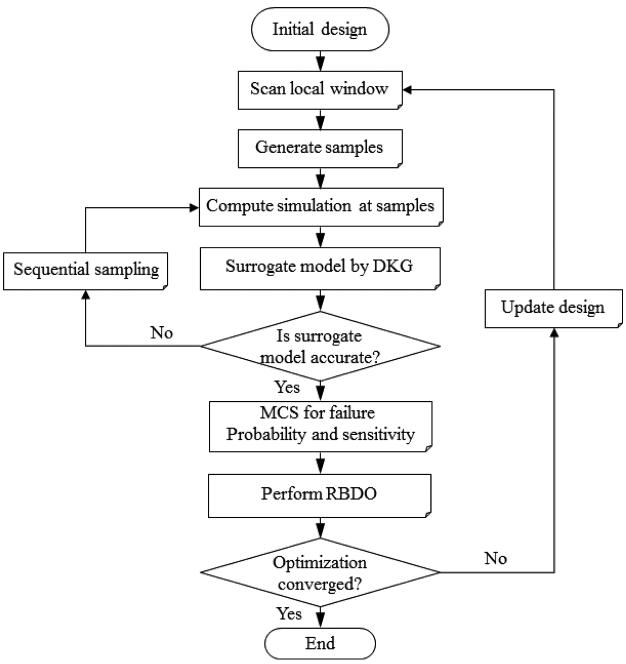

Fig. 1. Flowchart of the proposed sampling-based RBDO.

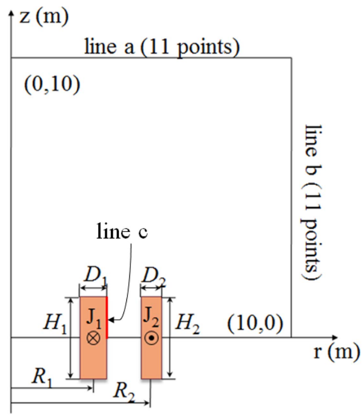

Fig. 2. (Color online) Configuration of the SMES device.

8) Perform RBDO formulation (1),

9) If convergence is satisfactory, stop. Otherwise, update a design point and go to step 2.

3. Case Studies

To investigate numerical efficiency and accuracy of the proposed RBDO method, the TEAM benchmark problem 22 of a SMES shown in Fig. 2 is considered [12]. The SMES model has six geometric variables and two current input variables. The benchmark problem proposes two design problems: one is to optimize three parameters with a multi-objective function, and the other is to optimize eight parameters with the multi-objective function subject to three constraints. These original design problems have been intensively tested with various deterministic design optimization (DDO) methods such as evolution strategy, genetic algorithm, particle swarm population, sensitivity-based method, etc. [16]. Herein, DDO is not dealt with because main concern of this paper lies on the comparison between different RBDO methods.

3.1. Design of Three Random Parameters

The original design problem with three parameters, R_2, D_2, and H_2, itself is not suitable for a RBDO test example because it has no constraint, on which a probabilistic condition is imposed. Therefore, the original problem was modified as follows:

$$\begin{aligned} &\text{minimize } f(\mathbf{d}) = \sum_{i=1}^{22} |B_{stray,i}(\mathbf{d})|^2, \quad \mathbf{d} = \mu(\mathbf{x}) \\ &\text{subject to } P_f(g_i(\mathbf{x}) > 0) - P_{t,i} \le 0 \quad i = 1,2,3,4 \\ &g_1(\mathbf{x}) = 1 - \left((E(\mathbf{x}) - E_o)/(0.05 \times E_o)\right)^2 \\ &g_2(\mathbf{x}) = (R_2 - R_1) - \frac{1}{2}(D_2 + D_1) \\ &g_{3,4}(\mathbf{x}) = -\left|\mathrm{J}_k\right| - 6.4\left|\mathrm{B}_{\max,k}\right| + 54.0 \quad k = 1,2 \end{aligned} \tag{9}$$

Table 1. Design variables and performances at Four different designs.

Design variables	d^L	SD	d^U	Initial	RBDO		
					RIA	PMA	Proposed
R_2 (mm)	2300	10	2400	2335	2348	2346	2345
D_2 (mm)	200	5	350	238	233	231	234
$H_2/2$ (mm)	800	10	950	926.5	933.5	943.5	943
B_{stray} (μT)	-	-	-	32	32	37	32
E (MJ)	-	-	-	173	181	181	179
$P_f(g_1)$	-	-	-	30.80	4.33	4.73	4.62
$P_f(g_2)$	-	-	-	1.66×10^{-2}	1.47×10^{-5}	0	0
$P_f(g_3)$	-	-	-	0	0	0	0
$P_f(g_4)$	-	-	-	0	0	0	0

Other six design variables were fixed as R_1=1,977 mm, D_1=404 mm, H_1=1,507 mm, J_1=16.30 A/mm^2, and J_2=16.19 A/mm^2, and B_{stray} is the average field value for 22 measurement points.

where $B_{stray,i}$ is the stray field calculated at the ith measurement point along line a and line b, E is the stored magnetic energy with the target value E_o of 180 MJ, and the target probability of failure $P_{t,i}$ is set to be 5% at the ith constraint (i.e. reliability of 95%).

It is assumed that the three random parameters comply with normal probabilistic distributions, of which standard deviation (SD) values are presented in Table 1. The RBDO problem of (9) was solved by using three different reliability analysis methods: RIA, PMA, and proposed sampling-based method. The proposed method was launched with 17 initial samples for surrogate models, and MCS combined with surrogate models was carried out with 500,000 samples at each intermediate design. Starting with the same initial point, three RBDO optima were obtained as seen in Table 1. It is observed that when compared with the initial design, the magnetic energy at the optimized designs reaches closer to the target value of 180 MJ, but a difference of 5 μT appears in the average stray field, B_{stray}, at the PMA-based RBDO optimum. As to the probability of failure evaluated, the energy constraint g_1 at the initial design has a relatively large value of 30.8%. It means that the initial design may violate the first constraint condition by 30.8% under the given statistical information of the random variables. On the other hand, all probability constraints at the three optima satisfy the target value of 5%. The numbers of FEA calls and iterative designs required for the three RBDO methods are compared with each other in Fig. 3. It is obvious that the proposed method shows the smallest numbers in both cases. From the results, it is evident that the proposed method drastically reduces the computational cost for RBDO, while maintaining the accuracy of reliability analysis.

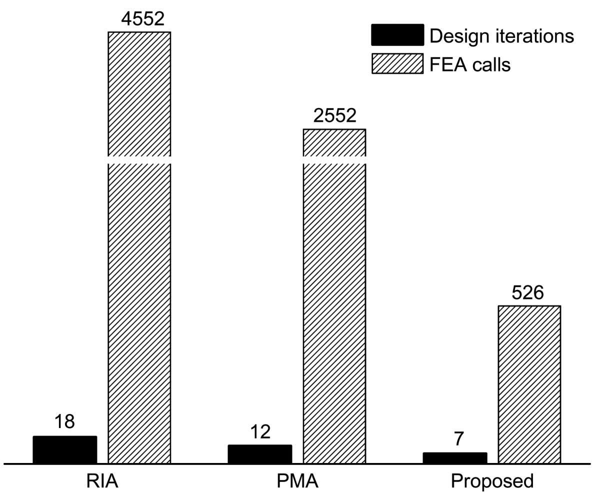

Fig. 3. Performance indicator of RBDO for three different reliability analysis methods.

3.2. Design of Eight Random Parameters

The RBDO formulation of the original SMES design problem with eight parameters is given by

$$\begin{aligned} &\text{minimize } f(\mathbf{d}) = B_{stray}^2(\mathbf{d}) / B_{norm}^2 + \left|E(\mathbf{d}) - E_0\right| / E_0 \\ &\text{subject to } P_f(g_i(\mathbf{x}) > 0) - P_{t,i} \le 0 \quad i = 1,2,3 \\ &g_1(\mathbf{x}) = (R_2 - R_1) - \frac{1}{2}(D_2 + D_1) \\ &g_{2,3}(\mathbf{x}) = -\left|\mathrm{J}_k\right| - 6.4\left|\mathrm{B}_{\max,k}\right| + 54.0 \quad k = 1,2 \end{aligned} \tag{10}$$

where B_{norm} is 200 μT, and the target probability of failure is 5% for all constraints. The first constraint g_1 prevents two magnets from overlapping each other, and the others

Table 2. Design variables and performances at Four different designs.

Design variables	Unit	d^L	SD	d^U	DDO	Proposed RBDO
R_1	mm	1000	10	2000	1296	1325
D_1	mm	100	6	800	583	631
$H_1/2$	mm	1000	10	1800	1089	1130
R_2	mm	1000	10	2000	1800	1842
D_2	mm	100	2	800	195	185
$H_2/2$	mm	1000	10	1800	1513	1545
J_1	A/mm^2	10.00	0.01	30.00	16.695	14.849
J_2	A/mm^2	-30.00	0.01	-10.00	-18.910	-19.629
B_{stray}	μT	-	-	-	15.8	40.3
E	MJ	-	-	-	179	179
$P_f(g_1)$	%	-	-	-	0	0
$P_f(g_2)$	%	-	-	-	67.29	0
$P_f(g_3)$	%	-	-	-	5.35	3.04
Iterations	-	-	-	-	-	11
FEA calls	-	-	-	-	-	6852

correspond to quench conditions of the superconducting magnets. All eight design variables are selected as random variables, and their statistical information is described in Table 2, where the DDO optimum is referred to [12].

Before performing RBDO, the probabilistic constraints were assessed at the DDO point by using MCS with 500,000 samples. Table 2 shows that while the failure probability of the third constraint slightly exceeds the target value, the second constraint has a large failure probability value of 67.29%. It implies there is a high possibility that the quench may occur specifically at the inner magnet in Fig. 2. In order to protect the magnet from such potential quench, RBDO was executed launching at the DDO point. Due to a heavy computational burden on the sensitivity-based RBDO methods, the problem of (10) was solved by only the proposed method, which started with 30 initial samples. After 11 iterative designs and 6,852 FEA calls, a RBDO optimum was obtained. As seen in Table 2, the optimized design satisfies all probabilistic constraints within the target value although B_{stray} becomes nearly three times larger than the initial one.

Fig. 4 compares the configuration of two superconducting magnets between DDO and RBDO designs. Two field distributions along line c depicted in Fig. 1 are presented in Fig. 5, where it is clear that the maximum field value appears at the center of the inner magnet. For better understanding, joint PDF and quench line relevant to g_2 are added in the figure. It implies that the failure occurs over the quench line, and the PDF area belonging to the failure region corresponds to the probability of failure. It is observed that the RBDO design has a safety margin enough to prevent the quench of the inner magnet.

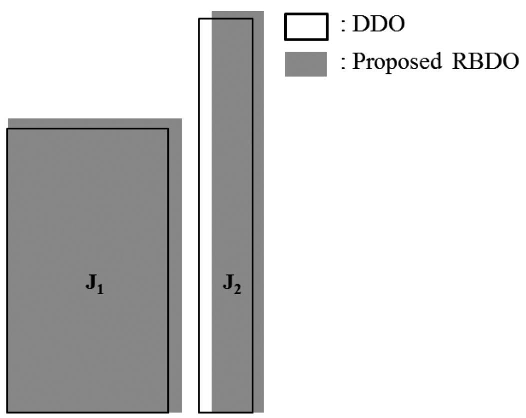

Fig. 4. Comparison of magnet dimensions between two designs.

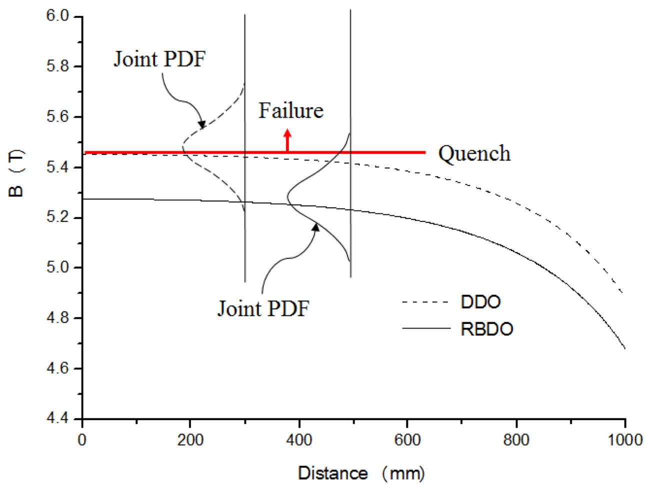

Fig. 5. (Color online) Field distribution along line c depicted in Fig. 2 and illustration of the failure probability for the quench condition.

4. Conclusion

This paper proposes a sampling-based RBDO method to effectively incorporate uncertain parameters into EM design problems. The results show that when compared with the sensitivity-based RBDO methods, the proposed method considerably reduces the numbers of FEA calls and iterative designs required for optimization without sacrificing the accuracy of reliability analysis.

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