ISSN (Print) 1226-1750
ISSN (Online) 2233-6656
Journal of Magnetics 19(1), 37-42 (2014) http://dx.doi.org/10.4283/JMAG.2014.19.1.037

A Type of Subsection Model for a Permanent Magnet Bar and its Leakage Permeance Calculation Method in an Open Magnetic Circuit

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(Received 17 September 2013, Received in final form 18 February 2014, Accepted 20 February 2014)

The equivalent model of a permanent magnet (PM) plays an important role in electromagnetic system calculation. A type of subsection model for a PM bar is established, to improve the accuracy of the traditional equivalent circuit method. The mathematical expression, and its end verification condition, are presented. Based on the analytical method and finite element method, the leakage permeance calculation of a PM bar in an open magnetic circuit is investigated. As an example, for a given certain type of PM bar, the magnetic flux of each section is validated by experiment, and by simulation. This model offers a foundation for building a high accuracy equivalent magnetic PM model in an electromagnetic system.

Keywords : analytical method, finite element method, magnetic field simulation, permanent magnet, leakage permeance

1. Introduction

In the field of electrical engineering, a permanent magnet (PM) is applied in more and more electrical products. As a key component, the PM's magnetic characteristics directly impact the performance of the product. How to deal with the PM model in the calculation of the electromagnetic system is the key factor of the attractive force calculation [1, 2]. Recently, there are mainly two ways to do so: one is the magnetic field simulation based on the finite element method (FEM) [3], and the other is the magnetic equivalent circuit (MEC) method [4]. The FEM has a high level of precision; however, the model is complex, and the amount of calculation is very large. Furthermore, it cannot show clear relations between the parameters, and it is not fit for the process of parameters design and tolerance design, which demand a large amount of calculation [5]. The MEC method has a simple model and a high calculation speed, and the relationship among various parameters in an electromagnetic system can be deduced from the model. Its disadvantage is its low accuracy. Some improvement of the MEC has been made by scholars; the main aim is to improve the accuracy of the MEC method, and keep its characteristic of fast calculation speed, to fit the demands of electromagnetic device calculation, and parameter design [6]. The infinitesimal element idea of FEM is introduced into the MEC, and an infinitesimal element solution is established, leading to an equation to be used in calculations [7]. Some studies have been performed aiming to improve the calculation methods of leakage permeance in certain electromagnetic devices [8, 9]. Some work about the PM division method, which is different from the traditional MEC, has been studied [10]. The basic research of the modeling method, and the leakage permeance calculation for a PM bar, need to be addressed.

In a traditional sense, a PM is often equivalent to one magnetic potential and one magnetic resistance. Actually, different parts of the PM have different operating points, particularly for the PM that has a nonlinear demagnetization curve. This type of PM is usually used in the electromagnetic apparatus of small size and high precision, for its high resistance to elevated temperatures and good machinability; for example, an aerospace relay. A more accurate MEC model for the PM can be established, based on parameters of subsections. Also, a suitable end verification condition will be deduced for calculation.

To calculate the parameters of a PM (section magnetic flux, operation point, leakage flux, etc.), the leakage permeance of the PM model in an open magnetic circuit is very important. It can be calculated through the graphical method. This method is based on the magnetic field dis-

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tribution graph drawn by analytical method, and in this method, the area is divided into tiny regular flux tubes. The permeance of each tiny flux tube is calculated, according to the series-parallel relationship of these tubes [11]. The graphical method can provide relatively accurate results, but needs a heavy work load. The analytical method is used to calculate the leakage permeance, which has a magnetic flux tube of crescent cross-section.

The calculation results of the PM subsection model using the above method are validated by test data, and 3-D FEM results. The PM subsection model improves the accuracy of the traditional MEC, and retains its characteristic of fast calculation speed. It can be used for building a high-accuracy MEC model in an electromagnetic system.

2. A Type of Subsection Model for a Permanent Magnet Bar

The model for a PM bar can be built based on parameters of subsections. Given the PM bar length L, and section area S, it is divided into n sections of equal size in pairs (two sub-sections) in the magnetizing direction, as shown in Fig. 1. The length of each section is $l_i = L/n$.

The parameter G_i $(1 \le i \le n)$ represents the leakage permeance of each subsection, and G_{end} represents the end leakage permeance. For permeance G, magnetic potential U, and magnetic flux ϕ, there is the equation $G = \phi/U$. For any section i between its two subsections, which are i–$(i$–$1)$ and i'–$(i$–$1)'$, the magnetic flux of the inner side surface $(i$–$1)$- $(i$–$1)'$ is ϕ_{i-1}, and the magnetic potential is U_{i-1}; the magnetic flux of the outer side surface i–i' is ϕ_i, and the magnetic potential is U_i. The magnetic potential increment of this section ΔU_i is:

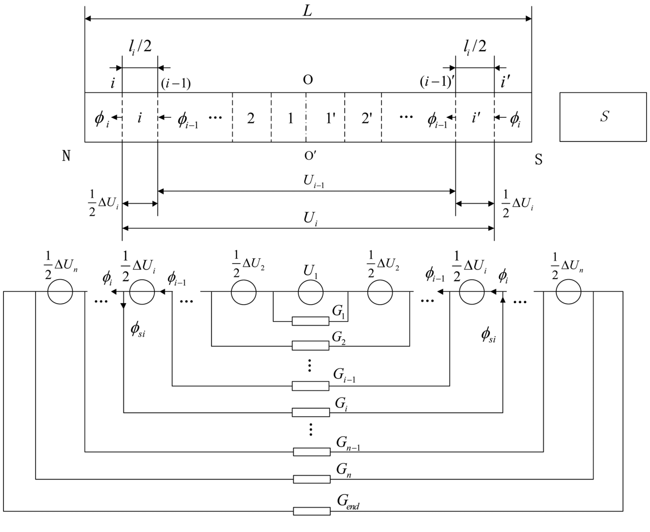


Fig. 1. PM subsectional model, and its equivalent magnetic circuit.

$$\Delta U_i = U_i - U_{i-1} \tag{1}$$

The average magnetic flux $\overline{\phi}_i$ and average magnetic flux density $\overline{B}_i$ of this section are defined as:

$$\overline{\phi}_i = \frac{1}{2}(\phi_{i-1} + \phi_i) \tag{2}$$

$$\overline{B}_i = \frac{\overline{\phi}_i}{A} = \frac{1}{2}(B_{i-1} + B_i) \tag{3}$$

where, $B_i = \phi_i/A$, and $B_{i-1} = \phi_{i-1}/A$. Following the demagnetization curve formula $B = f(H)$ of PM material (Fig. 3), the magnetic field intensity $\overline{H}_i$ can be calculated by $\overline{B}_i$. So the magnetic potential increment of segment i is:

$$\Delta U_i = -\overline{H}_i l_i \tag{4}$$

Therefore,

$$U_{i-1} = -(\overline{H}_1 l_1 + \overline{H}_2 l_2 + \ldots + \overline{H}_{i-1} l_{i-1}) = -\sum_{\zeta=1}^{j-1} \overline{H}_\zeta l_\zeta \tag{5}$$

where, ζ represents the subsection number $(1 \le \zeta \le j - 1)$. Considering (1), (4) and (5), the average magnetic potential $\overline{U}_i$ of section i is:

$$\overline{U}_i = \frac{1}{2}(U_{i-1} + U_i) = \begin{cases} U_{i-1} + \frac{1}{2}\Delta U_i = -\left(\sum_{\zeta=1}^{i-1} \overline{H}_\zeta l_\zeta + \frac{1}{2}\overline{H}_i l_i\right) \\ U_i - \frac{1}{2}\Delta U_i = -\left(\sum_{\zeta=1}^{i} \overline{H}_\zeta l_\zeta - \frac{1}{2}\overline{H}_i l_i\right) \end{cases} \tag{6}$$

According to formula (6), the magnetic leakage flux ϕ_{si} of section i is:

$$\phi_{si} = \overline{U}_i G_i = \begin{cases} -G_i\left(\sum_{\zeta=1}^{i-1} \overline{H}_\zeta l_\zeta + \frac{1}{2}\overline{H}_i l_i\right) \\ -G_i\left(\sum_{\zeta=1}^{i} \overline{H}_\zeta l_\zeta - \frac{1}{2}\overline{H}_i l_i\right) \end{cases} \tag{7}$$

For $\phi_{i-1} = \phi_i + \phi_{si}$, by substitution of (2), (3), and (7), it can be shown that:

$$\overline{B}_i - B_i = B_{i-1} - \overline{B}_i = -\frac{G_i}{2A}\left(\sum_{\zeta=1}^{i-1} \overline{H}_\zeta l_\zeta + \frac{1}{2}\overline{H}_i l_i\right) \tag{8}$$

For the first section, $i = 1$; according to formula (8), therefore,

$$B_1 = \overline{B_1} + \frac{G_1}{4A}\overline{H_1}l_1 \tag{9}$$

where, G_1 is the leakage permeance of the first subsection.

Thus, the entire PM bar model is as follows:

$$\begin{cases} B_1 = \overline{B_1} + \dfrac{G_1}{4A}\overline{H_1}l_1 \\ B_{i-1} - \overline{B_i} = -\dfrac{G_i}{2A}\left(\displaystyle\sum_{\zeta=1}^{i-1}\overline{H_\zeta}l_\zeta + \frac{1}{2}\overline{H_i}l_i\right) \quad (i = 2, 3, \ldots, n) \\ B_i = 2\overline{B_i} - B_{i-1} \\ \overline{B_i} = f(\overline{H_i}) \end{cases} \tag{10}$$

The end verification condition of this model is:

$$B'_n = \frac{G_{end}\overline{U_n}}{A} = \frac{G_{end}}{A}(\overline{H_1}l_1 + \overline{H_2}l_2 + \ldots + \overline{H_n}l_n) \tag{11}$$

where, B'_n represents the end verification magnetic intensity. The calculation stops when $(B_n - B'_n) < \varepsilon$, where ε is the error.

It can be deduced from the model that the leakage permeance G_i has a decisive influence on the calculation accuracy.

3. Leakage Permeance Calculation Method in an Open Magnetic Circuit

3.1. Leakage Permeance Calculation based on the Distribution of Leakage Magnetic Field Lines

A PM bar in an open magnetic circuit is divided into subsections; the relationship between variables, and the distribution of magnetic field lines for one pair section are roughly drawn in Fig. 2.

Most cross-sections of leakage magnetic flux tubes are close to crescent shape, except for the two ends, and the center part of the PM. Because the leakage flux near the PM center is relatively small, it is here neglected. This paper focuses on the leakage permeance calculation of a flux tube with crescent cross-section.

The integration calculation formula of leakage permeance is:

$$G = \mu_0 \iiint_V \frac{dV}{L_P^2} \tag{12}$$

where, μ_0 represents the air permeability, dV represents an infinitesimal volume, and L_P represents the equivalent length of the flux tube. For subsection i, the shape of the flux tube is the part between two coaxial faces with width W. The two coaxial faces are both surrounded by two arcs, which are not homocentric [12]. So the calculation formula for leakage permeance, G_i, can be integrated as:

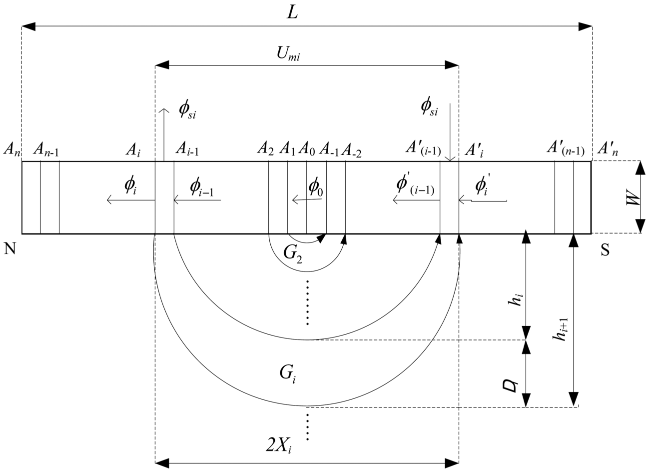


Fig. 2. Leakage permeance calculation schematic of a bar PM in an open magnetic circuit. W: width of PM; A_i: section area of PM along the direction of length $L(i = -n, 1 - n, 0,1,...n)$; ϕ_i: magnetic flux across section A_i; ϕ_{si}: leakage flux of i block PM; D_i: the distance between inner and outer boundary of i crescent cross section; h_i: the height of inner boundary of i crescent cross section; X_i: the distance between section A_i and section A_0; and U_{mi}: magnetic potential of section A_i.

$$G_i = \mu_0 \frac{W}{3} \arctan \frac{X_i D_i}{X_i^2 + D_i h_i + h_i^2} \tag{13}$$

Formula (13) needs the values of D_i and h_i, corresponding to different magnetic field lines. The values of D_i and h_i can be obtained from the ferrograph of PM. Also, the 2-D FEM method can be used to obtain them. By putting h_i and $D_i = h_{i+1} - h_i$ (as shown in Fig. 3) into formula (13) with known X_i, the leakage permeance of section i can be worked out. Because there are four side faces to be considered, the total leakage permeance should be summed together.

3.2. Analysis of Leakage Permeance in an Open Magnetic Circuit for the FEM

To verify the validity of this PM bar model, test data and the FEM method are used. For the FEM method, the results need to be analyzed, which analysis follows the procedure below.

The flux of any section parallel to the center section can be obtained from the FEM results. As for the model above, the whole FEM PM bar can be divided into n test sections. The leakage flux of PM section i can be obtained from:

$$\phi'_{si} = \phi'_{i+1} - \phi'_i \tag{14}$$

When there are enough PM sections divided, the average magnetic flux density of each section approximates:

$$B'_i = \frac{\phi'_i}{S} \tag{15}$$

For subsection i, the magnetic field intensity H'_i corresponding to B'_i is achieved, by referring to the demagnetization curve of PM:

$$H'_i = f'(B'_i) \tag{16}$$

where, function f' is the inverse function of the demagnetization curve function f.

$$U_{mab} = U_{ma} - U_{mb} = \int_a^b H'_i \cdot \mathrm{d}l \tag{17}$$

The magnetic potential U_{mab} between two sections, whose symmetric center is the center section of the PM, can be obtained by formula (17). Then, the leakage permeance G'_i of PM section i is:

$$G'_i = \frac{\phi'_{si}}{U_{mab}} \tag{18}$$

Finally, the curve of leakage permeance G'_i versus position X_i can be obtained, to compare with the PM subsection calculation results.

4. Calculation Example and Experimental Validation

According to the above PM bar model, the leakage permeance and parameters of the PM subsection model in an open magnetic circuit can be calculated. For example, taking one type of Fe-Cr-Co bar with dimensions of 60 × 16.2 × 3.3 mm, its demagnetization curve is as shown in Fig. 3.

4.1. PM Subsection Model Permeance Calculation

The distribution of PM bar magnetic field lines according to the ferrograph and 2-D simulation results is shown in Fig. 4.

According to $D_i = h_{i+1} - h_i$, the relationship between X_i and h_i can be obtained (Fig. 5).

Because the PM bar has four side faces, the total leakage magnetic permeance is the sum of four parts. Fig. 6 shows the total and four side faces' leakage magnetic permeance G_{si}, versus position X_i.

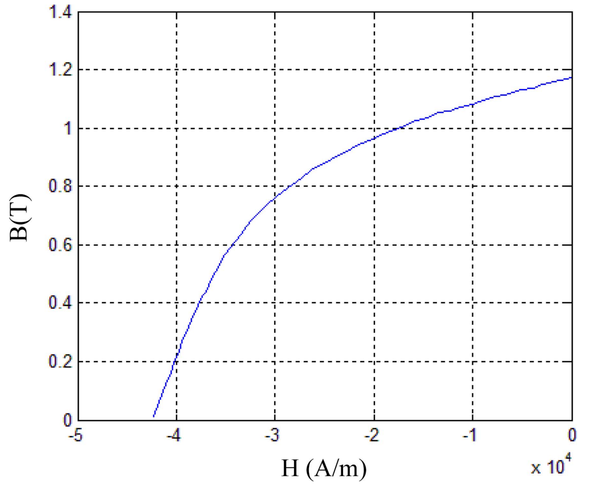
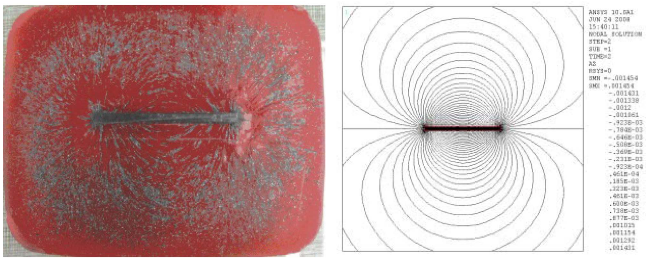


Fig. 3. (Color online) Demagnetization curve of PM Fe-Cr-Co.



(a) Ferrograph of a PM (b) 2-D simulation of a PM

Fig. 4. (Color online) Leakage magnetic field lines of a PM bar.

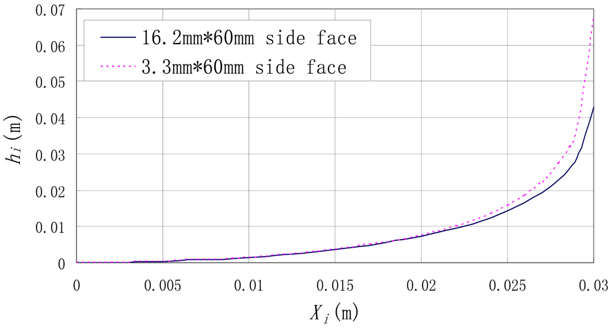


Fig. 5. (Color online) Parameter h_i, versus X_i.

4.2. Experimental and FEM Validation

The flux measuring equipment shown in Fig. 7 is designed to measure the sectional flux value of PM along the axial direction. The flux coil is fixed on the sectional plane of PM, and then the PM is released with high speed. The coil will measure the variation of current, and the flux difference is recorded with a flux meter. The flux coil fits the outer shape of the PM, and to ensure accuracy, is mounted on the PCB board. Then, the leakage magnetic flux of each section can be obtained, based on formula (14). As it is difficult to measure magnetic potential, the

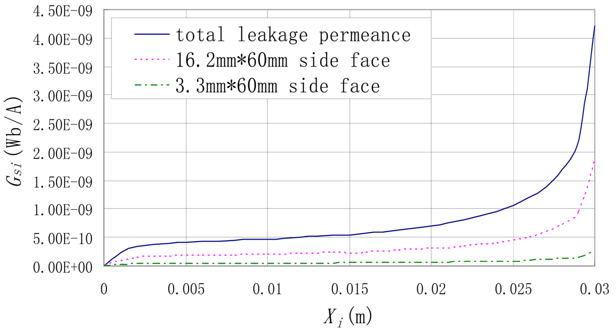


Fig. 6. (Color online) Total and four side faces' leakage permeance, versus X_i.

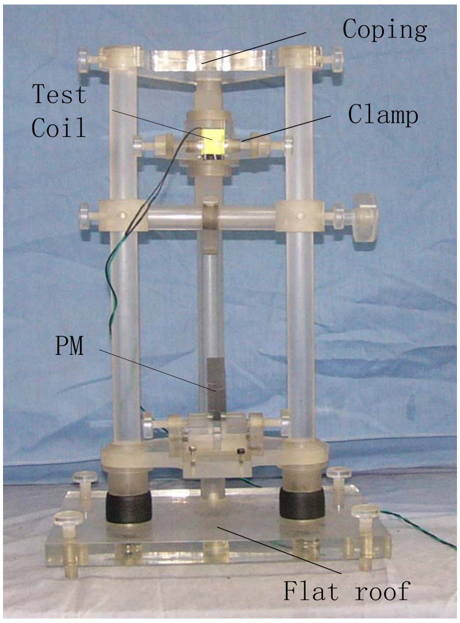


Fig. 7. (Color online) The sectional flux measuring equipment.

FEM simulation method mentioned above is used. According to formula (18), the leakage permeance based on measurement of the result can be obtained; then it is compared with the leakage permeance calculated by the 3-D FEM, and by the analytical method (Fig. 8).

Fig. 8 shows that the leakage permeance calculated by the simulation results, and the leakage permeance calculated by the measurement results, are close to each other, which proves that the leakage permeance calculated by the 3-D FEM result is relatively accurate. When X_i ranges from 0 m to 0.028 m, the leakage permeance calculated by the analytical formula matches well with the leakage permeance calculated by the measurement result; when X_i ranges from 0.028 m to 0.030 m, the leakage permeance calculated by the analytical formula and by the measurement result are different. The leakage permeance results can be used to calculate the PM subsection model mentioned above.

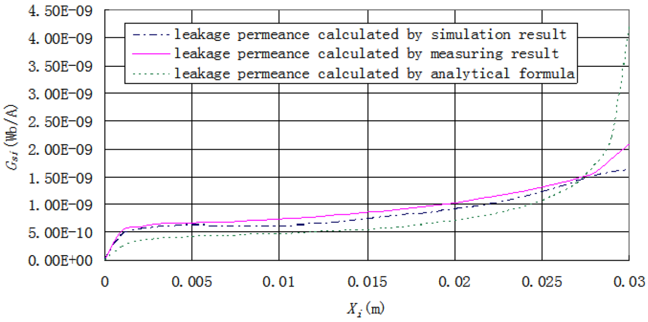


Fig. 8. (Color online) Comparison of the leakage permeance.

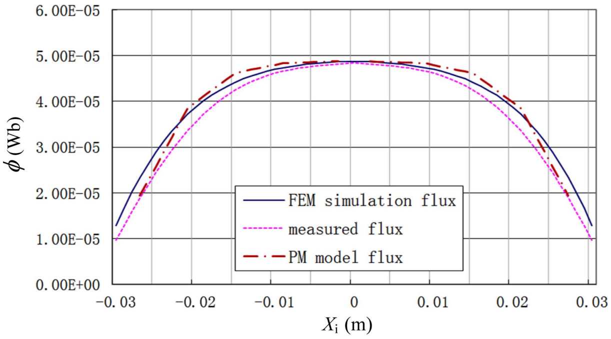


Fig. 9. A (Color online) Comparison of the PM model, the FEM, and the measured flux.

The section number is set to 10. The calculation model follows formula (10). The flux values of each section are shown in Fig. 9, in comparison with the FEM simulation results, and the test data.

In Fig. 9, it can be deduced that when the section number is set to 10, the PM subsection model section flux calculation results are very close to the test data, with a maximum error of 9.76%. The time cost of the 3-D FEM model (element number 56697, time: 316 s) is far more than this subsection model (time: 5 s). The modeling process and post-processing for the FEM method is complex. With this model, the inner section flux of a PM bar can be calculated by the MEC method. It is believed that as the section number *n* becomes larger, the results become more accurate.

5. Conclusion

1) A subsection model for a PM bar has been built, based on parameters of subsections. The end verification condition is also deduced for calculation.

2) The leakage permeance of this PM bar subsection model in an open magnetic circuit is calculated with the analytical method. The formula is deduced to calculate

the leakage permeance, which has a crescent cross-section magnetic flux tube.

3) This PM bar subsection model and its leakage permeance calculation method is superior in calculation time, to the FEM model; and in accuracy, to the traditional MEC model. This model proves to have the ability to calculate part of the inner magnetic parameters of a PM bar in a simple way, and it can be used for building a high-level accuracy MEC model in an electromagnetic system that contains a PM.

Acknowledgments

The authors express their thanks for the support of the NSFC (National Natural Science Foundation of China), Project 51177021 and 51207028.

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