Effect of a Magnetic Field on Mixed Convection of a Nanofluid in a Square Cavity

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The problem of mixed convection in a differentially heated lid-driven square cavity filled with Cu-water nanofluid under effect of a magnetic field is investigated numerically. The left and right walls of the cavity are kept at temperatures of T_h and T_c respectively while the horizontal walls are adiabatic. The top wall of the cavity moves in own plane from left to right. The effects of some pertinent parameters such as Richardson number (ranging from 0.1 to 10), the volume fraction of the nanoparticles (ranging 0 to 0.1) and the Hartmann number (ranging from 0 to 60) on the fluid flow and temperature fields and the rate of heat transfer in the cavity are investigated. It must be noted that in all calculations the Prandtl number of water as the pure fluid is kept at 6.8, while the Grashof number is considered fixed at 10^4. The obtained results show that the rate of heat transfer increases with an increase of the Reynolds number, while but it decreases with increase in the Hartmann number. Moreover it is found that based the Richardson and Hartmann numbers by increase in volume fraction of the nanoparticles the rate of heat transfer can be enhanced or deteriorated compared to the based fluid.

Keywords : magnetic field, mixed convection, square cavity, numerical simulation

1. Introduction

Mixed convection occurs in many industrial applications such as electronic equipment, solar collector and crystal growth in liquids [1]. When the cavity is filled with an electric conductive fluid the flow and temperature fields in a cavity can be controlled using a magnetic field. By enforcing a magnetic field, the Lorentz force is generated in the electric conductive fluid which interacts with buoyancy force and reduces the velocities. Oreper and Szekely [2] investigated the effect of a magnetic field on free convection heat transfer in a rectangular enclosure and showed that the magnetic field strength is one of the most important parameters for crystal formation and suppresses the free convection currents. In a numerical study Rudraiah et al. [3] investigated the effect of a transverse magnetic field on free convection heat transfer in a differentially

heat rectangular with isothermal side walls and adiabatic horizontal walls. Their results showed that a circulating flow is formed with a relatively weak magnetic field. More ever they found that with increasing the magnetic field, the convective heat transfer decreases. Krakov et al. [4] investigated numerically and experimentally effects of a uniform magnetic field on natural convection in a cubic enclosure. They found that a set of numerous convective structures exists in the cube and what structure is more stable depends on the comparison of magnetic and gravity forces. Wang et al. [5] reported results of a numerical study on magnetohydrodynamic natural convection in a porous media filled square cavity. Kandaswamy et al. [6] studied numerically magnetohydrodynamic natural convection in a square cavity with partially thermally active side walls. They considered nine dierent combinations of the relative positions of the active portions. Nithyadevi et al. [7] using a numerical simulation investigated effect of time periodic boundary conditions on magnetohydrodynamic natural convection in a square cavity with partially heated and cooled side walls. Recently Mahmoodi and Talea'pour [8] investigated numerically magnetohydrodynamic free convection in a square cavity with hot left wall, cold top wall and insulated right and bottom

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wall. They found that a clockwise primary vortex is formed inside the cavity regardless the Rayleigh number and the Hartmann number.

In the present work effect of a uniform and steady magnetic field on mixed convection of nanoparticles of Al_2O_3 suspended in pure water in a lid-driven differentially heated square cavity is investigated numerically. The effects of the Hartmann and Rayleigh numbers and nanoparticles volume fraction on the fluid flow and heat transfer inside the enclosure are investigated. The results are presented in terms of streamlines and isotherms inside the cavity and average Nusselt number of the hot wall.

2. Mathematical Modeling

A schematic geometry of the square cavity with boundary conditions considered in the present paper is shown in Fig. 1. The height and the width of the cavity are denoted by H. The left wall is kept at a T_h, while, the right wall is kept at cold temperature T_c and the horizontal walls are adiabatic. The magnetic field of strength B_0 is applied parallel to x-axis. The cavity is filled with Al_2O_3-water that is considered Newtonian and incompressible. The fluid flow is assumed to be laminar. The thermophysical properties of the nanoparticles and pure water are presented in Table 1. The thermophysical properties of the nanofluid are considered constant with the exception of the density which varies according to the Boussinesq approximation.

The continuity, momentum and energy equations for laminar, steady state, two-dimensional free convection with a magnetic field in x-direction, are as follow:
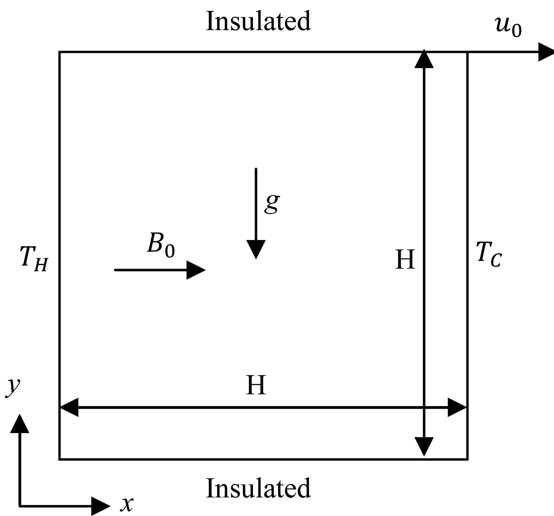

Fig. 1. a schematic view of the square cavity and boundary conditions considered in the present study.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\frac{\mu_{eff}}{\nu_f \rho_{nf}}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\frac{\mu_{eff}}{\nu_f \rho_{nf}}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right)$$
$$+ \frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}Ri\theta - \frac{Ha^2}{Re}V \tag{3}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\alpha_{eff}}{Pr\,Re}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

The dimensionless parameters in the above equations are as following:

$$X = \frac{x}{H}, \ Y = \frac{y}{H}, \ U = \frac{u}{u_0}, \ V = \frac{v}{u_0}, \ P = \frac{p}{\rho_{nf} u_0^2}, \ \theta = \frac{T-T_c}{T_h-T_c} \tag{5}$$

Where Ra, Pr and Ha are the Rayleigh, Prandtl and Hartmann numbers and are defined as

$$Re = \frac{u_0 H}{\nu_f}, \ Ri = \frac{g\beta_f(T_H-T_C)H}{u_0^2}, \ Pr = \frac{\nu_f}{\alpha_f}, \ Ha = B_0 H\sqrt{\frac{\sigma}{\rho\nu}} \tag{6}$$

Where, ν is the kinematic viscosity. The details of the derivation the equations have been presented by Mahmoodi and Talea'pour [8]. The relations of the nanofluid have been presented by Mahmoodi [9].

To computation of the rate of heat transfer, Nusselt number along the hot wall of the cavity is used. The local Nusselt number of the hot wall is

$$Nu_{local} = -\frac{K_{nf}}{K_f}\frac{\partial \theta}{\partial X}\Big|_{X=0} \tag{7}$$

3. Numerical Approach

The governing equations are discretized using the finite volume method. The diffusion terms in the governing equations are approximated by a second order central

Table 1. Thermophysical properties of water and Cu nanoparticles at 25ºC.

Physical properties	Water	Cu
c_p (J/kg K)	4179	383
ρ (kg/m³)	997.1	8954
k (W/m K)	0.613	400
β (K⁻¹)	21×10^{-5}	1.67×10^{-5}

Table 2. Comparison of the present results for the average Nusselt number of the heated wall with the results of Pirmahammadi *et al.* [12].

Ra	Ha	Nu	
		Pirmahammadi *et al.* [12]	Present study
10^4	0	2.29	2.289
	50	1.06	1.061
	100	1.02	1.019
10^5	0	4.62	4.631
	25	3.51	3.507
	100	1.37	1.365

difference scheme while a hybrid scheme which is a combination of central difference scheme and upwind scheme is applied to discretize convective terms. In order to couple the velocity field and pressure in the momentum equations, the SIMPLER algorithm is used. The set of algebraic equations are solved iteratively using TDMA algorithm [10].

In order to validate the numerical procedure, the results obtained by the present code for a differentially heated square enclosure under a magnetic field are compared with the results of Pirmohammadi *et al.* [11] for the same problem. Table 2 shows a comparison for the average Nusselt number obtained by the present code with the results of Pirmoammadi *et al.* for different Rayleigh numbers and Hartmann numbers. As can be observed from the table, very good agreements exist between the two results.

In order to determine a proper grid for the numerical simulation, a grid independence study is undertaken for magnetohydrodynamic mixed convection inside the cavity considered in Fig. 1 at Ri = 1 and Ha = 30. It was found that an 80×80 uniform grid is used for all the results to be presented in the following.

4. Results and Discussion

Figures 2 and 3 illustrate variations of streamlines and isotherms versus Richardson and Hartmann numbers inside the cavity for pure fluid and nanofluid with φ = 0.1. The results are presented for the Richardson number ranging from 0.1 to 10 and the Hartmann number varying from 0
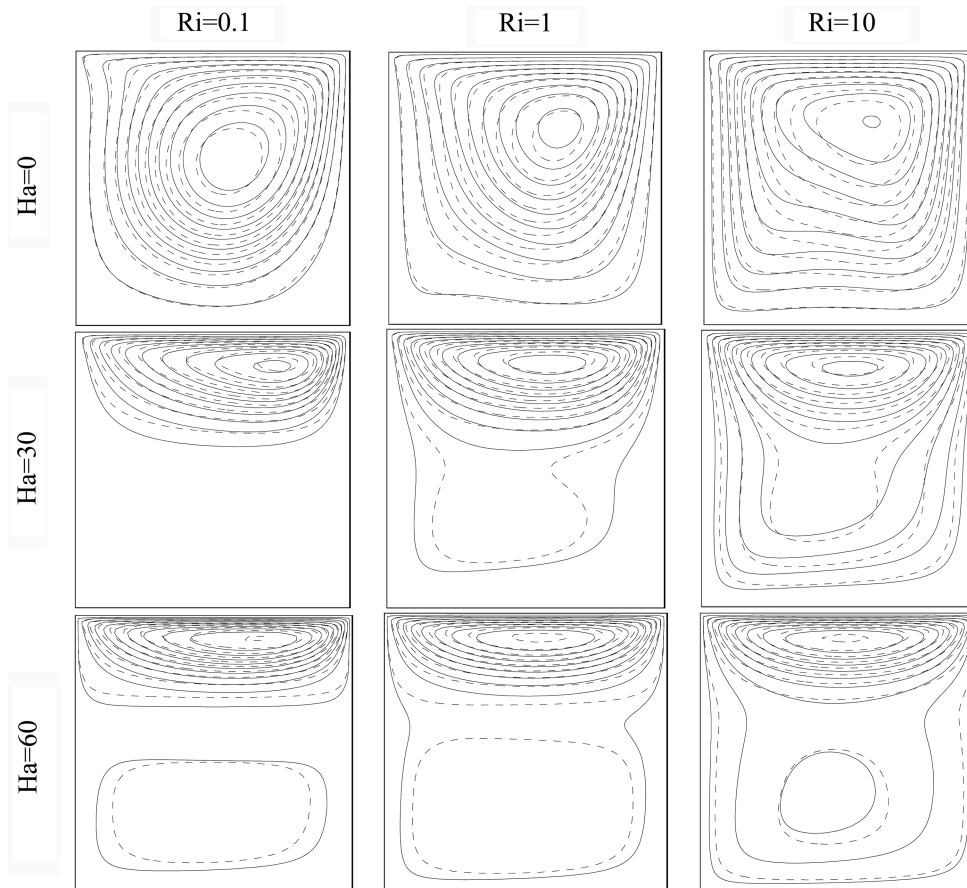

Fig. 2. Streamlines for Cu–water nanofluid with $\varphi = 0.1$ (dashed line) and pure fluid (solid line) inside the cavity for different Richardson and Hartmann numbers at $Gr = 10^4$.
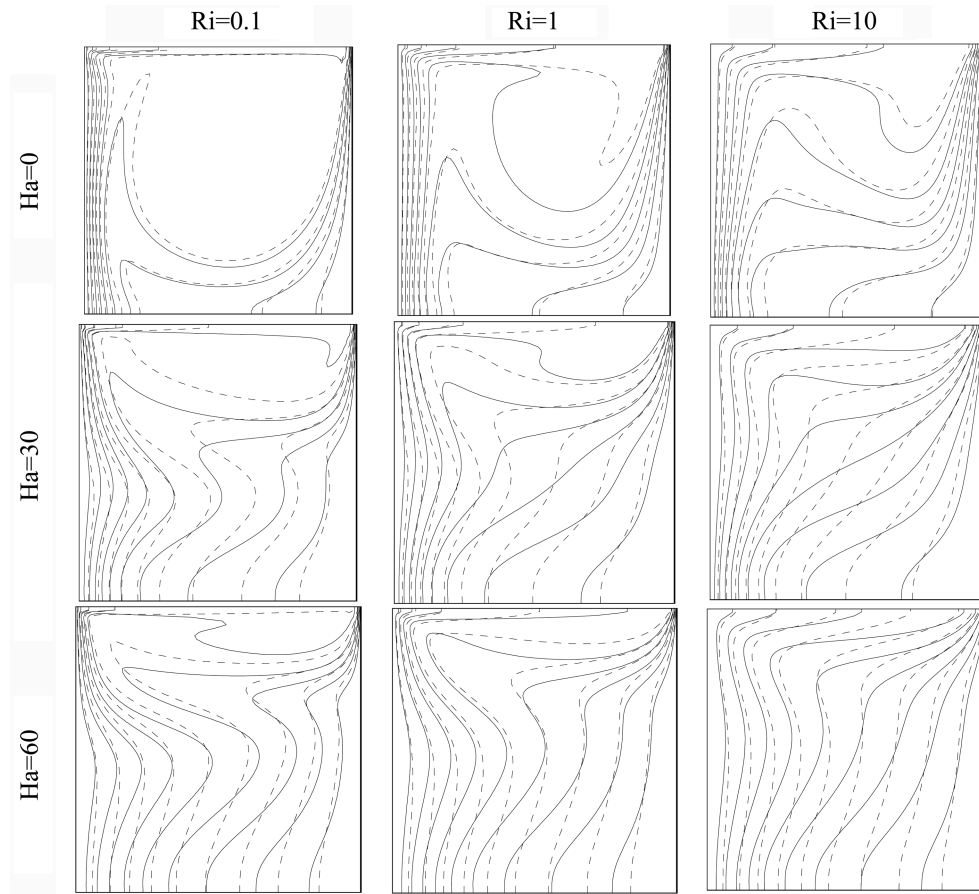
Fig. 3. Isotherms for Cu–water nanofluid with $\varphi = 0.1$ (dashed line) and pure fluid (solid line) inside the cavity for different Richardson and Hartmann numbers at $Gr = 10^4$.

to 60. As can be seen in the figure in the absence of the magnetic field (Ha = 0) a fully developed clockwise eddy occupies major portion of the cavity. At Ri=10 that is a buoyancy dominated regime the core of the eddy is located at upper portion of the cavity near the top wall. As the Richardson number decreases the effect of shear force due to the moving top wall increases, hence the core of the eddy moves downward. As can be seen in the figure a similar behavior is found for both nanofluid and pure fluid. As the Hartmann number increases to 30, the core of the eddy is located at the vicinity of top wall for all Richardson numbers considered. The only difference is the as the Richardson number decreases the lower portion of the cavity becomes nearly stagnant. At Ri = 0.1 and for the strongest considered magnetic field (Ha = 60) a secondary smaller eddy in formed in the lower portion of the cavity. As the Richardson number increases (increase in effect of buoyancy force) these eddies coalesce and form a double core large eddy.

As can be observed from the isotherms in the Fig. 3 at Ha = 0 and Ri = 0.1 high gradient thin region exits at the vicinity of isothermal wall which is due to domination of force convection and shear force of the top wall. As the Richardson number increases the effect of buoyancy force increases and the thermal boundary layers thicken. When an external magnetic field exist the velocity of the fluid and flow intensity decrease. The phenomenon motivates the heat transfer to occur due to conduction. It can be seen from uniformly distributed isotherms in the cavity. As can be seen from the figure the high temperature gradient regions exist only in upper corner of the cavity.

Variations of average Nusselt number versus Richardson number, Hartmann number and volume fraction of the nanoparticles are presented in Fig. 4. It can be seen from the figure that at each value of Hartmann number the average Nusselt number increase with increase in volume fraction of the nanoparticles. Moreover when the Richardson number increases the average Nusselt number increases too. Furthermore it can be seen that at all values of Richardson number considered the existence of the magnetic field the rate of heat transfer decreases. Also as the magnetic field become stronger its effect on decrease of
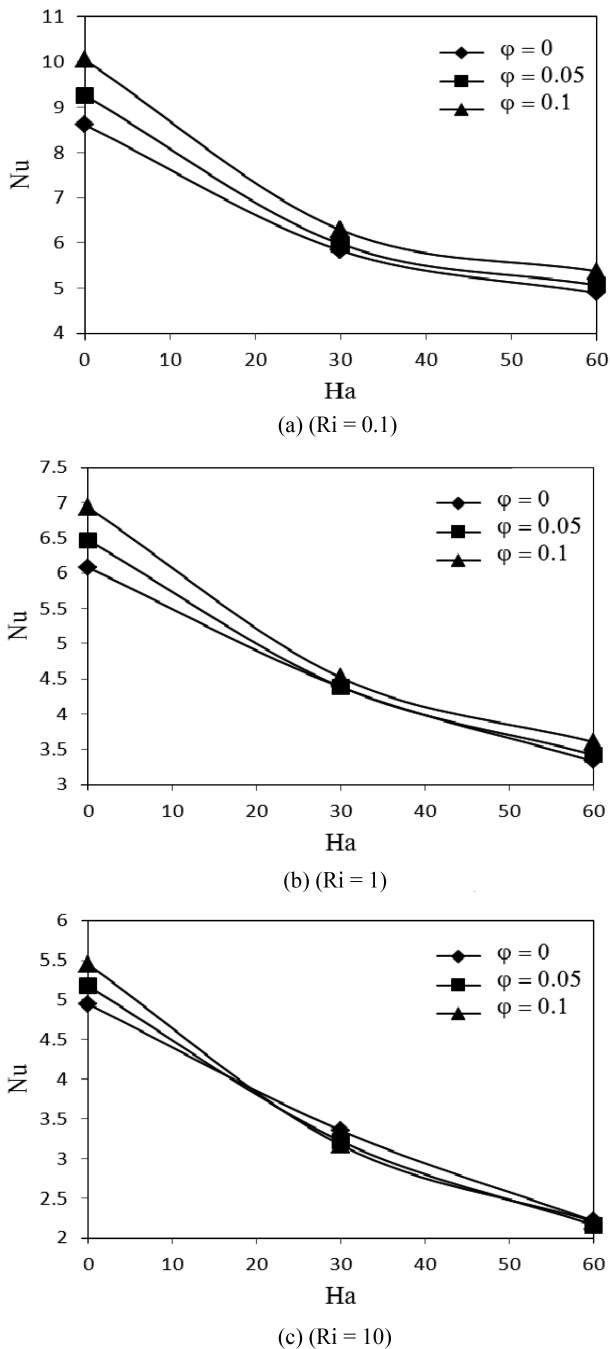
Fig. 4. Variations of average Nusselt number with the Hartmann numbers for different values of volume fraction of nanoparticles at different Richardson numbers and $Gr = 10^4$.

conditions the rate of heat transfer decreases with increase in volume fraction of the nanoparticles.

5. Conclusion

Using the finite volume method, the Magnetohydrodynamic mixed convection of Cu-water in top lid-driven square enclosure cooled from the right wall, heated from the left wall and adiabatic horizontal walls was studied numerically. A parametric study was performed and the effect of the Richardson number, the Hartmann number and the volume fraction of the nanoparticles on the fluid flow and heat transfer were investigated. It was found the heat transfer mechanisms and the flow characteristics inside the enclosure depend strongly upon both the strength of the magnetiv field and the Richardson number. It was found that the application of a longitudinal magnetic field results in a force opposite to the flow direction that tends to drag the flow. Moreover it was observed that, for high Rchardson numbers, by increasing Hartmann number, free convection is suppressed and heat transfer occurs mainly through conduction. Also it was found that the existence of the nanoparticled in the base fluid can enhanced the rate of heat transfer.

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heat transfer increases. The special case is at Ri = 10 in which in the existence of the magnetic field the volume fraction of the nanoparticles has a reverse effect. In these