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Sampling-Based Sensitivity Approach to Electromagnetic Designs Utilizing Surrogate Models Combined with a Local Window

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This paper proposes a sampling-based optimization method for electromagnetic design problems, where design sensitivities are obtained from the elaborate surrogate models based on the universal Kriging method and a local window concept. After inserting additional sequential samples to satisfy the certain convergence criterion, the elaborate surrogate model for each true performance function is generated within a relatively small area, called a hyper-cubic local window, with the center of a nominal design. From Jacobian matrices of the local models, the accurate design sensitivity values at the design point of interest are extracted, and so they make it possible to use deterministic search algorithms for fast search of an optimum in design space. The proposed method is applied to a mathematical problem and a loudspeaker design with constraint functions and is compared with the sensitivity-based optimization adopting the finite difference method.

Keywords : electromagnetics, metamodeling, optimization, sensitivity analysis

1. Introduction

Recently, some research in electromagnetics has focused on the metamodeling for electromagnetic optimization problems which usually require high computational cost. Among all the metamodeling methods such as the least-squares regression, support vector regression and radial basis functions, the Kriging method has gained significant attention due to its capability and accuracy of dealing with highly nonlinear problems. Generally, the method consists of two parts which realize a response function of interest: the mean structure and a zero-mean stationary Gaussian stochastic process [1-8].

Regarding the electromagnetic (EM) application, the first attempt was made to optimize the TEAM workshop problem 25 with the use of the Kriging-based surrogate models [1]. Since then, several articles have been published so far. For the same problem, J. D. Lavers *et al*. [2] compared three sequential methods of least square, Kriging and linear Bayesian, and K. R. Davey [3] introduced the Latin hypercube sampling technique combined with pattern search algorithm. K. Hameyer *et al*. [4] adopted the Kriging models in conjunction with evolution strategy to reduce the torque ripple of a switched reluctance motor. In [5], the optimal design of a brushless DC motor was conducted by the adaptive response surface method with the reduced design space at candidate optimal points. The reduced space is somewhat similar to the local window concept but its main role lies in improving the accuracy of the response surface generated on the entire design space. Most research works used the Kriging method to generate surrogate models in the entire design space, so-called global window, and then obtained the optimum through the models combined with stochastic optimization methods, such as evolution strategy, genetic algorithm, simulated annealing, etc., which need a lot of design iterations than the sensitivity-based optimization methods. When using the Kriging method with the stochastic search algorithms, it is worth noticing that only the chance of a global optimum is enhanced, not guaranteed. Moreover, it is obvious that, as the number of design variables increases, a computational burden still occurs because lots of sampling points are required for the surrogate modeling and optimization process especially on the entire design space.

As an effort of enhancing the efficiency of the Kriging-based optimization method, this paper proposes a new sampling-based optimization method for EM designs where

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design sensitivities are extracted from the elaborate surrogate models based on the universal Kriging (UKG) method and a local window concept. After inserting sequential samples to satisfy a certain convergence criterion, the elaborate surrogate model for each true performance function is generated within a relatively small design area, called a hyper-cubic local window, at an intermediate design point. From Jacobian matrices of the local surrogate models, the accurate design sensitivity values are easily extracted, and so they make it possible to use deterministic search algorithms for fast search of an optimum in the design space. The proposed method is applied to a mathematical problem and a loudspeaker design with constraint functions and is compared with the sensitivity-based optimization adopting the finite difference method (FDM).

2. Kriging Model and Its Derivative

In this section, the basic theory of UKG including its derivative is summarized briefly [6-8]. In the Kriging method, the outcomes are considered as a realization of a stochastic process. The goal is to estimate a response $\mathbf{y} = [y(\mathbf{x}_1), y(\mathbf{x}_2), \cdots, y(\mathbf{x}_n)]^T$ with $y(\mathbf{x}_i) \in \mathbf{R}^1$ based on n sample points, $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n]^T$ with $\mathbf{x}_i \in \mathbf{R}^m$. The response consists of a summation of two parts as

$$\mathbf{y} = \mathbf{F}\beta + \mathbf{e}$$
$$\text{where } \mathbf{F} = [f_k(x_i)] \; i = 1, \cdots n, \; k = 1, \cdots K \tag{1}$$
$$\beta = [\beta_1, \cdots, \beta_k], \; \mathbf{e} = [e(\mathbf{x}_1), \cdots, e(\mathbf{x}_n)]^T.$$

The first term of the right side of (1), called the mean structure of the response, is intended to follow the general tendency of the function to be modeled. It is generally composed of the first/second-order basis functions $f_k(x_i)$ and the vector of regression coefficient β, which is obtained from the generalized least square method. The second term $\mathbf{e}$ is a realization of the stochastic process. It is assumed to have zero mean $E[\mathrm{e}(\mathbf{x}_i)]=0$ and covariance structure $E[\mathrm{e}(\mathbf{x}_i)\ \mathrm{e}(\mathbf{x}_j)] = \sigma^2 R(\theta, \mathbf{x}_i, \mathbf{x}_j)$, where σ^2 is the process variance, θ is the correlation parameter vector estimated by applying the maximum likelihood estimator (MLE) and R is the correlation function of the stochastic process. The term $\mathbf{e}$ makes it possible to follow the fluctuations around the general tendency. In most engineering applications, the correlation function is set to be a Gaussian form expressed as follows

$$R(\theta, \mathbf{x}_i, \mathbf{x}_j) = \prod_{l=1}^{nr} \exp(-\theta_l (x_{i,l} - x_{j,l})^2) \tag{2}$$

where $x_{i,l}$ is the lth component of variable $\mathbf{x}_i$.

Under the decomposition of (1) and the optimal θ to maximize MLE, the noise-free unbiased response $\hat{y}$ at a new point of interest denoted by $\mathbf{x}_0$ is written as a linear predictor

$$\hat{y}(\mathbf{x}_0) = \mathbf{w}_0^T \mathbf{y} \tag{3}$$

where $\mathbf{w}_0 == [w_1(\mathbf{x}_0), w_2(\mathbf{x}_0), \cdots, w_n(\mathbf{x}_0)]^T$ means the $n \times 1$ weight vector for prediction at the point. It is obtained using the unbiased condition $E[\hat{y}(\mathbf{x}_0)] = E[y(\mathbf{x}_0)]$ as

$$\mathbf{w}_0 = \mathbf{R}^{-1}\left(\mathbf{r}_0 + \frac{1}{2\sigma^2} F\lambda\right) \tag{4}$$

where $\mathbf{R}$ is the symmetric correlation matrix with the ijth component $R_{i,j} = \mathrm{R}(\theta, \mathbf{x}_i, \mathbf{x}_j)$, $\mathbf{r}_0 = [\ \mathrm{R}(\mathbf{q}, \mathbf{x}_1, \mathbf{x}_0), \cdots, \mathrm{R}(\theta, \mathbf{x}_n, \mathbf{x}_0)]^T$ is the correlation vector between $\mathbf{x}_0$ and samples $\mathbf{x}$ and λ is the Lagrange multiplier.

After substituting (4) into (3), the prediction of Kriging model which interpolates the n sample points is expressed as

$$\hat{y}(\mathbf{x}_0) = \mathbf{w}_0^T \mathbf{y} = \left(\mathbf{r}_0 + \frac{\mathbf{F}\lambda}{2\sigma^2}\right)^T \mathbf{R}^{-1} \mathbf{y} = \mathbf{f}_0^T \beta + \mathbf{r}_0^T \mathbf{R}^{-1}(\mathbf{y} - \mathbf{F}\beta) \tag{5}$$

where $\sigma^2 = 1/\mathrm{n}(\mathbf{y}\text{-}\mathbf{Fb})\mathbf{R}^{-1}(\mathbf{y}\text{-}\mathbf{F}\beta)$ and $\beta = (\mathbf{F}^T\mathbf{R}^{-1}\mathbf{F})^{-1}(\mathbf{F}^T\mathbf{R}^{-1}\mathbf{y})$ are obtained from the generalized least square regression. From (5), the derivative $\hat{y}'$ of the prediction model at $\mathbf{x}_0$ is given by

$$\hat{y}'(\mathbf{x}_0) = \mathbf{J}_\mathbf{f}^T \beta + \mathbf{J}_r^T \mathbf{r}_0 \tag{6}$$

where $\mathbf{J}_\mathbf{f}^T$ and $\mathbf{J}_r^T$ denote the Jacobian transformation of $\mathbf{f}_0$ and $\mathbf{r}_0$, respectively.

3. Implementation of Sampling-Based Design Optimization

To efficiently carry out the sampling-based optimization method utilizing the design sensitivity information, three strategies of the hyper-cubic local window, sampling and sample reuse are explained, and accordingly the program architecture for its numerical implementation is presented.

3.1. Hyper-cubic local window

Since the design sensitivity of (6) corresponds to a local quantity at a design point of interest, the hyper-cubic local window is much more suitable for obtaining an accurate sensitivity value than the global window. Therefore the surrogate model at each intermediate design point is generated during optimization as shown in Fig. 1 where g_1 and g_2 represent constraint functions to be realized including an objective function. The window size, R_i, can be decided as:

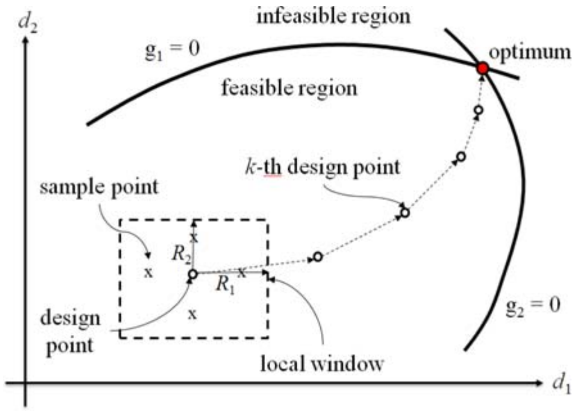

Fig. 1. (Color online) Illustration of hyper-cubic local window and sensitivity-based searching technique.

$$R_i = c(d_i^U - d_i^L) \quad i{=}1,2,\cdots nd \tag{7}$$

where c is the coefficient which is usually between 2-5%, d_i is the ith design variable in the nd-dimensional space and the superscripts, U and L, are the upper and lower bounds, respectively.

3.2. Sampling

After deciding the local window, evenly distributed N_r initial samples are generated on the window based on the Latin Centroidal Voronoi Tssellations (LCVT) [7, 8], and then the surrogate model is produced. The minimum number of the initial samples is given by

$$C_{n+P}^{P} \le N_r \tag{8}$$

where P is the highest order of the basis functions used in the model. The accuracy of the surrogate model generated with the initial samples is estimated by

$$\eta = \frac{mean(M\hat{S}E(\mathbf{x}_i))}{Var(y(\mathbf{x}_j))} \quad \text{for } i{=}1{\sim}S, j{=}1{\sim}Nr$$

$$M\hat{S}E(\mathbf{x}_i) = \frac{1}{S}\sum_{i=1}^{S}\left(\frac{\hat{y}(\mathbf{x}_i) - y(\mathbf{x}_i)}{y(\mathbf{x}_i)}\right)^2 \tag{9}$$

where $Var(y(\mathbf{x}_j))$ is the variance of N_r true responses at the samples, S is the total number of testing points generated using LCVT, and $M\hat{S}E(\mathbf{x}_i)$ is the predicted mean square error. If the accuracy of a surrogate model is not satisfactory ($h \le 1\%$), more samples are sequentially inserted within the local window until the surrogate model satisfies the target accuracy condition.

3.3. Sample Reuse

During the design iteration, the local window is always scanned to check whether samples exist before generating the initial samples. This case occurs when the current and the previous local windows overlap each other partially. In the case, existing samples belonging to both the windows are included in the initial samples, and so the less number of samples is generated in the current window. Especially around the optimum point, the number of inserted samples is much less than the number of existing samples. That can result in significant save of the computation time for executing the sampling-based design optimization.

3.4. Program Architecture

Utilizing the sensitivity values (6) of a current design, which are extracted from the locally accurate surrogate models corresponding to the objective and constraint functions, a next improved design is sought with the sequential quadratic programming (SQP) algorithm. The program flowchart of the proposed sampling-based design optimization method using the sensitivity information is shown in Fig. 2. The program consists of two parts: the main program was realized with Matlab, and only the part of computer simulations at samples marked with the dotted box in Fig. 2 was executed externally. It means the two parts are totally separated and the information on the sampling points and their simulation results is communicated with each other only when necessary.

4. Design Examples

To prove the validity of the proposed method, two

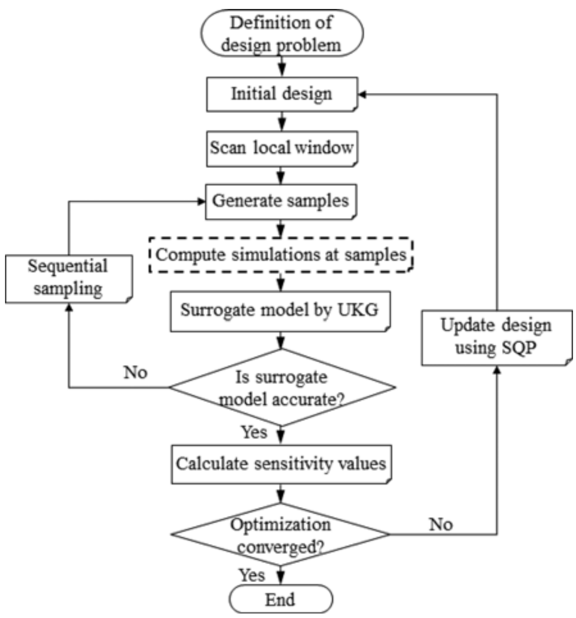

Fig. 2. Flowchart of the proposed sampling-based optimization method.

numerical examples are tested: The first is a two-dimensional (2-D) mathematical example, and the second is a 12-D loudspeaker design problem with respect to the design space. The two problems were solved using two different optimization methods: the first is a sensitivity-based method adopting FDM and SQP searching algorithm provided from Matlab; and the other is the proposed sampling-based method with the first-order basis function and the same SQP.

4.1. 2-D Mathematical problem

Let's consider a 2-D deterministic optimization problem of which the mathematical formulation is given as

$$\begin{aligned} &\min.\ f(\mathbf{d})=-(d_1+d_2-10)^2/30-(d_1-d_2+10)^2/120 \\ &\text{subject to} \\ &\quad g_1(\mathbf{d})=1-d_1^2 d_2/5 \qquad (10) \\ &\quad g_2(\mathbf{d})=1-(d_1+d_2-5)^2/30-(d_1-d_2-10)^2/120 \\ &\quad g_3(\mathbf{d})=1-80/(d_1^2+8d_2+5). \end{aligned}$$

The constraint functions and the initial design are illustrated in Fig. 3 where contour lines belong to the objective function and DDO denotes the deterministic design optimum to be sought. Starting with 5 initial samples, elaborate surrogate models were generated at each intermediate design based on the UKG and the hyper-cubic local window for the four true functions (i.e. one objective and three constraints), respectively. Total 99 new samples were sequentially inserted until reaching the DDO point. Fig. 3 includes the traces of intermediate designs marked with asterisks, and the enlargement design space is shown in Fig. 4 where samples are marked with empty circles. As shown in Fig. 4, the sample reuse process is mainly activated around the optimum, and accordingly total 57 samples were reused for constructing surrogate models during the whole optimization process.

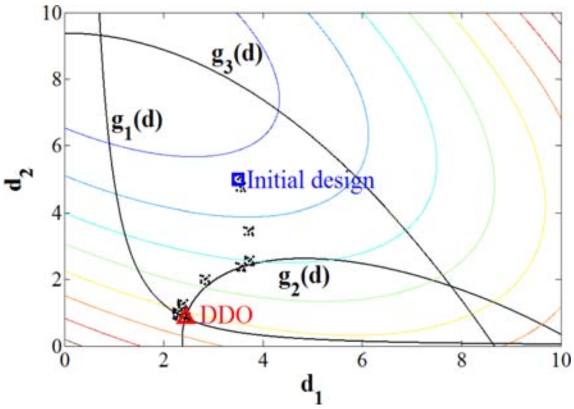

Fig. 3. (Color online) Shapes of objective/constraint functions and traces of design points.

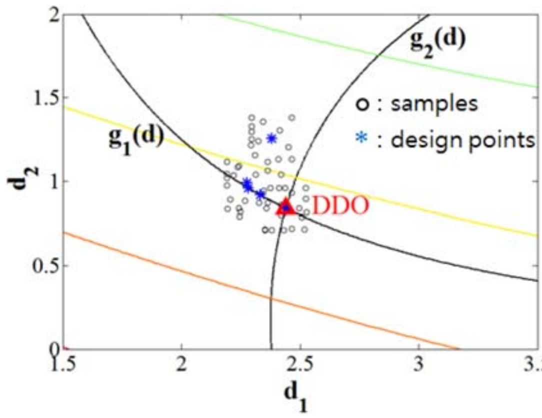

Fig. 4. (Color online) Sampling and design points around the optimum.

The performances between the two different optimization methods are compared with each other in Table 1. Both the methods searched for the exact optimum after same 12 design iterations but the proposed method required more than two times the number of true function simulations of the FDM-based method. The difference of the simulation numbers comes from the different ways of numerically calculating the sensitivity values. It means while FDM uses the one-dimensional scheme, the proposed method utilizes the multi-dimensional surface for extracting the values. Hereby, it can be inferred that the sampling-based sensitivity computation will be comparable to the FDM-based one as design variables increase.

4.2. 12-D Loudspeaker design

The loudspeaker design problem in [9-11] is selected to show the applicability of the proposed method to EM device designs. Fig. 2 shows the configuration of the loudspeaker with 12 design variables. The objective function f is defined to minimize the loudspeaker mass M, and the constraint function g is set to keep the average flux density of the air gap being more than B_0=1.8 T, as

Table 1. Performance indicators between two different methods for a mathematical example.

Design variables	d^L	Initial design	Optimum (FDM-based)	Optimum (sampling-based)	d^U
d_1	0	3.5	2.44	2.44	10
d_2	0	5.0	0.84	0.84	10
f(d)	–	–0.677	–2.627	–2.627	–
Iterations	–	–	12	12	–
no. simulations	–	–	39	104	–

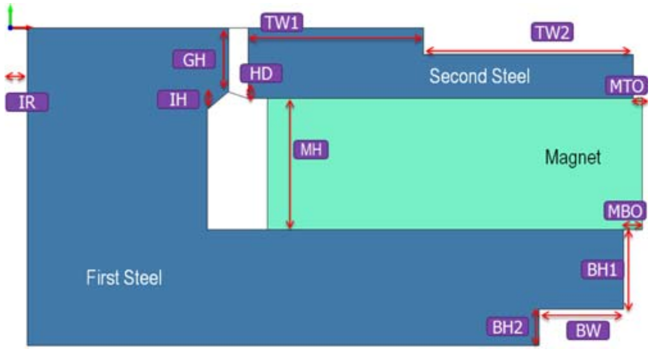

Fig. 5. (Color online) Two-dimensional axisymmetric configuration of a loudspeaker.

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{d})=\mathrm{M}(\mathbf{d}) \\ &\text{subject to} \;\; g(\mathbf{d})=\mathrm{B}_0-\mathrm{B}(\mathbf{d}) \geq 0 \end{aligned} \tag{11}$$

where d is the design variable vector, and mass density values of 7,390 kg/m^3 and 7,600 kg/m^3 are used for the mass calculation of permanent magnet and steel, respectively. To take into account the nonlinear property of the steel yoke, the estimation of the objective and constraint functions requires executing an EM simulator. Here, a commercial simulator, called MagNet VII based on the finite element method [12], was used and was easily incorporated with the proposed optimization method.

For generating the surrogate models of the two performance functions, the hyper-cubic local window started with 25 initial samples, and 751 sequential samples were used to accomplish the optimization. The comparison between the two optimization methods is presented in Table 2 where the two optimum points are quite different with each other. It implies that, due to relatively large number of design variables, the problem itself has local minima near the constraint boundary. It is also observed that the total iteration number of the proposed method is small by 30% of that of the FDM-based one even though both methods use the same SQP searching algorithm and initial design. Moreover, the method yields the best objective function value (i.e. the smallest mass of the loudspeaker) among the three different designs while satisfying the constraint condition. It is inferred that the proposed sampling-based method produces more accurate sensitivity values than the FDM-based method. In terms of the simulation number, the proposed method still requires more EM simulations by about 40% than the FDM-based one but the relative difference ratio of the simulation number of the 12-D design problem to the FDM-based one is reduced remarkably by nearly 230% when compared with the 2-D problem. In Fig. 6, the two optimized loudspeaker designs are compared with each other, of which the mass reduces by more than 2.4 kg to the initial one.

Table 2. Performance indicators between two different methods for a loudspeaker design problem.

Design variables	d^L	Initial design	Optimum (FDM-based)	Optimum (sampling-based)	d^U
BH1 (mm)	2.03	9.63	7.47	5.18	12.70
BH2 (mm)	2.03	6.15	7.67	5.03	12.70
BW (mm)	3.81	12.12	12.70	11.96	12.7
HD (mm)	0	2.64	0.10	0.48	5.08
IH (mm)	1.02	4.80	1.93	1.60	7.62
IR (mm)	1.02	4.97	3.96	4.04	8.13
MBO (mm)	0.76	1.50	4.06	3.53	4.06
MH (mm)	3.05	15.75	12.88	9.58	17.78
MTO (mm)	0.76	2.97	3.18	0.79	5.08
TH (mm)	2.03	3.10	2.08	4.01	4.57
TW1 (mm)	4.83	22.12	11.02	13.90	22.86
TW2 (mm)	4.83	19.28	8.69	11.38	22.86
M(d) (kg)	–	3.57	1.16	1.02	–
B(**d**) (T)	–	1.73	1.80	1.80	–
Iterations	–	–	24	17	–
no. simulations	–	–	549	776	–

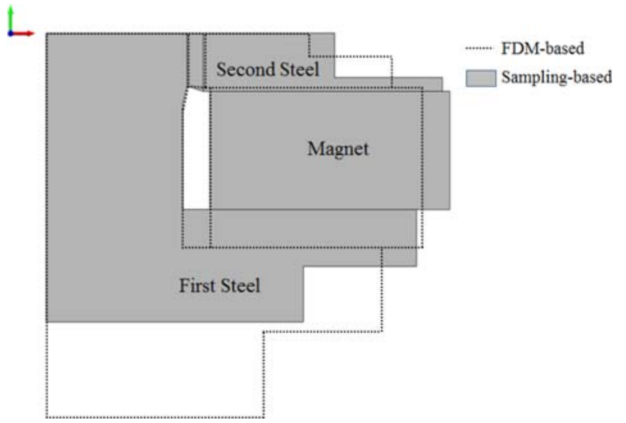

Fig. 6. (Color online) Comparison of two different optimized loudspeaker designs.

5. Conclusion

In this paper, the sampling-based optimization method, where design sensitivities are obtained from the elaborate surrogate models based on the hyper-cubic local window, is proposed. The method has been successfully applied to two design examples, and their results show the proposed method is comparable to the FDM-based optimization method in terms of accuracy and efficiency. The method will be very useful for dealing with EM design problems especially with high dimensional design variables.

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