Journal of Magnetics 17(4), 291-297 (2012)

ISSN (Print) 1226-1750
ISSN (Online) 2233-6656
http://dx.doi.org/10.4283/JMAG.2012.17.4.291

Efficient Methodology for Reliability Assessment of Electromagnetic Devices Utilizing Accurate Surrogate Models Based on Dynamic Kriging Method

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(Received 1 November 2012, Received in final form 17 December 2012, Accepted 17 December 2012)

This paper presents an efficient methodology for accurate reliability assessment of electromagnetic devices. To achieve the goal, elaborate surrogated models to approximate constraint functions of interest are generated based on the dynamic Kriging method and a hypercube local window. Then, the Monte Carlo simulation scheme is applied to the surrogate models. This leads to reducing computational cost dramatically without degrading accuracy of the reliability analysis. The validity of the proposed method is tested and examined with a mathematical example and a loudspeaker design.

Keywords : dynamic Kriging, electromagnetic fields, Monte Carlo simulation, reliability, robustness.

1. Introduction

For last few decades, computer-aided simulation tools such as finite element method, boundary element method, finite difference method, etc. have been widely used in the field of electromagnetic (EM) appliances because they can lead to saving development cost and time for either new or improved EM devices. However there is still a consistent demand for more efficient numerical modeling techniques when engineers often encounter time-consuming design problems, such as deterministic or probabilistic design optimization, which require a number of numerical simulations [1]. Recently, as one of the promising techniques which have a low computational cost with a good accuracy of numerical solutions, the Kriging-based surrogate modeling has drawn EM engineers' attention [2-5].

The Kriging method was first developed in geostatistics and has been successfully applied to other engineering such as mechanics, chemistry and so on [2-7]. Generally, the method consists of two parts which approximate a response function of interest: the mean structure and a zero-mean stationary Gaussian stochastic process. The ordinary Kriging method (OKG) assumes that the mean structure is zero or constant, and the universal Kriging method (UKG) constructs the mean structure using the first/second-order polynomials. As to EM application, the first attempt was made to optimize the TEAM workshop problem 25 with the use of the Kriging models for objective function [2]. Since then, several articles have been published so far. J.D. Lavers, *et al*. in [5] compared three sequential methods of least square, Kriging and linear Bayesian using a design problem. K. R. Davey in [3] introduced the Latin hypercube sampling method combined with pattern search algorithm. K. Hameyer, *et al*. in [4] adopted the Kriging models in conjunction with an evolution strategy to reduce the torque ripple of a switched reluctance motor. Most of previous research works used OKG or UKG to generate surrogate models and obtained deterministic optimum designs utilizing the models combined with stochastic methods such as evolution strategy, genetic algorithm, simulated annealing, etc.

The motivation of this work comes from following two questions on the previous research. The performance of EM devices normally has a highly nonlinear and implicit function with respect to design variables. On the other hand, the probabilistic optimization problems generally require much more expensive computational cost than the deterministic ones. From these facts, the first question is as follows: Is ORG or UKG most suitable for approximating such EM performance functions? The second is as: Is there any adequate problem which shows a maximum profit of the Kriging-based surrogate modeling?

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As an effort to seek an answer to the above questions, this paper presents an efficient methodology for reliability assessment of EM devices which adopts Monte Carlo simulation (MCS) combined with dynamic Kriging-based surrogate models. To obtain an elaborate surrogate model, the dynamic Kriging (DKG) method in conjunction with a hypercube local window is introduced. In the terms of the accuracy of DKG, it has already been proven by Choi, *et al*. in [6, 7] where DKG generates more accurate surrogate models for highly nonlinear functions compared with the existing Kriging methods. Meanwhile, for the probabilistic optimization, MCS is an indispensable tool because its result is considered as a reference when executing approximated reliability analysis methods such as first/second-order reliability analysis or dimension reduction method. However, for reliable results of MCS, several hundreds of thousands of numerical solutions are usually needed. That causes a very significant computational burden to engineers. To overcome this problem, the original MCS scheme is successfully incorporated with the DKG models. The method leads to reducing computational cost dramatically without degrading accuracy of the reliability analysis. The validity of the proposed method is tested and examined with a mathematical example and a loudspeaker design.

2. Dynamic Kriging Method

When compared to the traditional Kriging methods such as OKG and UKG, DKG yields more accurate surrogate models whenusing the genetic algorithm (GA) for the best basis function set and using the pattern search for optimum correlation parameters [6, 7]. In this section, the basic concepts of DKG are summarized briefly.

2.1. Formulation

In the Kriging method, the outcomes are considered as a realization of a stochastic process. The goal is to estimate a response $\mathbf{y} = [y(\mathbf{x}_1), y(\mathbf{x}_2), \cdots, y(\mathbf{x}_n)]^T$ with $y(\mathbf{x}_i) \in \mathbf{R}^1$ based on n sample points, $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n]^T$ with $\mathbf{x}_i \in \mathbf{R}^m$. The response consists of a summation of two parts as

$$\begin{aligned} &\mathbf{y} = \mathbf{F}\boldsymbol{\beta} + \mathbf{e} \\ &\text{where } \mathbf{F} = [f_k(x_i)] \quad i = 1, \cdots n, k = 1, \cdots K \\ &\boldsymbol{\beta} = [\beta_1, \cdots, \beta_k], \quad \mathbf{e} = [e(\mathbf{x}_1), \cdots, e(\mathbf{x}_n)]^T. \end{aligned} \tag{1}$$

The first term of the right side of (1), called the mean structure of the response, is intended to follow the general tendency of the function to be modeled. It is generally composed of user-defined basis functions $f_k(x_i)$ and the vector of regression coefficient $\boldsymbol{\beta}$. The second term $\mathbf{e}$ is a realization of the stochastic process. It is assumed to have zero mean $E[\mathrm{e}(\mathbf{x}_i)] = 0$ and covariance structure $E[\mathrm{e}(\mathbf{x}_i)\mathrm{e}(\mathbf{x}_j)] = \sigma^2 R(\theta, \mathbf{x}_i, \mathbf{x}_j)$, where σ^2 is the process variance, θ is the correlation parameter vector estimated by applying the maximum likelihood estimator (MLE) and R is the correlation function of the stochastic process. The term $\mathbf{e}$ makes it possible to follow the fluctuations around the general tendency. In most engineering applications, the correlation function is set to be a Gaussian form expressed as

$$R(\boldsymbol{\theta}, \mathbf{x}_i, \mathbf{x}_j) = \prod_{l=1}^{nr} \exp\left(-\theta_l (x_{i,l} - x_{j,l})^2\right) \tag{2}$$

where $x_{i,l}$ is the lth component of variable $\mathbf{x}_i$.

Under the decomposition of (1), and the optimal θ to maximize MLE, the noise-free unbiased response $\hat{y}$ at a new point of interest denoted by $\mathbf{x}_0$ is written as a linear predictor

$$\hat{y}(\mathbf{x}_0) = \mathbf{w}_0^T \mathbf{y} \tag{3}$$

where $\mathbf{w}_0 = [w_1(\mathbf{x}_0), w_2(\mathbf{x}_0), \cdots, w_n(\mathbf{x}_0)]^T$ means the $n \times 1$ weight vector for prediction at the point. It is obtained using the unbiased condition $E[\hat{y}(\mathbf{x}_0)] = E[y(\mathbf{x}_0)]$ as

$$\mathbf{w}_0 = \mathbf{R}^{-1}\left(\mathbf{r}_0 + \frac{1}{2\sigma^2} F\boldsymbol{\lambda}\right) \tag{4}$$

where $\mathbf{R}$ is the symmetric correlation matrix with the ijth component $R_{i,j} = \mathrm{R}(\theta, \mathbf{x}_i, \mathbf{x}_j)$ and $\mathbf{r}_0 = [\mathrm{R}(\theta, \mathbf{x}_1, \mathbf{x}_0), \cdots, \mathrm{R}(\theta, \mathbf{x}_n, \mathbf{x}_0)]^T$ is the correlation vector between $\mathbf{x}_0$ and samples $\mathbf{x}$ and λ is the Lagrange multiplier. After substituting (4) into (3), the prediction of Kriging model which interpolates the n sample points is expressed as

$$\begin{aligned} \hat{y}(\mathbf{x}_0) &= \mathbf{w}_0^T \mathbf{y} = \left(\mathbf{r}_0 + \frac{1}{2\sigma^2}\mathbf{F}\lambda\right)^T \mathbf{R}^{-1}\mathbf{y} \\ &= \mathbf{f}_0^T \boldsymbol{\beta} + \mathbf{r}_0^T \mathbf{R}^{-1}(\mathbf{y} - \mathbf{F}\boldsymbol{\beta}) \end{aligned} \tag{5}$$

where $\sigma^2 = 1/\mathrm{n}(\mathbf{y}\text{-}\mathbf{F}\beta)\mathbf{R}^{-1}(\mathbf{y}\text{-}\mathbf{F}\beta)$ and $\beta = (\mathbf{F}^T\mathbf{R}^{-1}\mathbf{F})^{-1}(\mathbf{F}^T\mathbf{R}^{-1}\mathbf{y})$ are obtained from the generalized least square regression.

Under the assumption of the Gaussian process, the 1-α lever prediction interval of the response is given by

$$\hat{y}(\mathbf{x}_0) - Z_{1-\alpha/2}\sigma_p(\mathbf{x}_0) \le y(\mathbf{x}_0) \le \hat{y}(\mathbf{x}_0) + Z_{1-\alpha/2}\sigma_p(\mathbf{x}_0) \tag{6}$$

where $Z_{1\text{-}\alpha/2}$ is the 1-α quantile of the standard normal distribution and $\sigma_p^2 = \sigma^2(1 + \mathbf{w}_0^T\mathbf{R}\mathbf{w}_0 - 2\mathbf{w}_0^T\mathbf{r}_0)$. Therefore, the bandwidth of the prediction interval at the point $\mathbf{x}_0$ is

$$d(\mathbf{x}_0) = 2Z_{1-\alpha/2}\sigma_p(\mathbf{x}_0). \tag{7}$$

This prediction interval is used as an accuracy measure to decide if the surrogate model is accurate or not.

2.2. Best basis-function set using genetic algorithm

The basis function $f_k(x_i)$ in (1) do not change during the surrogate model generation process for the traditional Kriging methods where the polynomial order are fixed in advance of the model generation. However, it is obvious that the fixed-order basis functions may not be suitable to describe the nonlinearity of the mean structure for highly nonlinear problems. In some cases, it is also pointed out that the accuracy of the surrogate model may not improve by using higher-order terms.

Contrast to the traditional methods, DKG adopting GA selects the optimal basis function set at a prediction point in order that the generated surrogate model has the best accuracy. It means the best combination up to the highest-order basis function prescribed is decided by GA which screens all the foreseeable basis function sets under certain convergence criteria. With the highest-order P satisfying (8), (5) can be solved when the total number of basis functions is less than sampling points n,

$$C^{P}_{nd+P} \le n \tag{8}$$

where nd is the dimension of design variables engaged in the surrogate model. After deciding P, GA is applied to find the best basis function set efficiently.

2.3. Optimum correlation parameter using pattern search

DKG also utilizes the pattern search algorithm to find the optimal correlation parameter θ in (2) based on MLE. The MLE maximization problem for θ is written by

$$\text{find } \theta$$
$$\min\ \zeta(\theta) = \frac{1}{2}\ln(|\mathbf{R}|) + \frac{n}{2}\ln(\sigma^2) \tag{9}$$

where ζ(θ) is equivalent to the maximum likelihood estimator. Since it is not a gradient-based optimization method, the pattern search algorithm is powerful enough to find the optimum which satisfies (9).

3. Reliability Assessment Using MCS Based on DKG Surrogate Model

To efficiently carry out MCS for accurate reliability assessment, the DKG method has to be combined with MCS successfully. In this section, the DKG-based surrogate model using a hypercube local window in the normalized standard design space is first explained and then the flowchart of the proposed MCS based on the surrogate model is presented.

3.1. Hypercube local window for surrogate model generation

Since the reliability analysis is carried out with prescribed variations of random design variables, a local window is more preferable to generate surrogate models than a global window that covers entire design space. The hypercube local window concept for generating surrogate models is illustrated in Fig. 1 where g represents a constraint function to be realized and the failure surface (limit state function) distinguishes between feasible and infeasible regions. The window size R in the standard normal U-space where all random variables have the standard normal distribution can be decided as

$$R = c_R\beta_t \tag{10}$$

where c_R is the coefficient, which is usually between 1.0 and 2.0, and β_t is the target reliability index defined by designers. Hence, sample points are first generated in the U-space and then the points are transformed into the X-space of actual design domain. It means the responses corresponding to the sample points are calculated in the X-space.

After deciding the local window, evenly distributed N_r initial samples are generated on the window based on the Latin Centroidal Voronoi Tssellations (LCVT) [6, 7] and then the surrogate model is produced. The accuracy of the surrogate model generated with the initial samples is estimated by

$$\eta = \frac{mean(M\hat{S}E(\mathbf{x}_i))}{Var(y(\mathbf{x}_i))} \quad \text{for } i = 1\sim S,\ j = 1\sim n \tag{11}$$

where $Var(y(\mathbf{x}_j))$ is the variance of n true responses at the sample points and S is the total number of testing points generated using LCVT. The predicted mean square error, $M\hat{S}E$ from the Kriging model in (1) is written as

$$M\hat{S}E(\mathbf{x}_i) = \left(\frac{d(\mathbf{x}_i)}{2Z_{1-\alpha/2}}\right)^2. \tag{12}$$

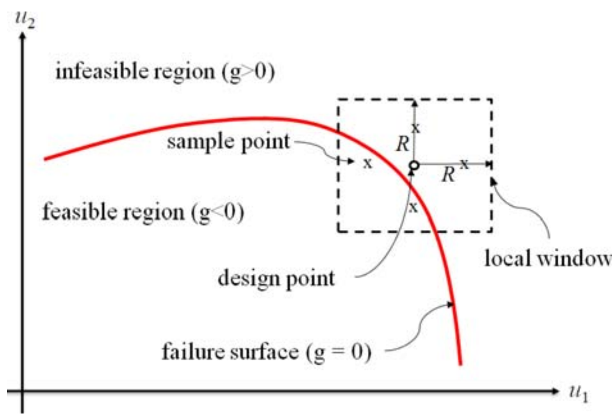

Fig. 1. (Color online) Hypercube local window concept: g is the constraint function.

If the accuracy of a surrogate model is satisfactory ($\eta \leq$ 1%), the model is used for reliability assessment using MCS. Otherwise, more samples are sequentially inserted within the local window until the surrogate model satisfies the target accuracy condition.

3.2. Proposed MCS based on surrogate model

The original MCS is the most widely used method to estimate the reliability indices of engineering application. The underlying principle of MCS is to sample random design points based on the probability and both success (belongs to feasible region in Fig. 1) and failure (belongs to infeasible region in Fig. 1) states contribute to the reliability assessment as shown in Fig. 2(a) where g($\mathbf{x}_i$) is the constraint function of interest and n_f is the number of failure states. The main advantage of MCS lies on the simplicity in numerical implementation but it often requires expensive computational cost depending on the simulation time for a given design problem, g($\mathbf{x}_i$), once. Some research in the field of power system, which utilizes the genetic algorithm, particle swarm optimization or artificial immune system [8, 9], has been carried out to render the MCS computationally more efficient. However there is still a consistent need for computationally efficient and more accurate methods for reliability assessment of EM device designs.

The proposed MCS is combined with the accurate surrogate model, $\hat{g}(\mathbf{x}_i)$, based on the DKG method and the hypercube local window, to decide success/failure sates of the function at random samples. It can lead to dramatically reducing computational cost without degrading accuracy of reliability values. The proposed program architecture shown in Fig. 2(b) follows as:

1) Input the number N_r of initial sampling points, random variables and window size (R = 1.2 is used),

2) Executing computer simulation at given samples,

3) Generate a surrogate model based on DKG,

4) If the surrogate model satisfies the specified accuracy ($\eta \leq 1\%$), go to next step. Otherwise, insert sequential sample points and then go to 2),

5) Carry out the original MCS in Fig. 2(a) by using $\hat{g}(\mathbf{x}_i)$,

6) Evaluate the probability of failure according to $P_f = \frac{1}{N_{total}} \sum_{i=1}^{N_{total}} I_{\Omega_f}(\mathbf{x}_i)$ where the failure set is defined by $I_{\Omega_f} \equiv \{\mathbf{x} : g(\mathbf{x}_i) > 0\}$.

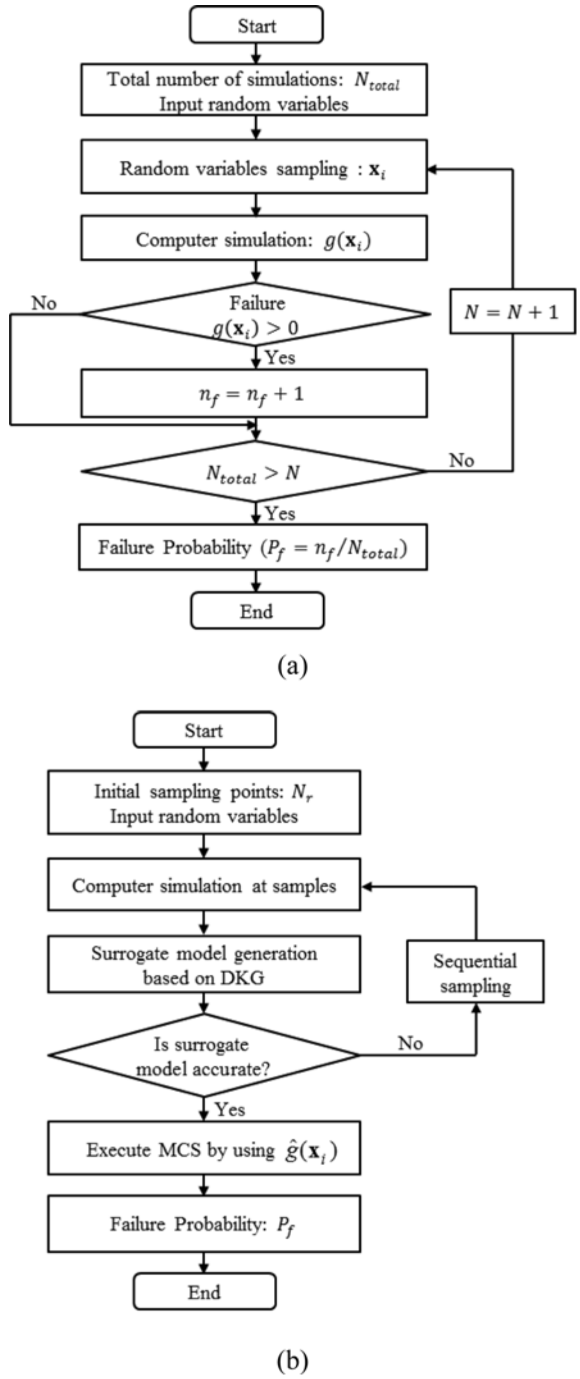

Fig. 2. Flowchart of the proposed reliability assessment: (a) original MCS, (b) proposed MCS based on DKG surrogate model.

4. Numerical Examples

To verify the efficiency and accuracy of the proposed method, two numerical examples are tested: The first is a two-dimensional (2-D) mathematical example and the second is a 4-D loudspeaker design problem with respect to the random design variable space. The failure probabilities (i.e. reliabilities) of constraint functions for the examples are calculated at three nominal design points which has been obtained from an initial design, deterministic design optimization (DDO) and reliability-based design optimization (RBDO) in the previous work [10]. Then the estimated probability values are compared with those of the original MCS.

4.1. 2-D Mathematical problem

Consider a 2-D probabilistic optimization problem of which the mathematical formulation is given as

$$\text{minimize } f(\mathbf{d}) = -\frac{(d_1 + d_2 + 10)^2}{30} - \frac{(d_1 - d_2 + 10)^2}{120} \quad (13)$$

$$\text{subject to } P(g_j(\mathbf{X}(\mathbf{d}) > 0) \le P_{f_j}^{tar} = 2.275\%, j = 1, 2, 3$$

where $f(\mathbf{d})$ is the objective function, g_j is the constraint function, $\mathbf{d}$ is the design variable vector given by $\mathbf{d} = \mu(\mathbf{X})$, and μ denotes the mean value vector of the random vector $\mathbf{X}$, respectively. The target failure probability value in (13) is corresponding to $\beta_t = 2$. The three constraint functions, which include high nonlinear terms with respect to the variables, are expressed by

$$g_1(\mathbf{X}) = 1 - \frac{X_1^2 X_2}{5}$$

$$g_2(\mathbf{X}) = 1 - \frac{(X_1 + X_2 - 5)^2}{30} - \frac{(X_1 - X_2 - 12)^2}{120} \quad (14)$$

$$g_3(\mathbf{X}) = 1 - \frac{80}{X_1^2 + 8X_2 + 5}.$$

The properties of two random variables and three nominal design points are presented in Table 1.

In this example, only the accuracy of the proposed method is compared with that of the original MCS because the constraint functions are given analytically as in (14). The constraint functions and nominal design points, denoted by initial design, DDO and RBDO, are illustrated in Fig. 3 where contour lines belong to the objective function. The failure probability values of the constraint functions were calculated at each design point by using the proposed method. For each constraint function, an elaborate surrogate model was generated based on the DKG and the local window. In each model, 5 initial samples were used in the local window and then new samples were sequentially inserted until the prescribed accuracy of the model was achieved. After the DKG-based surrogate model for each constraint was obtained with total 15 samples, each failure probability was computed by the MCS algorithm using not the true functions of (14) but their surrogate models. For reliable results on the probability of failure, 500,000 of sample states (i.e. success or failure) were checked in both cases of the proposed and the original MCS methods. The failure probability values of the three constraint functions are presented in Table 2 where the smaller the value is, the more robust the current design is to the variances of design variables. In the case of the proposed method, the true functions were used only 15 times and the surrogate models were used for the rest. The results show that the proposed method yields accurate probability values of which the maximum error is less than 1.12% on the basis of the original MCS values.

Table 1. Properties of random variables and three nominal design points.

Random variables	Initial design	DDO	RBDO	Distribution	Standard deviation
X_1	3.500	2.440	2.251	Normal	0.30
X_2	5.000	0.840	1.970	Normal	0.30
f(X)	−0.677	−2.627	−1.995	–	–

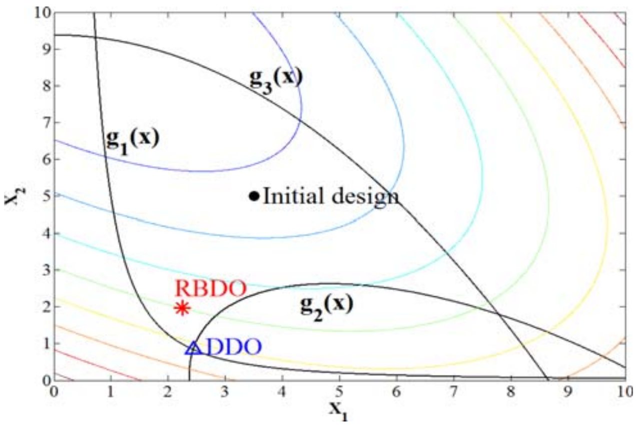

Fig. 3. (Color online) Shapes of constraint functions.

Table 2. Failure probability values at the initial, DDO and RBDO design points.

Random variables	Method	Initial design	DDO	RBDO
$P_f(g_1)$ (%)	MCS	0	52.84	2.57
	Proposed	0	52.75	2.60
$P_f(g_2)$ (%)	MCS	0	47.80	1.95
	Proposed	0	47.87	1.97
$P_f(g_3)$ (%)	MCS	0	0	0
	Proposed	0	0	0
Function calls	MCS	500,000	500,000	500,000
	Proposed	15	15	15

4.2. 4-D Loudspeaker design

The loudspeaker design problem subjected to variations of design variables in [11] is used to show the efficiency

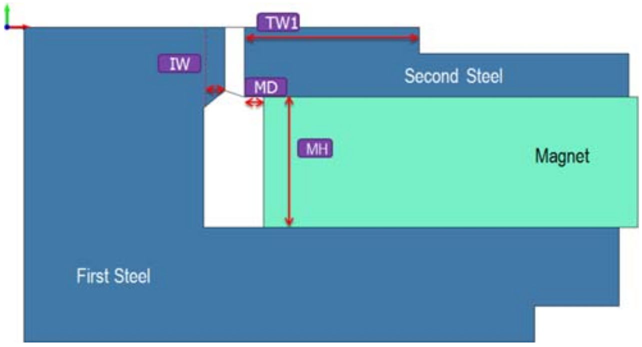

Fig. 4. (Color online) Axisymmetric configuration of a loudspeaker.

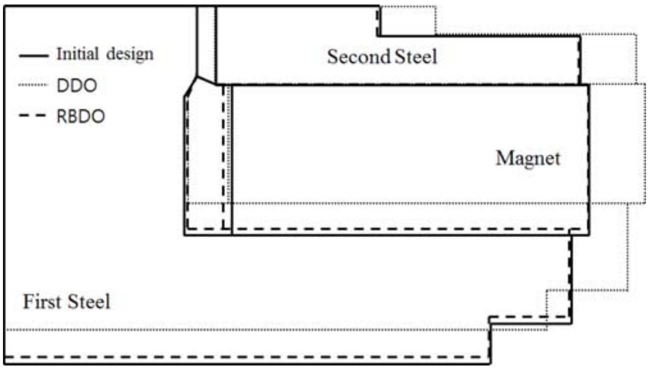

Fig. 5. Comparison of three different loudspeaker designs.

of the proposed method. For the simplicity, four design variables shown in Fig. 4 were selected as random variables. The objective function is to keep an average flux density B_0 of 1.8 T at the air gap. The loudspeaker mass with a tolerance of 5% with respect to the specified value M_0 of 7.5 kg is set as a constraint condition. The mathematical formulation of the design problem is written by

$$\begin{aligned} &\text{minimize } f(\mathbf{d}) = |B_0 - B(\mathbf{X})| \\ &\text{subject to } P(g_j(\mathbf{X}(\mathbf{d}) > 0) \leq P_f^{tar} = 5\% \\ &g(\mathbf{X}) = 1 - \left(\frac{M(X) - M_0}{0.05 \times M_0}\right)^2 \end{aligned} \tag{15}$$

where B(**X**) and M(**X**) are the average flux density of the air gap and the mass of the device at a current design point, respectively. The target failure probability value 5% (i.e. reliability of 95%) in (15) is corresponding to $\beta_t = 1.645$. The properties of variables and three nominal design points are presented in Table 3 which includes their performance values. In Fig. 5, the two optimized loudspeaker designs (DDO and RBDO designs) are compared with the respect to the initial one.

In this problem, the estimation of the objective and constraint functions requires executing an EM simulator in order to take into account the nonlinear property of the steel yoke. Here, a commercial simulator, called MagNet VII based on the finite element method [12], and a desktop computer equipped with an Intel Core i7 CPU of 3.2 GHz were used. For the surrogate models of the constraint, 9 initial samples were used in the local window and the new samples, 45, 25 and 30, were sequentially inserted at the initial, DDO and RBDO designs, respectively until the prescribed accuracy of the model was achieved. It implies that even though the same function is realized by the DKG method, the best surrogate model strongly depends on the local position of the design point. For the probability of failure of the constraint, different numbers of sample states were checked in the cases of the proposed and the original MCS methods because the original MCS took quite a lot of EM simulation time: 500,000 for the proposed MCS and 10,000 for the original MCS. For the reliability analysis of the given constraint, the failure probability value, EM simulation number and computation time are presented in Table 4 where, in case of the initial design, the proposed method carried out only 54 EM simulations and then the surrogate model was used for the MCS algorithm. The results show that the failure probability values between the proposed and the original MCS methods at the three nominal designs show a good agreement with each other. However, in terms of the com-

Table 3. Properties of random variables and three nominal design points.

Random variables	Unit	Initial design	DDO	RBDO	Distribution	Standard deviation
IW	mm	1.14	0.76	0.82	Normal	0.05
MD	mm	1.42	1.16	0.67	Normal	0.08
MH	mm	10.87	8.31	10.3	Normal	0.25
TW1	mm	14.86	20.00	14.69	Normal	0.38
B	T	1.76	1.80	1.80	–	–
Mass	kg	7.85	7.84	7.55	–	–

Table 4. Performance indicators at the initial, DDO and RBDO design points.

Random variables	Method	Initial design	DDO	RBDO
P_f(%)	MCS	44.45	43.79	6.54
	Proposed	44.62	43.76	6.13
Function calls	MCS	10,000	10,000	10,000
	Proposed	54	34	39
Computation Time (sec.)	MCS	42,298	42,005	41,911
	Proposed	487	355	379

putation time, the proposed method (about 5.9 minutes for the DDO point) was much faster by nearly 120 times than the original MCS (about 11.7 hours for the DDO point).

5. Conclusion

For accurate and efficient reliability assessment, this paper combined the original MCS scheme with the elaborate surrogate models based on the DKG method and hypercube local window. The method was successfully applied to the mathematical 2-D problem and the 4-D loudspeaker design. The results show that the proposed method leads to reducing computational cost dramatically without degrading accuracy of reliability values. The method will be very useful to estimate the reliability of EM devices designs in the future.

Acknowledgment

This work was supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2011-013-D00042). This research was also partially supported by the World Class University Program through the National Research Founation of Korea (NRF) grant funded by the Ministry of Education, Science and Technology (Grant Number R32-2008-000-10161-0 in 2009). These supports are greatly appreciated.

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