

# Analytical Study Considering Both Core Loss Resistance and Magnetic Cross Saturation of Interior Permanent Magnet Synchronous Motors

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This paper presents a method for evaluating interior permanent magnet synchronous motor (IPMSM) performance over the entire operation region. Using a d-q axis equivalent circuit model consisting of motor parameters such as the permanent magnetic flux, copper resistance, core loss resistance, and d-q axis inductance, a conventional mathematical model of an IPMSM has been developed. It is well understood that in IPMSMs, magnetic operating conditions cause cross saturation and that the iron loss resistance – upon which core losses depend – changes according to the motor speed; for the sake of convenience, however, d-q axis machine models usually neglect the influence of magnetic cross saturation and assume that the iron loss resistance is constant. This paper proposes an analysis method based on considering a magnetic cross saturation and estimating a core loss resistance that changes with the operating conditions and speed. The proposed method is then verified by means of a comparison between the computed and the experimental results.

**Keywords :** IPMSM, d-q axis equivalent model, magnetic saturation, cross saturation, (iron) core loss resistance

## 1. Introduction

Because of many advantages including robust structure, high efficiency, and high controllability, interior permanent magnet synchronous motors (IPMSMs) are currently used in a wide range of industrial applications [1-3].

As appropriate control of current vectors is necessary at high performance levels, several control methods have been proposed in order to reduce the power loss and to improve the IPMSM performance. Widely used control algorithms include maximum torque-per-ampere (MTPA) and flux weakening, both of which are based on the two-phase equivalent circuit model (d-q axis machine model).

As the key to high performance control lies in the motor parameters of the d-q axis machine model, it is necessary in designing and analyzing a machine to consider non-linearity in the levels of saturation and magnetic field distribution, which vary nonlinearly with the operating conditions [4].

In particular, inductance along the d- and q-axes varies with the respective currents along each axis. In addition, IPMSMs experience a magnetic cross saturation phen-

omenon resulting from current vector control and their core losses are depended on the motor speed [5, 6]. In previous studies [4, 7], an equivalent IPMSM magnetic circuit performance model was developed; however, neither magnetic saturation nor cross saturation effects were taken into account.

This paper is based on recent work to develop an analysis method for correctly determining the current vector in the d-q axis model of an IPMSM in which the cross saturation effect and the core loss are taken into account.

## 2. Modified D-Q Equivalent Circuit Model

Equivalent circuit analysis, which is frequently used to simulate the performance of an IPMSM, is based on the synchronous rotation of the d-q reference frame. The mathematical model of an equivalent circuit follows from a series of voltage equations [1, 7-9]:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R_s + pL_d & -\omega_s L_q \\ \omega_s L_d & R_s + pL_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_s \Psi_a \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (1)$$

$$T = P_n \left\{ \Psi_a I_a \cos \beta + \frac{1}{2} (L_q - L_d) I_a^2 \sin 2\beta \right\} \quad (2)$$

where :

$i_d, i_q$  = d- and q-axis component of armature current;

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- $I_e$  = phase armature current (rms);  $I_a = \sqrt{3} I_e$ ;
- $v_d, v_q$  = d- and q-axis component of terminal voltage;
- $R_s$  = armature winding resistance per phase;
- $\Psi_f$  = flux linkage peak of permanent magnet;
- $\Psi_a = \sqrt{3} \Psi_f$ ;
- $L_d, L_q$  = d- and q-axis armature self-inductance;
- $P_n$  = pole pair;  $p$  = differential operator ( $=d/dt$ );
- $\omega_s$  = angular frequency;
- $\beta$  = current angle;

These can be modified to represent an IPMSM by considering core losses:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \begin{bmatrix} i_{od} \\ i_{oq} \end{bmatrix} + \left(1 + \frac{R_s}{R_c}\right) \begin{bmatrix} v_{od} \\ v_{oq} \end{bmatrix} + p \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \begin{bmatrix} i_{od} \\ i_{oq} \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} v_{od} \\ v_{oq} \end{bmatrix} = \begin{bmatrix} 0 & -\omega_s L_q \\ \omega_s L_d & 0 \end{bmatrix} \begin{bmatrix} i_{od} \\ i_{oq} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_s \Psi_a \end{bmatrix} \quad (4)$$

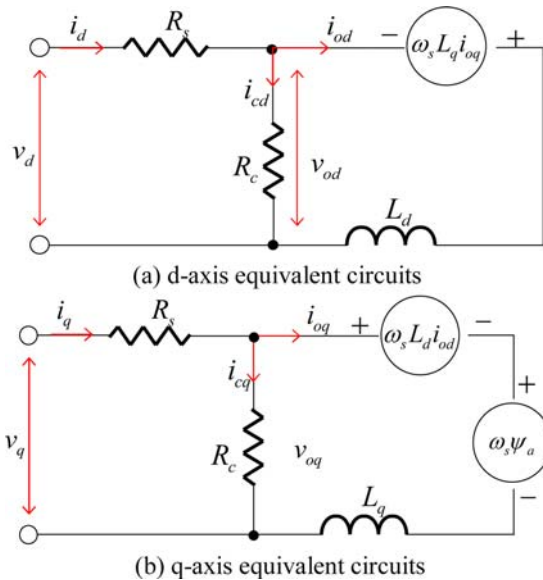


Fig. 1. (Color online) Equivalent circuits of IPMSM.

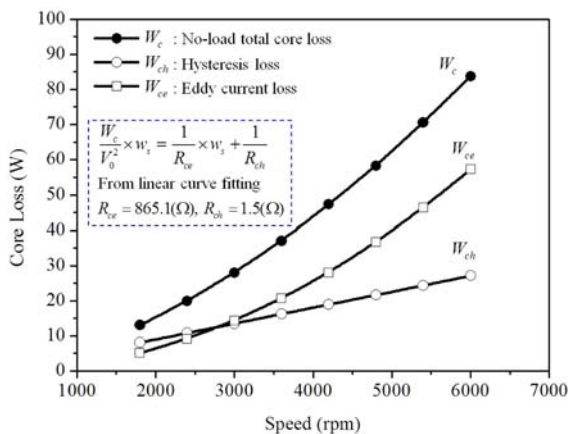


Fig. 2. (Color online) No load core loss and equivalent core loss resistance.

Table 1. Specification of prototype IPMSM.

Parameters	Value	Unit	Parameters	Value	Unit
Phase	3	phase	Back-emf	0.0214	V/rpm
Pole	4	pole	$R_s$	0.25	$\Omega$
Vdc	300	V	$R_{ce}, R_{ch}$	865, 1.5	$\Omega$

$$T = P_n \left\{ \Psi_a i_{od} + (L_d - L_q) i_{od} i_{oq} \right\} \quad (5)$$

where :

- $i_{cd}, i_{cq}$  = d- and q-axis component of core loss current;
- $i_{od}, i_{oq}$  = d- and q-axis component of load current;
- $R_c$  = total core loss resistance;

The core loss resistance can be modeled by means of either a parallel or a series equivalent circuit; in this paper, a parallel connection model is employed, as shown in Fig. 1. To simplify the characteristic analysis, the parallel-connected core loss resistance can generally be treated as a constant that can be extracted using

$$W_c = \frac{V_o^2}{R_c} \quad (6)$$

where :

- $V_o$  = no-load induced voltage;
- $W_c$  = no-load core loss;

However, as shown in Fig. 2, the no-load core loss consists of both the eddy current and the hysteresis losses and thus varies depending on the motor speed. To reflect this fact, the core loss resistance in a parallel equivalent circuit needs to be modeled correctly. In this study, the core loss resistance ( $R_c$ ) is divided into two parallel resistance components: the eddy current resistance ( $R_{ce}$ ) and the hysteresis loss resistance ( $R_{ch}$ ), as shown in Fig. 3. In order to express  $R_c$  as a function of the motor speed, the parallel hysteresis loss resistance component,  $R_{ch}$ , is multiplied by an angular frequency value,  $\omega_s$ . It is well known that the core eddy current loss is proportional to the square of the frequency, whereas the core hysteresis loss is di-

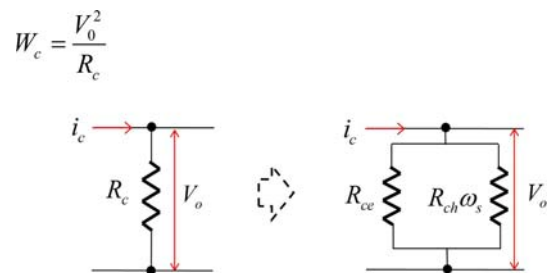


Fig. 3. (Color online) Modified segment of core loss resistance from d-q equivalent circuit.

rectly proportional to the frequency. The relationship between the core loss resistances and the no-load core loss can then be expressed as

$$W_c = \frac{V_0^2}{R_c} = \frac{V_0^2}{R_{ce}} + \frac{V_0^2}{\omega_s R_{ch}} = W_{ce} + W_{ch} \quad (7)$$

Through algebraic manipulation, Equation (7) can be rewritten as

$$\omega_s \frac{1}{R_{ce}} + \frac{1}{R_{ch}} = \omega_s \frac{W_c}{V_0^2} \quad (8)$$

By using the values obtained from the analysis for the no-load induced voltage and the no-load core loss, the unknown coefficients  $R_{ce}$  and  $R_{ch}$  can be approximated by means of a curve fitting method. A no-load core loss calculation for an electric motor is given in detail by other authors [8, 10]. On the basis of the parameters for a prototype IPMSM motor used as a compressor in an air conditioner, which are listed in Table I, the computed values of the core loss, eddy current resistance ( $R_{ce}$ ), and hysteresis loss resistance ( $R_{ch}$ ) are shown in Fig. 2.

### 3. Introduction Evaluation for Magnetic Cross Saturation

If the magnetic cross saturation effects are ignored, the constant values of inductance along the d-q axes, i.e.,  $L_d$  and  $L_q$ , can be calculated using the d- and q-axis currents. However, as the operating performance has a tendency to deteriorate and the control system may become unstable, accurate operating analysis of the IPMSM performance must take into account the cross-saturation effects of these inductances. In this study, the values for  $L_d$  and  $L_q$  are obtained directly from the armature linkage and magnet linkage fluxes, which in turn are obtained through finite element analysis (FEA). On the basis of Fig. 4 [3] and using (9),  $L_d$  and  $L_q$  can be expressed in terms of the armature current unit vectors  $i_d$  and  $i_q$  as

$$L_d = \frac{\psi_o \cos \alpha - \psi_a}{i_d}, L_q = \frac{\psi_o \sin \alpha}{i_q} \quad (9)$$

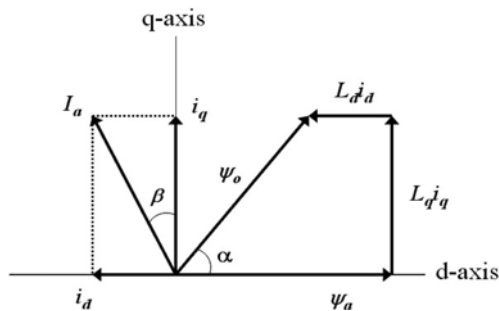


Fig. 4. Phasor diagram of IPMSM.

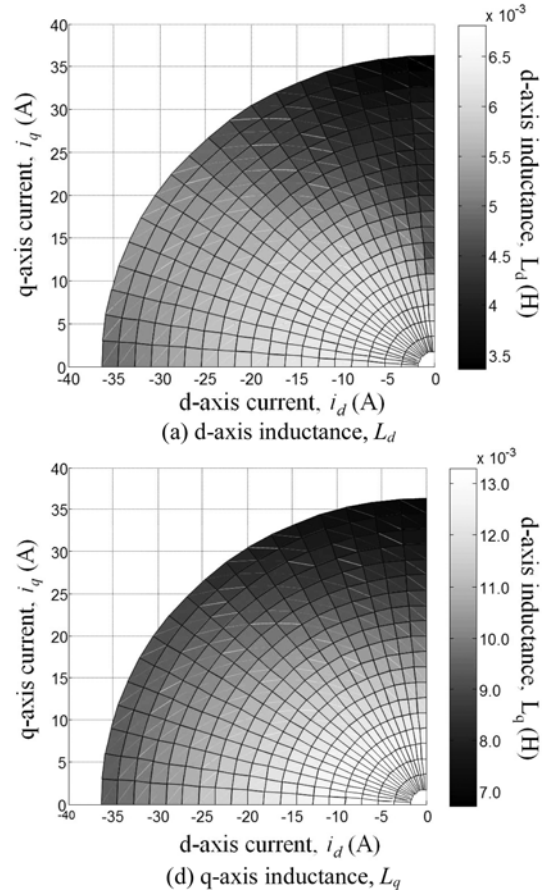


Fig. 5.  $L_d$  and  $L_q$  profile according to the cross saturation effect.

where :  $\psi_o$  = linkage flux of the armature current reaction,  
 $\alpha$  = phase difference between  $y_a$  and  $y_o$ .

Although this method is obviously not very new, its use here represents a novel approach to the challenge of evaluating a d-q equivalent circuit characterized by both magnetic cross saturation and two rotational-speed-dependent components of core loss resistance. Fig. 5 shows the values of the d-q axial inductances derived by incorporating magnetic cross saturation in this manner. By scatter-calculating inductance values using FEM, the respective inductances are derived through spline interpolation over the entire operational space.

These derived inductance values are then used in the analysis method described below.

### 4. Description of Proposed Analysis Method

As an IPMSM has reluctance torque-based saliency, each current vector can be manipulated in order to produce MTPA control over regions of constant torque and flux weakening control over regions of constant power. The conditions governed by the MTPA control can be derived

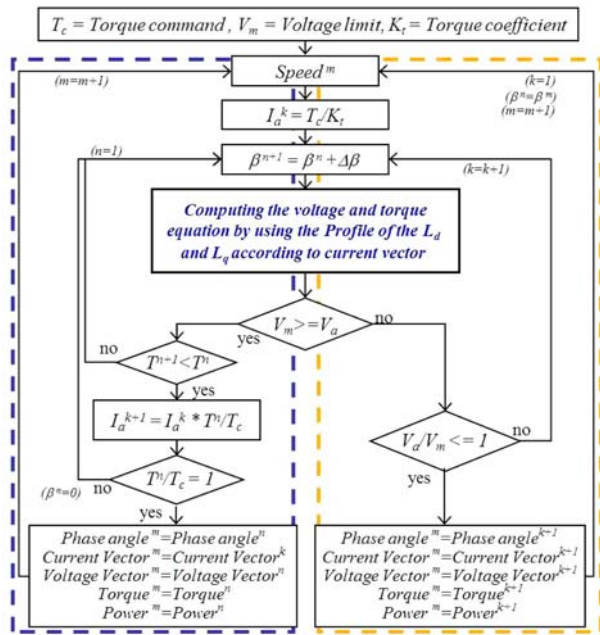


Fig. 6. (Color online) Flow-chart of the proposed evaluation of the d-q equivalent circuit.

by setting the derivatives of the torque equation with respect to  $\beta$  to zero whereas those conditions governed by flux weakening control are determined by satisfying a voltage limitation. However, as these mathematical formulations are either too complicated or insufficient for modeling both magnetic cross saturation and core losses in a d-q equivalent circuit model, evaluation of the model in this study is instead accomplished by using the fundamental voltage equation in tandem with the control region constraints. The proposed computational method is based on the iteration algorithm shown in Fig. 6, which can be solved by means of numerical optimization. In order to choose a suitable current vector that satisfies the operating region constraints, the proposed analysis method uses an inductance profile of magnetic cross saturation and core loss resistances; in each iteration step, it replaces the inductances (which correspond to the current vectors) by the voltage and torque equation.

### 5. Analysis Results and Discussion

Fig. 7 shows a photograph of the setup used in this study to verify the proposed analysis method; the IPMSM shown here serves as an air conditioner compressor. Figs. 8 and 9 show comparisons of the computed and the measured results over a range of speeds and at constant torques of 2 and 4 N-m, respectively. In Fig. 8, it can be seen that the analysis results agree well with the measured results, except in the high speed region, where more time

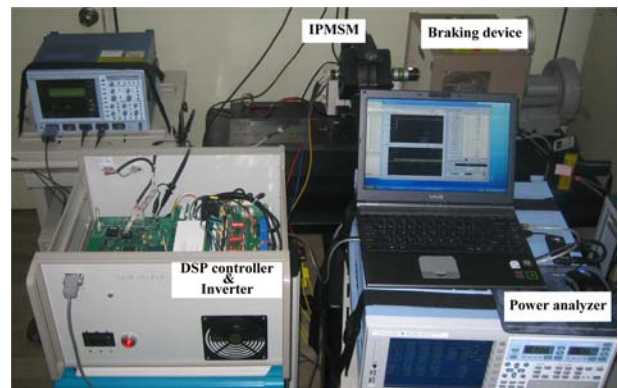


Fig. 7. (Color online) Bench testing set for the IPMSM.

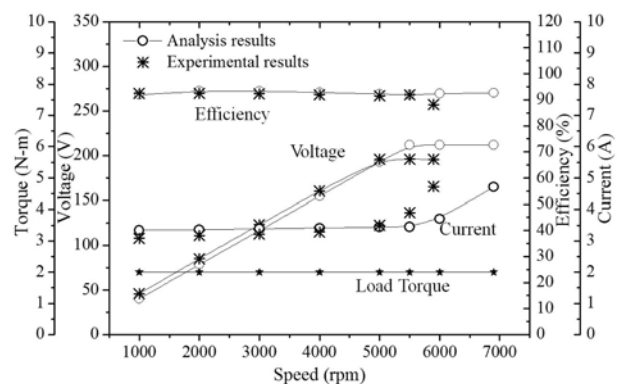


Fig. 8. Characteristics of the IPMSM at constant torque, 2 N-m.

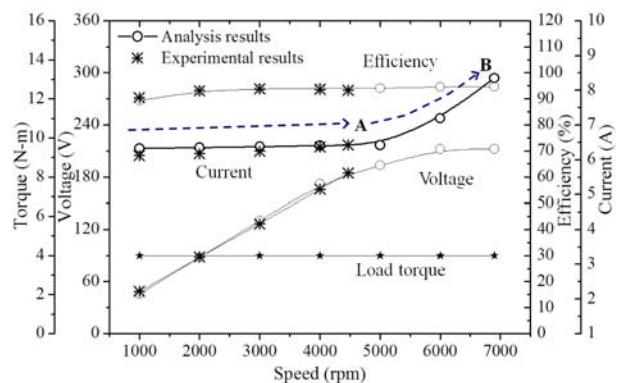


Fig. 9. (Color online) Characteristics of the IPMSM at constant torque, 4 N-m.

was needed to tune the proper control gain. From Fig. 9, it can be seen that, as the operating point moves to “A,” the current increases slightly to compensate for the current of the core loss component caused by an increased motor speed, and then, as the operating point moves to high speeds (“B”), the current surges in order to maintain a constant torque in the flux-weakening region. Figs. 10 and 11 show the loss and efficiency distributions over the entire range of operating points. These results, produced

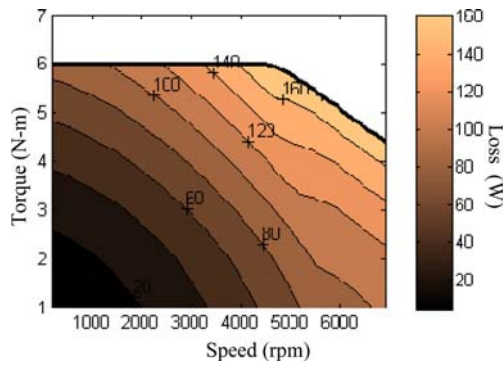


Fig. 10. (Color online) Loss distribution for overall operation point.

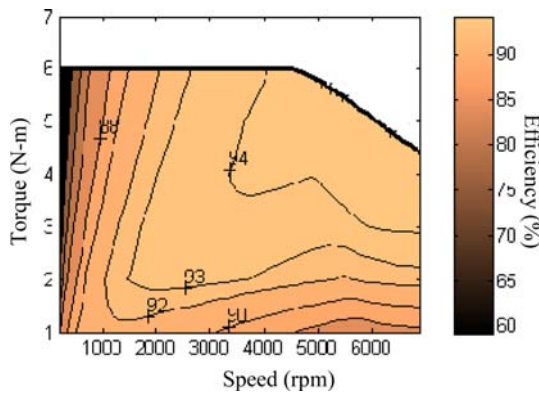


Fig. 11. (Color online) Efficiency distribution for overall operation point.

by the analysis method proposed in this paper, make estimation of the overall performance of the motor simple.

## 6. Conclusion

The influence of the magnetic cross saturation is considered by using the d-q axis inductance due to operating condition. Additionally, the core loss is reloaded at each load condition by using two components of the core loss resistance. The proposed analysis procedure can easily evaluate the overall performance of the IPMSM, has been successfully demonstrated. Therefore, it can be considered that the proposed method is useful for the analysis and design of the IPMSM.

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