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# Influence of Cogging Torque Reduction Method on Torque Ripple in a Surface-Mounted Permanent Magnet Synchronous Motor

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**The torque characteristics of a surface-mounted permanent magnet synchronous motor (SPMSM) are analyzed in this study. The harmonics of the back electromotive force (EMF) and cogging torque are analyzed by the finite element method to study their effects on the torque ripple. Although low cogging torque can be achieved by varying geometric parameters such as the permanent magnet (PM) offset and notch depth on the stator teeth, the torque ripple is increased in some cases. The analysis results show that the ripple of the generated torque is determined by not only the amplitudes but also the phases of harmonics for the back EMF and cogging torque.**

**Keywords :** back EMF, cogging torque, harmonics, synchronous motor, torque ripple

## 1. Introduction

Permanent magnet (PM) motors are used in a wide range of applications requiring precise position control. In the design aspects of PM motors, torque ripple is a major obstacle to achieving better control performance and should be minimized as much as possible. Cogging torque is a factor that influences the torque ripple and precision of PM motors. Many studies have been carried out to reduce cogging torque, and various methods have been introduced that changes the geometric shapes of the motor [1-3]. However, most studies assumed that reduced the cogging torque also decrease the torque ripples when the motor is in operation. This is not true if the shape of the back electromotive force (EMF) is influenced by geometric changes made to reduce the cogging torque.

In this study, the methods used to decrease cogging torque are analyzed to determine their influence on the torque characteristics. PM offset and notches on the stator teeth are cogging-torque-reducing methods [4-7]. The harmonics of the back EMF and cogging torque are analyzed by the finite element method, and their effects on the torque ripples are studied. Even though low cogging torque can be achieved by varying the PM offset and notch depth, this does not guarantee a low torque ripple in the generated torque. On the contrary, in some cases, the torque ripple becomes more severe with the changes in the back EMF shape caused by the geometric changes. The analysis results show that the phases of the harmonics for cogging torque and back EMF are dominant factors in determining the ripple of the generated torque with the amplitudes of harmonics of them.

## 2. Torque Characteristics

### 2.1. Analysis model

Fig. 1 shows the 1/4 cross-sectional view of an 8-pole 12-slot surface-mounted permanent magnet synchronous motor (SPMSM) under symmetry conditions for the flux distribution. The SPMSM has outer diameter of 160 and 290 mm for the rotor and stator, respectively. In this study, the characteristics of the torque are analyzed by varying the PM offset and notch depth. The PM offset is changed from 30 mm to 80 mm. When the motor has 80 mm of PM offset, the center of the outer radius of the PM coincides with the center of the rotor, and it deviates from the rotor center with decreasing PM offset. The notch depth, width and number of notches in the stator teeth influence the cogging torque. From [4], we adopt two notches on each tooth to reduce the cogging torque. They are equally spaced and as wide as the opening of the actual slots. The notch depth is changed from 0 mm, no-notch case, to 2 mm.

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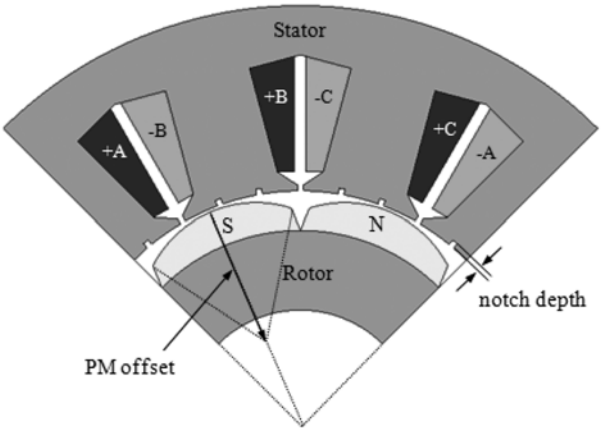

**Fig. 1.** 1/4 cross-sectional view of 8-pole 12-slot SPMSM.

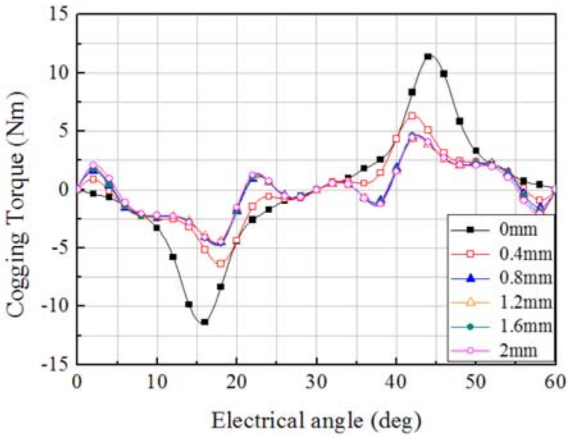

**Fig. 3.** (Color online) Cogging torque waveform with different notch depths.

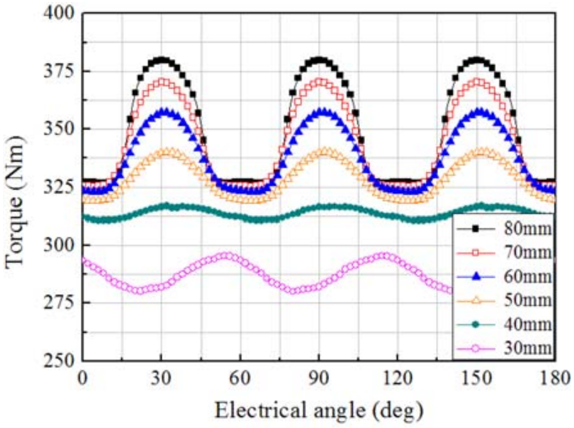

**Fig. 4.** (Color online) Torque waveform with different PM offsets.

### 2.2. Cogging and generated torques

Cogging torque is generated by variation in the magnetic energy due to changes in the permeance according to the rotor position [4].

Figs. 2 and 3 show the cogging torque waveforms of different PM offsets and notch depths, respectively. The amplitudes of the cogging torque can be reduced by changing these two variables. Varying the PM offset decreases the cogging torque more rapidly than changing the notch depth. If the notches are not deep enough, they cannot perform as actual slots, which is why the cogging torque cannot be reduced as much.

Figs. 4 and 5 show the generated torque waveforms for variations of the notch depth and PM offset. The input current has a sinusoidal waveform and is applied in phase with the back EMF waveform of the two variables. For the PM offset, the amplitude of the torque is proportional to the size of the PM on the rotor. As the PM offset is increased, the volume of the PM is also increased, so the generated torque has a higher average value. However, the torque ripple has a minimum value in the middle of the PM offset range, and not at the ends. Unlike the PM

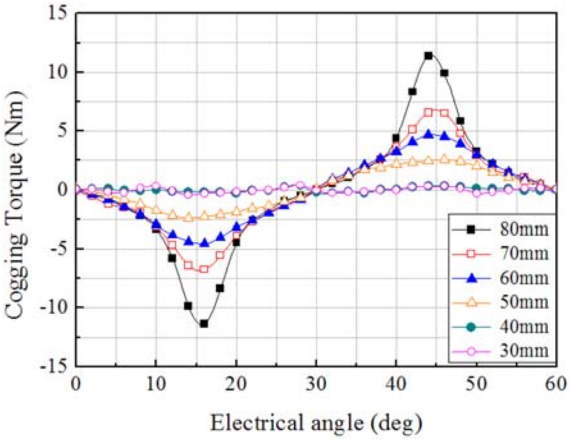

**Fig. 2.** (Color online) Cogging torque waveform with different PM offsets.

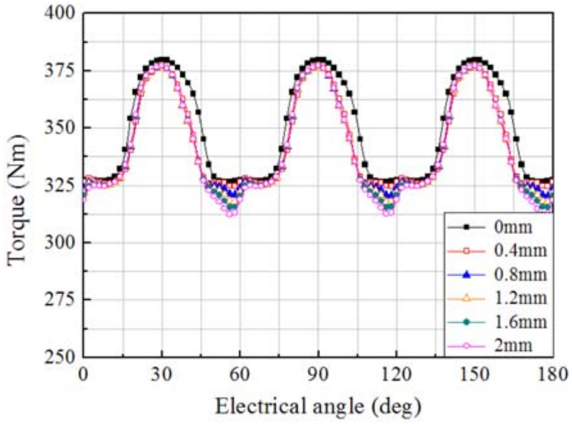

**Fig. 5.** (Color online) Torque waveform with different notch depths.

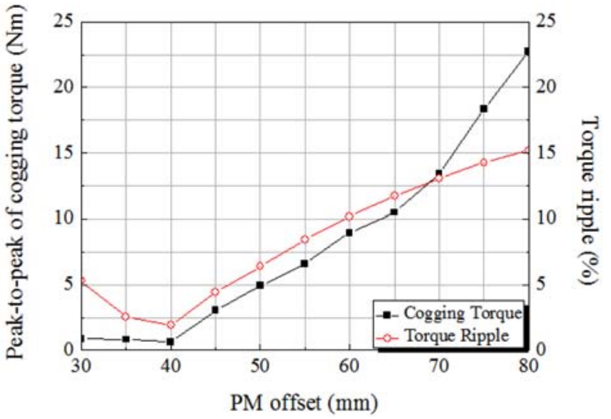

**Fig. 6.** (Color online) Peak-to-peak value of cogging torque and torque ripple for PM offset.

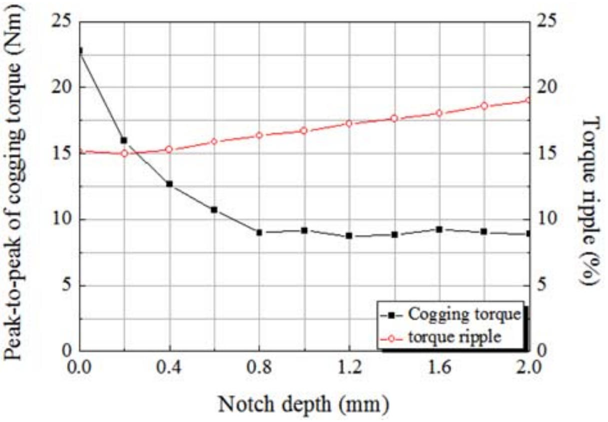

**Fig. 7.** (Color online) Peak-to-peak value of cogging torque and torque ripple for notch depth.

offset, changing the notch depth causes little change in the average torque, and torque ripple slowly increases with the notch depth variations. As the notch depth is increased, asymmetric saturation of the stator teeth becomes distinct, which increases the torque ripple.

Figs. 6 and 7 compare the peak-to-peak values of the cogging torque and torque ripple with the variations in the PM offset and notch depth, respectively. The most notable point in the curves in Fig. 6 appears at the PM offset range of 30 to 40 mm. The peak-to-peak value of the cogging torque decreases rapidly with the PM offset and becomes saturated after 40 mm of PM offset. The generated torque ripple shows the same trend until the PM offset reaches 40 mm. However, further reducing the PM offset, causes the torque ripple to increase, unlike the saturated value of the cogging torque. For the notch depth, the inverse relation between the peak-to-peak value of the cogging torque and torque ripple is dominant for the entire range of notch depth variation, as shown in Fig. 7.

In general, the torque ripple decreases with the cogging torque reduction. However, as shown in the analysis results, the relation between the cogging torque and torque ripple does not always follow this trend. The torque ripple is not proportional to the peak-to-peak value of cogging torque. This is related to the amplitudes and phases of the harmonics of the back EMF and cogging torque, which are changed by the shape of the stator and PM on the rotor.

## 3. Analysis of torque ripple

### 3.1. Effect of back EMF

In a synchronous machine, when the phase current, $i$ flows through the back EMF source, $e$, the instantaneous power is converted to mechanical power and is equal to the product of $e$, $i$. In three-phase motors with balanced windings, the back EMFs and input currents of each phase have the same shape and are offset by 120 electrical degrees from each other. In the ideal case, if the waveforms of the back EMF and input current are sinusoidal, and there is no phase difference between them, the motor produces constant torque without torque ripple. However, torque ripple occurs when the back EMF contains harmonic components. The magnitude of the torque ripple is influenced by the amplitudes and phases of each harmonic component of the back EMF.

The back EMF and input current of the three-phase motor are written as

$$e_a = \sum_{n=1,3,5,\ldots} e_{an}\cos n\omega t,\quad i_a = I_m\cos\omega t \tag{1}$$

$$e_b = \sum_{n=1,3,5,\ldots} e_{bn}\cos n\left(\omega t - \frac{2}{3}\pi\right),\quad i_b = I_m\cos\left(\omega t - \frac{2}{3}\pi\right) \tag{2}$$

$$e_c = \sum_{n=1,3,5,\ldots} e_{cn}\cos n\left(\omega t + \frac{2}{3}\pi\right),\quad i_c = I_m\cos\left(\omega t + \frac{2}{3}\pi\right) \tag{3}$$

where $n$ is an odd integer greater than 1 due to the halfway symmetry of the back EMF shape, $e_{an}$, $e_{bn}$, $e_{cn}$ are the amplitudes of the $n^{th}$ harmonic of the back EMF in each phase, and $I_m$ is the maximum value of the input current.

The total output power is obtained from the summation of $e \times i$ in each phase as follows

$$\begin{aligned}\Sigma e\times i &= e_a i_a + e_b i_b + e_c i_c\\ &= \frac{1}{2}I_m\sum_{n=1,3,5,\ldots} e_{an}(\cos(n-1)\omega t + \cos(n+1)\omega t)\\ &+ \frac{1}{2}I_m\sum_{n=1,3,5,\ldots} e_{bn}\left(\cos(n-1)\left(\omega t - \frac{2}{3}\pi\right) + \cos(n+1)\left(\omega t - \frac{2}{3}\pi\right)\right)\\ &+ \frac{1}{2}I_m\sum_{n=1,3,5,\ldots} e_{cn}\left(\cos(n-1)\left(\omega t + \frac{2}{3}\pi\right) + \cos(n+1)\left(\omega t + \frac{2}{3}\pi\right)\right)\end{aligned} \tag{4}$$

As the amplitude of back EMFs and the mechanical power are proportional to rotating speed, the harmonic components in the sum of $e \times i$ occur in the generated torque profile as well.

In (4), the coefficients of the $n^{\text{th}}$ harmonics of the torque are determined by the $(n-1)^{\text{th}}$ and $(n+1)^{\text{th}}$ harmonics in the back EMF. Therefore, the torque ripple is influenced by both the amplitudes and phases of the harmonic components in the back EMF. In other word, the cross product of the back EMF and current harmonics makes the torque harmonics at frequencies equal to the sum and difference of the current and individual back EMF harmonics. In a balanced three-phase system, the motor torque only has harmonics that are multiples of six, and the other harmonic components are zero due to the three-phase symmetry.

Figs. 8 and 9 show the back EMF waveform for variations in the two design variables; the PM offset and notch depth. In each case, the other parameter is set to a basic value, a notch depth of 0 mm and PM offset of 80 mm. As shown by Figs. 8 and 9, the PM offset has the dominant influence on the shape of the back EMF. The back EMF waveforms of the PM offset approximate the sinusoidal waveform; however, the back EMF waveforms of the notch depth distorted such that they resemble a trapezoidal waveform.

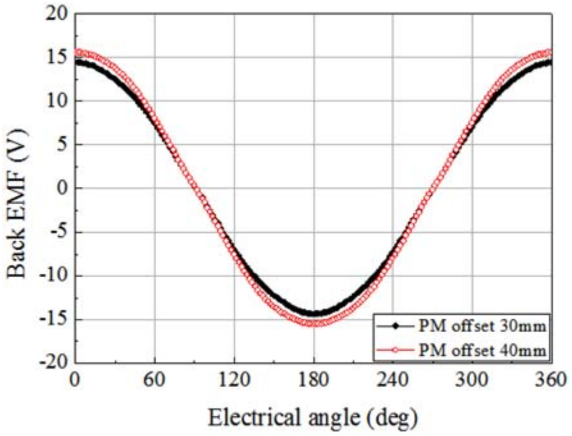

**Fig. 8.** (Color online) Back EMF waveform for PM offsets of 30 and 40 mm.

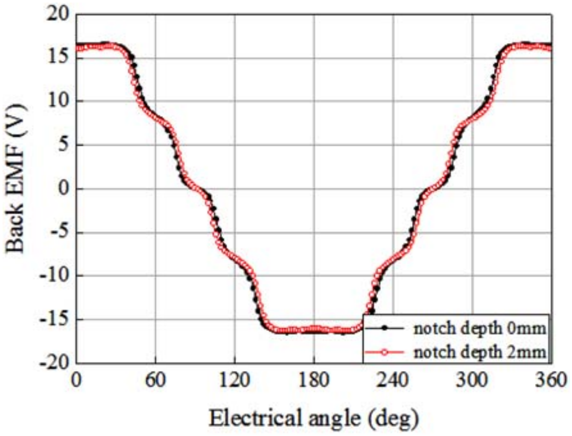

**Fig. 9.** (Color online) Back EMF waveform for notch depths of 0 and 2 mm.

Tables 1 and 2 show the amplitude and phase of each harmonic component and the total harmonic distortion (THD) of the back EMF curves in Figs. 8 and 9. The triplen harmonics do not influence the torque ripple and they are not listed in the table.

For the PM offset, the amplitudes of the harmonics are relatively small values compared with the variation in the notch depth. However, a decrease in the fundamental value reduces the average torque.

Evaluating (4) for various n gives the harmonic content of the generated torque. When the sum of the adjacent harmonics to each multiple of six harmonics, (e.g., the 5th and 7th or the 11th and 13th) have small values, the corresponding harmonic components in the torque ripple, (the 6th or 12th) also have small values. As one example, when comparing the two PM offset cases, the torque amplitude at 6th harmonic is relatively small at a PM offset of 40 mm due to the phase reverse between the 5th and 7th harmonics in the back EMF shape.

When the notch depth varied, there is no phase change

**Table 1.** Harmonics and THD of the back EMF for the PM offset.

| Harmonic orders | PM offset | |
|---|---|---|
| | 30 mm | 40 mm |
| 1 | 14.32 | 15.62 |
| 5 | 0.26 | –0.18 |
| 7 | 0.07 | 0.06 |
| 11 | 0.36e-2 | 0.03 |
| 13 | –0.48e-2 | –0.02 |
| THD (%) | 1.88 | 1.25 |

**Table 2.** Harmonics and THD of the back EMF for the notch depth.

| Harmonic orders | Notch depth | |
|---|---|---|
| | 0 mm | 2 mm |
| 1 | 17.41 | 17.04 |
| 5 | –1.07 | –0.42 |
| 7 | –0.31 | –0.92 |
| 11 | 0.80 | 0.82 |
| 13 | –0.58 | –0.41 |
| THD (%) | 8.56 | 8.01 |

between each harmonic component. Thus, torque harmonics are only determined by comparing the magnitudes of the back EMF harmonics.

### 3.2. Effect of cogging torque

The cogging torque also has the amplitude and phase of the harmonic components, and it is superimposed on the electromagnetic torque. If the magnetic circuit is not saturated, both torque components are added arithmetically. Thus, the total generated torque contains harmonic components of both the cogging and electromagnetic torques and each torque harmonic is influenced by the amplitudes and phases of the two torque components.

### 3.3. Ripples of generated torque

Tables III and IV show the harmonic amplitude and phase of the cogging torque ($T_c$), electromagnetic torque ($T_e$), and total generated torque ($T_t$) for the PM offset and notch depth, respectively. The total generated torque calculated by finite element analysis is almost the same as the sum of the cogging and electromagnetic torques. By comparing each case, several facts for the torque ripple can be deduced.

The first deduction is regarding the influence of the cogging torque on the torque characteristics. The cogging torque affects the torque ripple in two ways. First, the ripple comes from the cogging torque itself. It also has the amplitude and phase in the harmonic components and is added to the electromagnetic torque to produce the total generated torque. Except for when the PM offset is 30 mm, the ripple of the total generated torque is reduced by adding cogging torque relative to the electromagnetic torque. Even if the reduced peak-to-peak value in the total generated torque is small, the important consideration here is that the amplitude of the cogging torque itself is not the sole factor for the torque ripple; the phase should also be considered. It is also found that the geometric changes to reduce cogging torque influence the variation in electromagnetic torque. This is clearly shown in Table 4. The peak-to-peak value of the cogging torque is definitely reduced by the notches on the stator teeth. However, this geometric change influences the back EMF shape and electromagnetic torque, hence, the ripple of the total generated torque. In contrast to the 60% reduction in cogging torque caused by changing the notch depth to 2 mm, the generated torque ripple is increased by 3.5%.

The second deduction is regarding the phases of harmonics in the back EMF. When comparing the two PM offset cases, the amplitude of the electromagnetic torque at the 6th harmonic for a 40 mm offset is small compared to that for a 30 mm PM offset. This is because of the phase reverse between the 5th and 7th harmonics in the back

**Table 3.** Harmonics of $T_c$, $T_e$, and $T_t$ of the PM offset.

| Harmonic orders | PM offset | | | | | |
|---|---|---|---|---|---|---|
| | 30 mm | | | 40 mm | | |
| | $T_c$ | $T_e$ | $T_t$ | $T_c$ | $T_e$ | $T_t$ |
| 0 | 0 | 287.67 (100%) | 287.67 (100%) | 0 | 313.74 (100%) | 313.74 (100%) |
| 6 | 0.10 | 7.45 (2.6%) | 7.47 (2.6%) | 0.12 | –2.93 (0.9%) | –2.88 (0.9%) |
| 12 | 0.02 | 0.37 (0.1%) | 0.38 (0.1%) | 0.04 | 0.24 (0.08%) | 0.25 (0.08%) |
| 18 | –0.15 | –0.17 (0.06%) | 0.17 (0.06%) | –0.15 | –0.17 (0.06%) | –0.19 (0.06%) |
| Peak-to-peak (Nm) | 0.89 | 14.46 | 14.51 | 0.62 | 6.09 | 6.08 |
| Ripple (%) | – | 5.026 | 5.041 | – | 1.941 | 1.938 |

**Table 4.** Harmonics of $T_c$, $T_e$, and $T_t$ of the notch depth.

| Harmonic orders | Notch depth | | | | | |
|---|---|---|---|---|---|---|
| | 0 mm | | | 2 mm | | |
| | $\mathrm{T_c}$ | $\mathrm{T_e}$ | $\mathrm{T_t}$ | $\mathrm{T_c}$ | $\mathrm{T_e}$ | $\mathrm{T_t}$ |
| 0 | 0 | 349.49 (100%) | 349.49 (100%) | 0 | 341.31 (100%) | 341.31 (100%) |
| 6 | 6.32 | –30.10 (8.6%) | –30.07 (8.6%) | 2.25 | –30.43 (8.9%) | –30.39 (8.9%) |
| 12 | –0.49 | 4.80 (1.4%) | 4.76 (1.4%) | 0.23 | 8.11 (2.4%) | 8.20 (2.4%) |
| 18 | –2.74 | 5.07 (1.4%) | 4.47 (1.3%) | –1.47 | 2.92 (0.8%) | 1.53 (0.4%) |
| Peak-to-peak (Nm) | 22.75 | 54.47 | 54.39 | 8.94 | 64.83 | 64.81 |
| Ripple (%) | – | 15.585 | 15.562 | – | 18.995 | 18.989 |

EMF with a 40 mm PM offset. This is also the main reason for the reduced peak-to-peak value in the total generated torque for a 40 mm PM offset. For the notch depth, the same phenomenon occurs in the 12th harmonic of the electromagnetic torque.

The last deduction is regarding the relation between the THD value of the back EMF and the ripple of the electromagnetic torque. For the notch depth, the ripple of the electromagnetic torque is small even if the magnitude of the THD of the back EMF has a larger value. This shows that each harmonic component in the generated torque is influenced by not only the magnitude but also the phase of each harmonic of the back EMF. As the THD value only shows the magnitude of the harmonic components relative to the fundamental, the influence of the phase of the harmonics cannot be included.

Based on (4), the electromagnetic torque can be expressed as

$$\begin{aligned}T_e = \frac{1}{2}I_m\Big[&(e_{a1}+e_{b1}+e_{c1}) + (e_{a5}+e_{a7})\cos 6\omega t\\&+ (e_{b5}+e_{b7})\cos 6\left(\omega t - \frac{2}{3}\pi\right)\\&+ (e_{c5}+e_{c7})\cos 6\left(\omega t + \frac{2}{3}\pi\right)\\&+ (e_{a11}+e_{a13})\cos 12\omega t \qquad (5)\\&+ (e_{b11}+e_{b13})\cos 12\left(\omega t - \frac{2}{3}\pi\right)\\&+ (e_{c11}+e_{c13})\cos 12\left(\omega t + \frac{2}{3}\pi\right) + \cdots\Big]\\= \frac{1}{2}I_m\Big[&(e_{a1}+e_{b1}+e_{c1})\\&+ ((e_{a5}+e_{a7}) + (e_{b5}+e_{b7}) + (e_{c5}+e_{c7})\cos 6\omega t)\\&+ ((e_{a11}+e_{a13}) + (e_{b11}+e_{b13}) + (e_{c11}+e_{c13})\cos 12\omega t) + \cdots\end{aligned}$$

In balanced winding, the coefficients of the nth harmonics of each phase back EMF have the same amplitudes. So, the electromagnetic torque can be written as

$$T_e = \frac{3}{2}T_m[e_{a1}(e_{a5}+e_{a7})\cos 6\omega t + (e_{a11}+e_{a13})\cos 12\omega t + \cdots] \quad (6)$$

When the absolute value of the sum of the back EMF harmonics producing each multiple of six harmonics in the generated torque is small, the corresponding harmonic in the electromagnetic torque is also small.

## 4. Conclusion

This paper examined the effect of the amplitudes and phases of the back EMF and cogging torque harmonics on the torque ripple in a SPMSM. The methods introduced for reducing cogging torque changes the shape of back EMF as well as cogging torque itself and do not guarantee a reduction in the torque ripple. The THD value of the back EMF also does not indicate the amount of torque ripple. For the torque ripple, the phase of harmonics, along with the amplitude of the cogging torque and back EMF is the dominant factor for determining the ripple of the generated torque.

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