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Reliability-Based Design Optimization of a Superconducting Magnetic Energy Storage System (SMES) Utilizing Reliability Index Approach

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A reliability-based optimization method for electromagnetic design is presented to take uncertainties of design parameters into account. The method can provide an optimal design satisfying a specified confidence level in the presence of uncertain parameters. To achieve the goal, the reliability index approach based on the first-order reliability method is adopted to deal with probabilistic constraint functions and a double-loop optimization algorithm is implemented to obtain an optimum. The proposed method is applied to the TEAM Workshop Problem 22 and its accuracy and efficiency is verified with reference of Monte Carlo simulation results.

Keywords : electromagnetic fields, optimization, reliability theory, robustness

1. Introduction

Due to a growing demand for high-performance and high-reliability electromagnetic (EM) devices or equipments, attention has recently focused on dealing with uncertain design parameters such as manufacturing errors, operating conditions, material properties, etc. [1-6]. The methodology for treating optimization problems in the presence of uncertainties of design parameters/variables can be categorized into two approaches. The first is robust design optimization (RDO) based on several approaches, such as the worst-case analysis, gradient index, six sigma, etc., to improve the product quality by minimizing variability of the output performance functions [1-3]. However, it does not address the quantitative assessment of performance reliability (i.e. at what confidence or probability level the robustness of an EM design is achieved). The second is reliability-based design optimization (RBDO) utilizing the probabilistic reliability analysis to achieve product reliability at a given probabilistic level [4, 5]. Until now, most of the reported attempts for EM designs fall into RDO. Although the reliability analysis has been applied to a superconducting magnetic energy storage system (SMES) design [6], it is to merely assess the robustness of performance functions considered at different design points and not to execute design optimization of SMES.

For the first time, RBDO based on the first-order reliability method (FORM) is applied to an EM device in this paper. The method estimates the probability of failure (i.e. a nominal design point dose not satisfy the performance condition given) by the first-order Taylor series approximation of the performance function when probabilistic information of random variables is known. Failure probability is expressed in terms of the integral of joint probability density function (PDF) with respect to random variables and it corresponds to the most difficult part in

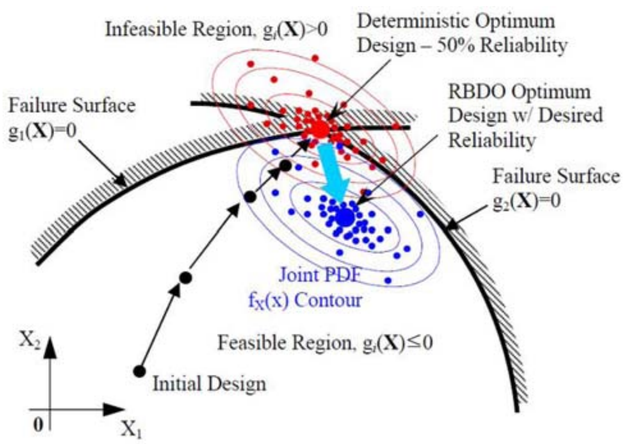

Fig. 1. (Color online) DO and RBDO optimum from reliability point of view: g_1 and g_2 are the constraint functions [4].

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implementing RBDO. To efficiently calculate the failure probability, the reliability index approach (RIA) is employed.

By incorporating reliability concept into design optimization, the probabilistic feasibility of constraints in RBDO can be much improved. For instance, Fig. 1 illustrates that the design points are modified to achieve the desired reliability when an optimum obtained by deterministic optimization (DO) has a relatively low reliability due to random variables.

RBDO formulation based on RIA was successfully applied to the TEAM Workshop Problem 22 [7]. Using the proposed method, RBDO optimum of SMES windings was obtained where two design constraints (stored energy and geometrical constraint) maintain high reliabilities (greater than 95%) under the uncertainties of design variables. In addition, DO, RDO [2], and RBDO optima were compared to each other in terms of reliability of design. To validate the accuracy and efficiency of the RBDO model based on RIA, Monte Carlo simulation (MCS) was performed.

2. Reliability-Based Design Optimization

2.1. Definition of RBDO model

A deterministic optimization is generally formulated as:

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{d}) \\ &\text{subject to } g_i(\mathbf{d}) \geq 0,\ i = 1,\ldots, nd \\ &\qquad \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U,\ \mathbf{d} \in R^n \end{aligned} \tag{1}$$

where f(d) is the objective/cost function, g_i is the constraint function, **d** is the design variable vector, and $\mathbf{d}^L$ and $\mathbf{d}^U$ mean the lower and upper bound of **d**, respectively.

On the other hand, the probabilistic constraints are newly introduced to the RBDO formulation when considering uncertainties of the design variables [4].

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{d}) \\ &\text{subject to } P(g_i(\mathbf{X}) < 0) \leq P_{t,i},\ i = 1,2,\ldots np \\ &\qquad \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U,\ \mathbf{d} \in R^n \end{aligned} \tag{2}$$

where **d** is given by $\mathbf{d} = \mu(X)$ and μ denotes the mean value vector of the random vector **X**. Target failure probability $P_{t,i}$ is given for ensuring a certain safety/confidence level with respect to the ith constraint function. The failure probability P_f of constraint g_i is expressed as

$$P_f = 1 - R = P(g_i(\mathbf{X}) < 0) = \int_{g_i(\mathbf{X})<0} \int f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{3}$$

where R denotes reliability and $f_{\mathbf{X}}(\mathbf{x})$ is the joint PDF of **X**.

2.2. First-order reliability analysis in RIA

To evaluate the multiple integration of (3) effectively, the reliability analysis based on FORM requires following two procedures.

1) Transformation of a design space

Original random variables **X** are transformed to uncorrelated normal random variables **U** of which each has zero mean and unit standard deviation (SD). That is, the constraint function $g(\mathbf{X})$ in X-space is mapped onto $g(T(\mathbf{X})) \equiv g(\mathbf{U})$ in U-space. In the case of normal random variable X_k, the transformation is given as follows

$$U_k = \frac{X_k - \mu_k}{\sigma_k} \tag{4}$$

where μ_k and σ_k are the mean and SD of the random variable X_k. When the nonnormal random variables are involved, Rosenblatt transformation can be used [5].

2) Linear approximation

Using the first-order Taylor series, g is approximated by a linear function at the design point $\mathbf{u}^*$.

$$g(\mathbf{U}) \approx g(\mathbf{u}^*) + \nabla g(\mathbf{u}^*)^T (g(\mathbf{u}) - g(\mathbf{u}^*)) \tag{5}$$

where $\nabla g(\mathbf{u}^*)$ is the gradient vector of g and $\mathbf{u}^*$ is called as the most probable point (MPP) on the failure surface $g(\mathbf{u}^*) = 0$.

The first-order reliability index β in RIA is obtained by formulating a sub-optimization problem as:

$$\begin{aligned} &\text{minimize} \quad \|\mathbf{U}\| \\ &\text{subject to} \quad g(\mathbf{U}) = 0. \end{aligned} \tag{6}$$

MPP ($\mathbf{u}^*_{g(\mathbf{U})=0}$) is geometrically interpreted as the minimum distance point on the failure surface from the origin in U-space and the reliability index is defined by $\beta = \|\mathbf{u}^*_{g(\mathbf{U})=0}\|$ as shown in Fig. 2.

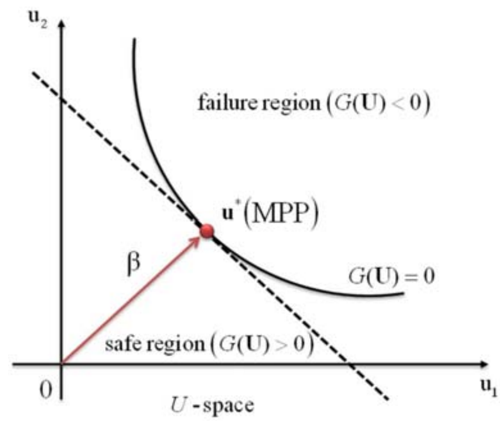

Fig. 2. (Color online) Geometrical interpretation of MPP and reliability index in U-space.

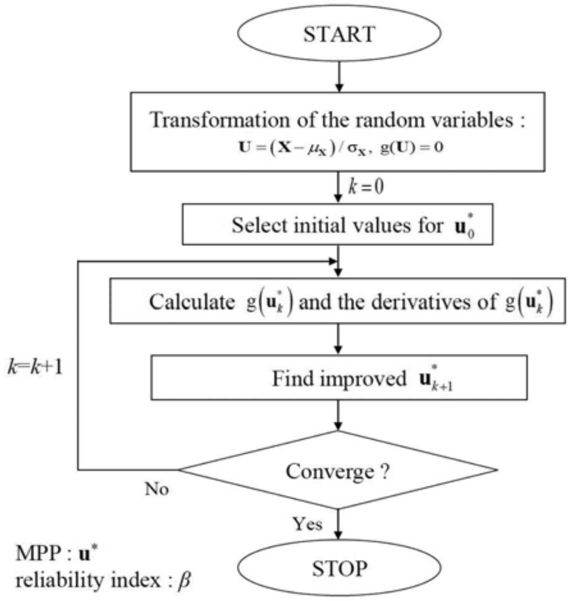

Fig. 3. Flowchart of HL-RF method.

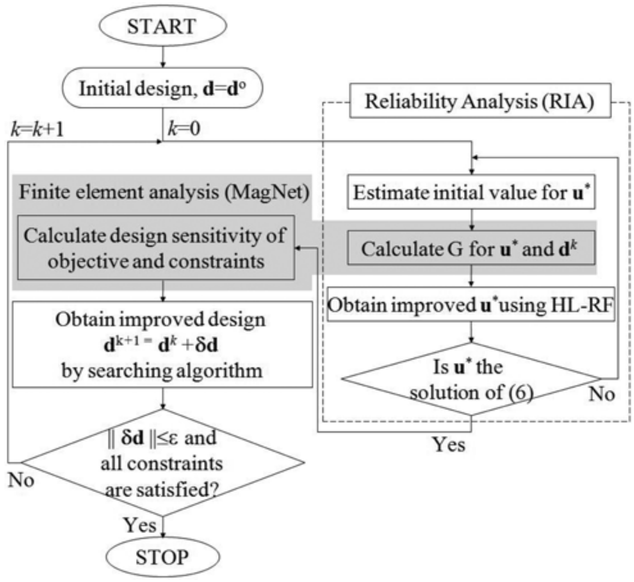

Fig. 4. Flowchart of RBDO based on RIA.

For solving (6), the Hasofer Lind and Rackwitz Fiessler (HL-RF) method [5] is employed here because of its simplicity and efficiency. The flowchart of the HL-RF method is illustrated in Fig. 3.

Finally, the probability of failure of (3) is approximated as

$$P_f = P(g_i(\mathbf{U}) \le 0) \approx \Phi(-\beta) \tag{7}$$

where $\Phi(\bullet)$ is the standard normal cumulative distribution function (CDF).

2.3. RBDO based on RIA

Adopting RIA, the RBDO formulation can be rewritten as:

$$\begin{aligned} &\text{find } \mathbf{d} \\ &\text{minimize } f(\mathbf{d}) \\ &\text{subject to } \Phi(-\beta_i(\mathbf{U})) \le \Phi(-\beta_{t,i}),\ i = 1,\ldots, np \\ &\qquad \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U,\ \mathbf{d} \in R^n \end{aligned} \tag{8}$$

where $\beta_{t,i}$ is the target reliability index. The probabilistic constraints of (8) is equivalently expressed as

$$\beta_i(\mathbf{U}) \ge \beta_{t,i}\,. \tag{9}$$

Finally, RBDO requires two kinds of optimization procedures simultaneously: One is for evaluating failure probability of each constraint and the other is for optimizing the cost function as satisfying the given constraints.

3. Numerical Implementation

As aforementioned, the implementation of RBDO consists of a double-loop optimization structure as shown in Fig. 4. It means the parametric optimization problem has sub-problems for reliability analysis for each iterative design. Therefore, the procedure of the proposed RBDO problem can be divided into two optimization procedures as follows:

1) inner loop: sub-optimization procedure for evaluating failure probability of each constraint (dotted box in Fig. 4),

2) outer loop: overall optimization procedure to optimize the cost function with constraints satisfaction.

For outer loop optimization, a traditional optimization algorithm, sequential quadratic programming (SQP) is adopted here. Usually, the computational demand may be high if many probabilistic constraints are imposed or the associated function evaluations are very expensive. Thus, an alternative method should be studied to improve an efficiency of a probabilistic design optimization.

4. Comparison of Different SMES Designs

A RBDO formulation for minimizing an objective function subject to a set of constraints is expressed as:

$$\begin{aligned} &\text{minimize } f(\mathbf{d}) = \sum_{i=1}^{21} |B_{stray,i}(\mathbf{d})|^2 \\ &\text{subject to } P(g_i(\mathbf{X}) < 0) - \Phi(-\beta_{t,i}) \le 0 \quad i = 1, 2 \\ &\qquad g_1(\mathbf{X}) = 1 - \left(\frac{E(\mathbf{X}) - E_o}{0.05 \times E_o}\right)^2 \\ &\qquad g_2(\mathbf{X}) = (R_2 - R_1) - \frac{1}{2}(D_2 + D_1), \\ &\qquad \mathbf{X} = [R_2, D_2, H_2],\ \mathbf{X}^L \le \mathbf{X} \le \mathbf{X}^U \end{aligned} \tag{10}$$

Table 1. Design variables and performance indicators at the deterministic, RDO and RBDO Optima.

Design variables	Unit	Initial design	DO optimum	RDO optimum [2]	RBDO optimum
R_1	mm	1977	1977	1977	1977
D_1	mm	404	404	404	404
H_1	mm	1507	1507	1507	1507
R_2	mm	2340	2347	2348	2350
D_2	mm	310	253	233	242
H_2	mm	1780	1732	1871	1800
J_1	A/mm^2	16.30	16.30	16.30	16.30
J_2	A/mm^2	16.19	16.19	16.19	16.19
B_{stray}	μT	6,772	157	34	86
Energy	MJ	180	174	181	178
Function calls	-	-	82	-	352

Table 2. Results of failure probability estimation at four different designs.

	Method	Initial design	DO optimum	RDO optimum	RBDO optimum
$P_f(g_1)$	RIA	10.1	30.2	4.0	1.2
(%)	MCS	11.5	29.4	6.2	4.5
Error (%)	-	6.1	-4.5	14.0	34.2
Function	RIA	40	12	20	16
calls (g_1)	MCS	10,000	10,000	10,000	10,000
$P_f(g_2)$	RIA	28.0	3.02×10^{-3}	1.76×10^{-5}	7.12×10^{-5}
(%)	Exact	28.0	3.02×10^{-3}	1.76×10^{-5}	7.12×10^{-5}
Error (%)	-	0	0	0	0
Function calls (g_2)	RIA	6	6	6	6

where $B_{stray,i}$ is the stray field calculated at the ith measurement point along line a and line b, E is the stored magnetic energy with a target value E_o of 180 MJ and the wanted confidence level $\beta_{t,i}$ is set to be 1.645 corresponding to the failure probability value of 5% (i.e. reliability of 95%). It is assumed that the random variables follow the normal distributions and the SD values of R_2, D_2 and H_2 are 10 mm, 5 mm, 10 mm, respectively.

The optimization problem was solved using two different optimization methods where design sensitivity values are calculated with finite differencing method (FDM). The first is a deterministic method without taking probability distributions of design variables into account; the second approach is the proposed RBDO. Starting with an initial design, the deterministic and RBDO optima are presented in Table 1. The result of RDO in the previous article [2], where the gradients of performance function required for the second-order sensitivity information were exploited, is added in Table 1 for proving the validity of RBDO. It is observed that total number of function calls of RBDO is more than four times as high as that of DO because of the inherent double-loop optimization structure. The RDO and RBDO optima produce better values of the stray fields than DO optimum. It is inferred that the deterministic optimum is trapped in one of the local minima near the constraint boundaries, while better optimal solutions are found as the feasibility robustness of the constraints is improved.

For the initial design and three different optima, the failure probability values estimated from MCS and RIA are listed in Table 2. Assuming MCS results to be reference values, the error of RIA is defined by

$$\text{Error}(\%) = \frac{\beta - \beta_{\text{MCS}}}{\beta_{\text{MCS}}} \times 100,\ \beta_{\text{MCS}} = -\Phi^{-1}(P_{f,\text{MCS}}) \quad (11)$$

where β, β_{MCS} are the reliability indexes obtained from RIA and MCS, respectively, and $P_{f,\text{MCS}}$ is the failure probability of MCS. The symbol $\Phi^{-1}(\bullet)$ denotes the inverse standard normal CDF.

As shown in Table 2, RIA provides acceptable results on the constraint g_1 even with less than 40 simulations for all cases. Although MPP is successfully found at the RBDO optimum, RIA gives a bit inaccurate estimation (error = 34.2%). It indicates that the first-order approximation does not properly express the nonlinear behavior of g_1.

Since g_2 is a linear function and all associated random variables are normal distributions, the failure probability of g_2 can be calculated analytically without executing EM or reliability analyses.

The TEAM benchmark problem 22 of SMES depicted in Fig. 5 is concerned with RBDO. For simplification of the design problem, a constraint of the current quench

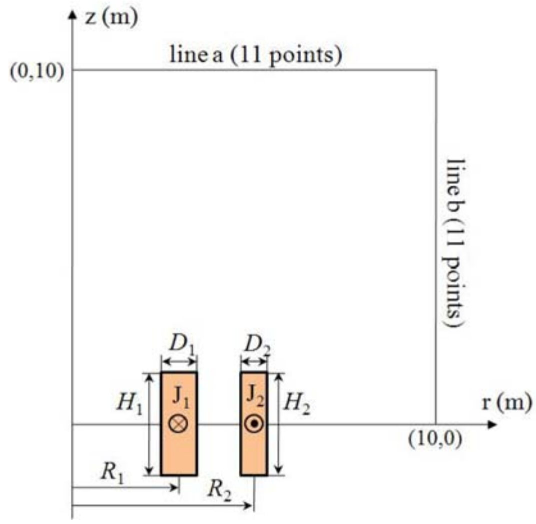

Fig. 5. (Color online) Configuration of the SMES device.

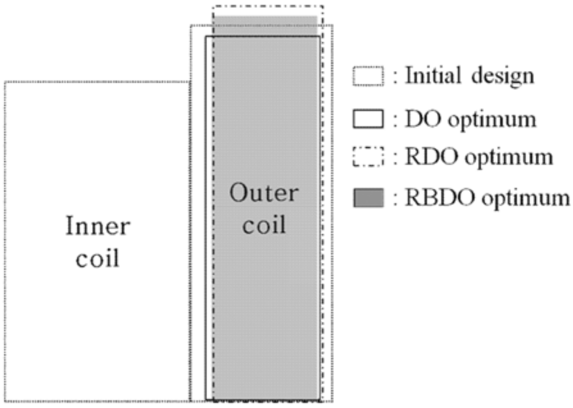

Fig. 6. Comparison of magnet dimensions after optimization.

condition on the superconductivity magnet is not considered here. Only three of total eight design variables, *R*2, *D*2 and *H*2, are selected as independent random variables.

$$P_f(g_2) = \Phi(-\mu_{g_2}/\sigma_{g_2}),$$
$$\mu_{g_2} = (\mu_{g_2} - R_1) - \frac{1}{2}(\mu_{D_2} + D_1), \quad (12)$$
$$\sigma_{g_2} = \sqrt{\sigma_{R_2}^2 + \frac{1}{4}\sigma_{D_2}^2}$$

where the mean and SD values of R_2, D_2 are denoted as μ_{R_2}, μ_{D_2}, σ_{R_2}, σ_{D_2}, respectively. The RIA converges to exact failure probability values with only 6 function evaluations.

At the DO optimum, the reliability of g_2 of initial design (82%) is remarkably increased to 99.997%. However, the violation probability of g_1 is deteriorated from 10.1% to 30.2%. On the other hand, the failure probabilities of g_1 and g_2 at the DO optimum are much improved at both the RDO and RBDO optima. It is revealed that RBDO yields a better optimal solution than RDO in terms of the reliability of the two constraint functions considered.

In Fig. 6, the dimensions of the three optimized magnets are compared with each other with respect to the initial ones. As shown in the figure, the distance between the two windings resulting from the initial and DO designs is too small to be fabricated in practice. On the other hand, both of RDO and RBDO optima produce more acceptable results by improving the probabilistic feasibility of constraints.

5. Conclusion

A reliability-based design optimization based on the reliability index approach has been successfully applied to the SMES design. The results reveal that the proposed method provides an optimum satisfying the specified confidence level in the presence of uncertain parameters.

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