

## The Dumb-bell Shaped Magnetostrictive/Piezoelectric Transducer

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**Traditional magnetostrictive/piezoelectric laminate composites are generally in the regular geometries such as rectangles or disks. To explore properties of the irregular geometry magnetostrictive/piezoelectric transducer in the fundamental resonant frequency, a step dumb-bell shaped Magnetostrictive (ME) transducer is presented in this study. Both analytical and experimental investigations are carried out for the dumb-bell shaped transducer in the fundamental frequency. Comparing with the traditional rectangular transducer, the theory shows the resonant frequency of dumb-bell shaped transducer is reduced 31%, and the experiment gives the result of that is 37% which is independent of dc magnetic fields. The ratio of magnetostrictive voltage coefficient (MEVC) between the dumb-bell shaped and rectangular shaped transducers in theory is 66% comparing with that of in experiment is varying from 140% to 33% when the dc field is increased from 0 Oe to 118 Oe.**

**Keywords :** Magnetostrictive (ME) effect, magnetostrictive/piezoelectric transducer, Ferromagnetic constant-elasticity alloy (FCEA), Polyvinylidene fluoride (PVDF), Magnetostrictive voltage coefficient (MEVC), fundamental resonant frequency

### 1. Introduction

The magnetostrictive (ME) effect is an electric polarization response to an applied magnetic field  $H$  (i.e., the direct ME effect), or a magnetization response to an applied electric field  $E$  (i.e., the converse ME effect). Magnetostrictive laminate composites are investigated for their giant ME effects at room temperature [1, 2]. The ME voltage of the Magnetostrictive/Piezoelectric laminated composites is arisen from the product property of the magnetostrictive phase and piezoelectric phase. The ME effect is usually described by ME voltage coefficient (MEVC) which is estimated from  $\alpha_{ME} = \delta E / \delta H = \delta V / (t_p \delta H)$ , where  $t_p$  is the thickness of the piezoelectric phase along the polarized direction,  $\delta H$  is the applied ac magnetic field,  $\delta V$  is the induced ME voltage.

The ME effect has a dramatic resonance enhancement at magneto-acoustic resonance. The MEVC in resonance is 1-2 orders of magnitude over that of nonresonant states [3, 4]. So the prior choice is to has the ME transducer operated in resonant states. The fundamental frequency of the transducer is inversely proportional to its geometry

size, which suggests that a lower resonant frequency needs a larger size. This is intolerable in some occasions. Applications need to decrease the fundamental resonant frequencies of ME transducers at a given size. A few such investigations have been performed. Wan, J. G. *et al.* present the unsymmetrical bilayer laminate composites which operated in the bending resonance mode [5]. Zhai, J. Y. *et al.* present unsymmetrical magnetic bias for the symmetric bimorph composite which enables it operated in a bending mode [6]. Mingsen Guo *et al.* adopt the annual bilayer composites which operated in a much lower bending mode comparing with the radical mode [7]. All these investigations are focused on the bending mode. It has not been published that investigations on the decrease of the resonance frequency for the ME transducer which operated in longitudinal mode. In this study, the traditional rectangular shaped ME transducer (Fig. 1(a)) is modified into a dumb-bell shaped one (Fig. 1(b)). This modification enabled the transducer to operate in a lower fundamental frequency without size extension or changing operating mode. This is proved by the theory and experiments. The influences of magnetic bias on MEVCs are investigated by experimental method, which is neglected in the theory analysis. The experiments show that the dumb-bell shaped transducer has higher MEVC at

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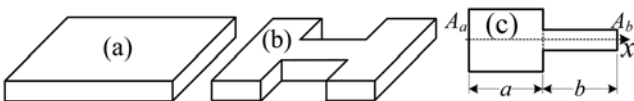
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weak magnetic bias and lower MEVC at strong magnetic bias than the rectangular shaped transducer.

## 2. The Dumb-bell Shaped ME Transducer

Regular geometry structures are the preferred form in the design of ME laminate transducers to simplify theory analysis and fabrications. The step dumb-bell shaped ME transducer (Fig. 1(b)) is designed as one irregular geometry structure to decrease the fundamental frequency. Half of the transducer (Fig. 1(c)) is considered for the magnetic-mechanical vibrations based on the geometrical symmetry. It operates in one quarter wave mode. The narrower end is fixed and the wider end is free. The cross-sectional area of the wider end is  $A_a = t_m w_a$ , and that of the narrower end is  $A_b = t_m w_b$ , where,  $t_m$  is the thickness of the transducer,  $a$  and  $b$  are the lengths of the wider end and narrower end, respectively.  $w_a$  and  $w_b$  are the widths of the wider end and narrower end, respectively. The scale factor of the shape is defined as  $r = A_a/A_b = w_a/w_b$ . When  $r = 1$ , the transducer is the rectangular shaped one. Its length is  $l = 2(a + b)$ .

For easy manufacturing, a type of ferromagnetic constant-elasticity alloy (FCEA) is chosen as the piezomagnetic phase [8]. The FCEA is commercially available (Chongqing Instrument Material Research Institute, China). The Polyvinylidene fluoride (PVDF) is chosen as the piezoelectric phase for its soft elastic constants, high resistivity, high piezoelectric voltage coefficient, and low density [9, 10]. The PVDF film is commercially available also (Jinzhou KeXin electronic materials Co. Ltd., China). Firstly, the FCEA and PVDF are processed into  $27 \text{ mm} \times 9 \text{ mm} \times 0.6 \text{ mm}$  and  $27 \text{ mm} \times 9 \text{ mm} \times 30 \text{ }\mu\text{m}$  rectangular shaped solids, respectively. The longitudinal direction of the FCEA plate is along its magnetized direction. Secondly, the rectangular shaped FCEA and PVDF are further processed into the dumb-bell shaped solids by cutting off two  $9 \text{ mm} \times 3 \text{ mm}$  small rectangular patches at the middle of both longitudinal edges. So the parameters of the designed dumb-bell shaped transducer are  $a = 9 \text{ mm}$ ,  $b = 4.5 \text{ mm}$ , and  $r = 3$ . The rectangular and dumb-bell shaped Magnetostrictive/Piezoelectric bilayer transducers are fabricated by laminating one FCEA plate and one PVDF film together with epoxy binder. The load influence



**Fig. 1.** The structures of transducers. (a) The rectangular shaped structure, and (b) the dumb-bell shaped structure, and (c) the dimensions of the half dumb-bell shaped structure.

of the PVDF on the FCEA is neglectable due to the low density, small stiffness and thickness of the PVDF. The ME transducers operate in longitudinal mode and their vibrate characters are determined by the FCEA.

## 3. Theoretical Analysis

The basic idea of analyze the transducer is firstly to establish the wave equation by the Newton's law. Then the fundamental resonant frequency and the solving of the wave equation can be derived. The induced ME voltage can be calculated from the piezoelectric constitutive equations and the wave solution. The MEVC is deduced finally.

As described in Fig. 1(c), the axial line of the transducer is set as  $x$  axis. The intersecting point of the step cross section and the axial line is chosen as the original point. The width direction is chosen as  $y$  axis, and the thickness direction is chosen as  $z$  axis. In the rectangular coordinate system, the linear magnetostrictive constitutive equations of the FCEA are

$$S_{1,m} = S_{11,m}^H T_{1,m} + d_{11,m} H_1 \quad (1)$$

$$B_1 = d_{11,m} T_{1,m} + \mu_{11}^T H_1 \quad (2)$$

where  $S_{1,m}$  and  $T_{1,m}$  are the strain and stress along the longitudinal direction,  $H_1$  and  $B_1$  are the magnetic field and magnetic field flux density,  $S_{11,m}^H$ ,  $d_{11,m}$  and  $\mu_{11}^T$  are the elastic compliance at constant  $H$ , longitudinal effective piezomagnetic coefficient, and magnetic permeability at constant stress, respectively. The linear piezoelectric constitutive equations of the PVDF film are

$$S_{1,p} = s_{11,p}^E T_{1,p} + d_{31,p} E_3 \quad (3)$$

$$D_3 = d_{31,p} T_{1,p} + \epsilon_{33}^T E_3 \quad (4)$$

where  $S_{1,p}$  and  $T_{1,p}$  are the mechanical strain and stress along the longitudinal direction,  $E_3$  and  $D_3$  are the electric field strength and electric displacement along the thickness direction,  $s_{11,p}^E$ ,  $d_{31,p}$  and  $\epsilon_{33}^T$  are the elastic compliance coefficient at constant electric field, piezoelectric voltage coefficient, and dielectric permittivity at constant stress, respectively. The vibration analysis of the transducer can be simplified into the vibration analysis of the piezomagnetic phase after neglect the load of the PVDF. The piezoelectric phase and the piezomagnetic phase are assumed to have the same displacement and strain distributions. These assumptions are used in the following analysis.

It is assumed that the applied ac magnetic field  $H_1$  is sinusoidal, and the transducer has the corresponding harmonic vibration along the longitudinal direction with

angle frequency  $\omega$ . When the harmonic time term is neglected, the wave equation of the even cross-section plate can be deduced by Newton's law from the constitutive equation (1):  $\partial^2 u / \partial x^2 + k^2 u = 0$ , where  $u$  is the displacement along  $x$  axis;  $k = \omega/c$  is the wave number;  $c = 1/\sqrt{\rho s_{11,m}^H}$  is the longitudinal wave velocity;  $\rho$  is the density of the FCEA. The structure of Fig. 1(c) can be viewed as two even parts, and its wave solution can be written as

$$\begin{cases} u_a = A_1 \cos(kx) + B_1 \sin(kx) & (-a < x < 0) \\ u_b = A_2 \cos(kx) + B_2 \sin(kx) & (0 < x < b) \end{cases} \quad (5)$$

The boundary conditions can be written as:

$$A_a T_{1,m}|_{x=-a} = 0, u_b|_{x=b} = 0 \quad (6)$$

The link conditions can be written as:

$$u_a|_{x=0} = u_b|_{x=0}, A_a T_{1,m}|_{x=0} = A_b T_{1,m}|_{x=0} \quad (7)$$

When  $H_1=0$ , the frequency equation of the dumb-bell shaped structure can be written as follows according equations (5)-(7):

$$\tan(ka) \tan(kb) = r \quad (8)$$

When  $r=1$ , equation (8) is simplified into the frequency equation of the rectangular shaped structure which is the same as the classical theory derived:

$$\cos(ka + kb) = 0 \quad (9)$$

Equation (8) gives the fundamental frequency  $f_D$  of the step dumb-bell shaped transducer, and equation (9) gives the fundamental frequency  $f_R$  of the rectangular shaped one. The percentage of frequency decrease is:

$$\eta_f = (f_R - f_D) / f_R \quad (10)$$

The derivation procedure demonstrated that the percentage of frequency decrease  $\eta_f$  is only related to the transducer's geometry parameters and longitudinal wave velocity in the FCEA.

The wave solution of the dumb-bell shaped ME transducer can be solved from equation (1), (5)-(7)

$$\begin{cases} u_a = \frac{\cos(kb)\sin(kx) - r\sin(kb)\cos(kx) + (r-1)\sin(kb)\cos(ka+kx)}{k\Delta} d_{11,m} H_1 & (-a < x < 0) \\ u_b = \frac{r\sin(kx-kb) + (r-1)\cos(ka)\sin(kb-kx)}{k\Delta} d_{11,m} H_1 & (0 < x < b) \end{cases} \quad (11)$$

where  $\Delta = \cos(kb)\sin(kb) - r\sin(ka)\sin(kb)$ .

By eliminating the stress in equations (3) and (4), it is given

$$s_{11}^E D_3 = d_{31,p} S_{1,p} + (s_{11}^E \varepsilon_{33}^T - d_{31,p}^2) E_3 \quad (12)$$

Under the open circuit condition, integrating equation (12) over the area of the electrode, we get

$$d_{31,p} \iint_A S_{1,p} dA + (s_{11}^E \varepsilon_{33}^T - d_{31,p}^2) \iint_A E_3 dA = 0 \quad (13)$$

After calculate and simplify, the analytic expression of the MEVC is obtained as follows.

$$\alpha_{ME} = \frac{r\sin(ka-kb) + (2r^2 - r + 1)\cos(ka)\sin(kb) + 2r(1-r)\sin(kb)}{k[\Delta + i\Gamma](ar+b)} \frac{d_{31,p} d_{11,m}}{(d_{31,p}^2 - s_{11}^E \varepsilon_{33}^T)} \quad (14)$$

where,  $\Gamma$  is the damping parameter [3],  $i$  is the unit of imaginary number.

Equation (14) gives the MEVC  $\alpha_{ME}^D$  of the dumb-bell shaped ME transducer at fundamental frequency. When  $r=1$ , the MEVC  $\alpha_{ME}^R$  of the rectangular shaped one can be calculated also. The ratio of MEVCs between the dumb-bell shaped and rectangular shaped ME transducers in resonance can be calculated as

$$\eta_V = \alpha_{ME}^D / \alpha_{ME}^R \quad (15)$$

If material parameters of the two kinds of transducers are the same, the calculation of  $\eta_V$  based on equation (14) and (15) only related to geometry parameters and wave velocity.  $\eta_f$  and  $\eta_V$  are comparable results between the two kinds of transducers.

The geometry parameters have been given in part 2. The parameter  $\Gamma$  is given by experiments. Material parameters of the FCEA used in calculations are [8]:  $s_{11,m}^H = 5.37 \times 10^{-12} \text{ m}^2/\text{N}$ ,  $\rho = 8 \times 10^3 \text{ kg/m}^3$ ,  $s_{11}^E = 5 \times 10^{-10} \text{ m}^2/\text{N}$ ,  $d_{11,m} = 0.75 \text{ nm/A}$ . The parameters of the PVDF are:  $d_{31,p} = 25 \text{ pC/N}$ ,  $\varepsilon_{33}^T / \varepsilon_0 = 10$ ,  $\varepsilon_0$  is the dielectric constant in vacuum. It can be calculated that  $\eta_f \approx 31\%$  and  $\eta_V \approx 66\%$ . The results show that the frequency decrease is at the cost of the MEVC decrease. The theoretical MEVC spectra of transducers are shown in Fig. 2 and compared with the

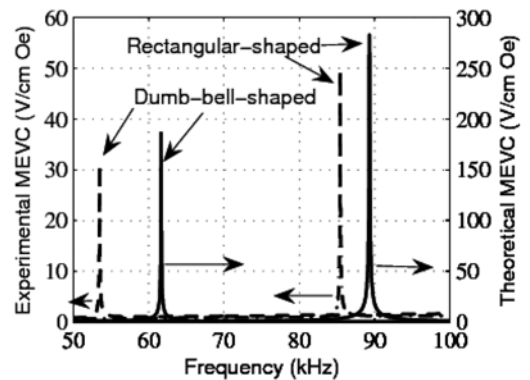


Fig. 2. The comparing of theoretical (right axis) and experimental (left axis) results for transducers. The dc magnetic field is 30 Oe during the given experiment.

experimental results at magnetic bias of 30 Oe. The magnetic bias has influence on the physical parameters which determined the performance of the ME transducer [11, 12]. The following experiments will show the magnetic bias influences on above results.

### 4. Experimental Results

The design and fabrication of the ME transducers have been mentioned in details in part 2. To verify the theoretical results in part 3, the resonant frequencies and MEVCs of the designed ME transducers are measured at different magnetic bias field. During ME measurements, dc and ac magnetic fields are applied along the longitudinal direction of the laminates. A pair of permanent annular magnets are used to provide  $H_{dc} = 0-118$  Oe, and a solenoid is used to generate  $H_{ac} = 1$  Oe. A lock-in amplifier is used to generate a controllable input current to the solenoid at vary frequencies, and at the same time it is used to detect the induced ME voltage by difference measurement.

Fig. 2 shows the experimental and theoretical MEVC spectra of transducers. The experimental spectrum of amplitude is measured at dc field of 30 Oe. Due to the neglecting of load effects of the PVDF and epoxy adhesive in theory analysis, both theoretical values of the MEVCs and resonant frequencies are higher than that of the experimental results. But the ratio  $\eta_V$  in theory or in experiment for the two kinds of transducers is almost the same value.

Physical parameters of the ME transducers are related to dc magnetic fields. For example, the effective piezomagnetic coefficient is the function of dc magnetic bias fields [13]. It is necessary to consider the influences of magnetic bias on the resonant frequencies and MEVCs. This will give a more comprehensive view about the comparisons between the two kinds of transducers. The comparisons of experimental results are shown in Fig. 3 and Fig. 4.

Fig. 3 shows that the resonant frequencies have shifted only 20-40 Hz when dc fields varied from 0 Oe to 118 Oe. The frequency shift is caused by the Delta-E effect [14, 15]. The slight different of resonant frequencies have little influence on the frequency decrease of the dumb-bell shaped transducer, and  $\eta_V$  is the constant value 37% which is a little higher than previous calculation. This demonstrated that the resonant frequency is mainly determined by its geometry sizes and boundary conditions. As magnetic bias increased from 0 Oe to 43 Oe, the resonant frequency shift tendency of the transducers are the same. As the magnetic bias getting stronger, the shift tendency

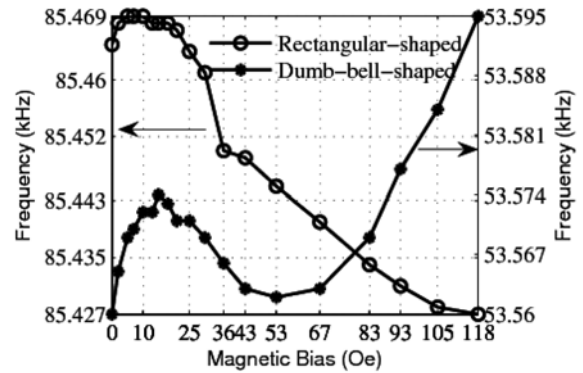


Fig. 3. Magnetic bias field dependence of the resonant frequencies for transducers.

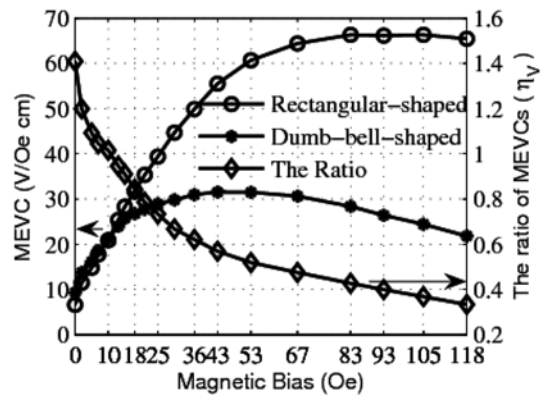


Fig. 4. Magnetic bias field dependence of the MEVC magnitudes in resonance for transducers.

is inverted.

Fig. 4 shows that the varieties of MEVCs at different dc fields are significant for both transducers. As the magnetic bias fields increasing from 0 Oe to 118 Oe, the MEVCs of both structures are increased to their maximum values, and the ratio  $\eta_V$  decreased monotonously from 140% to 33%. Fig. 4 shows  $\eta_V = 84.4\%$  when the magnetic bias is 18 Oe. At dc field of 30 Oe, the experimental value of  $\eta_V$  agreed well with the previous calculations as shown in Fig. 2. The rapid change of  $\eta_V$  demonstrated that the effective piezomagnetic coefficients of the two structures have different change rates with vary dc fields. The ratio of MEVCs is the results of the ratio of effective piezomagnetic coefficients. At strong dc fields, the MEVC of the dumb-bell shaped transducer has a sharp decrease comparing with the rectangular shaped one. This is because the dumb-bell shaped transducer has a higher demagnetization factor. The demagnetization factor determined the variety of the effective piezomagnetic coefficients and effective magnetic fields [16]. The strong dc fields caused the decrease of the effective piezomagnetic coefficients

and effective magnetic fields which caused the decrease of the MEVC.

The theory analysis indicates that the decrease of the fundamental frequency of the dumb-bell shaped transducer is accompanied with the decrease of the MEVC. But as experimental results shown in Fig. 4, the dumb-bell shaped transducer has higher MEVC than the rectangular one at weak magnetic bias. So the irregular transducer has lower resonant frequency and higher MEVC when magnetic bias is absent or weak.

## 5. Conclusions

This paper analyzed and compared the fundamental resonant frequencies and MEVCs of the step dumb-bell shaped and rectangular shaped ME transducers. Both theory analysis and experiments show that the dumb-bell shaped transducer could decrease the resonant frequency more than 31% comparing with the rectangular shaped one without size extension. This gives a new method for miniaturize ME transducers. The MEVC of the dumb-bell shaped transducer is almost equal to that of rectangular one at dc field less than 18 Oe. The theory and experiments are expected to extend for analyzing other irregular geometry structures such catenoid dumb-bell or butterfly shaped transducers [17]. These irregular structures are expected to have their own advantages.

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