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A Hybrid ON/OFF Method for Fast Solution of Electromagnetic Inverse Problems Based on Topological Sensitivity

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A new hybrid ON/OFF method is presented for the fast solution of electromagnetic inverse problems in high frequency domains. The proposed method utilizes both topological sensitivity (TS) and material sensitivity (MS) to update material properties in unit design cells. MS provides smooth design space and stable convergence, while TS enables sudden changes of material distribution when MS slows down. This combination of two sensitivities enables a reduction in total computation time. The TS and MS analyses are based on a variational approach and an adjoint variable method (AVM), which permits direct calculation of both sensitivity values from field solutions of the primary and adjoint systems. Investigation of the formulations of TS and MS reveals that they have similar forms, and implementation of the hybrid ON/OFF method that uses both sensitivities can be achieved by one optimization module. The proposed method is applied to dielectric material reconstruction problems, and the results show the feasibility and effectiveness of the method.

Keywords : inverse problems, ON/OFF method, topological sensitivity

1. Introduction

Electromagnetic inverse problems with high resolution grids require a very long time to solve when stochastic methods are used. Recently, the material sensitivity (MS) formulation based on a variational approach was derived for use in the high frequency domain [1]. Combined with deterministic optimization methods, MS can achieve a considerable decrease in the computation time for the electromagnetic inverse problem when compared to stochastic methods. However, when MS is used, material properties such as permittivity can vary continuously between two extreme values (empty space and real material). This allows a smooth design space, but also introduces intermediate materials which are often represented as gray cells in the material distribution. In many practical cases, the presence of these intermediate materials often leads to a slow convergence of the objective function, though it is still faster than stochastic methods. As such, these cells have to be removed at the end using specific techniques such as a boundary identification method.

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To avoid these grayscale elements, an ON/OFF method has been used for optimal designs in the low-frequency domain [2-4]. In this method, the sign of material sensitivity in each cell is used to determine the ON/OFF status of that cell. Although the ON/OFF methods have been shown to be effective for low-frequency systems, they require the sorting of elements in the order of sensitivity, and an annealing loop in a certain iteration to determine which elements are to be changed. Also, they have not been applied to problems in high frequency domains.

The concept of topological sensitivity (TS) was first introduced by Schumacher *et al.* in 1994 [5]. The physical meaning of TS is totally different from that of MS. TS gives information on the opportunity to create a small hole in the design domain, whereas MS represents how the incremental change of material property affects the objective function. Cea *et al.* applied the concept to a high frequency domain Helmholtz equation in 2000 [6], and Masmoudi *et al.* extended the formulation for a hole with a dielectric boundary, as well as one with a metallic boundary, and provided inverse problem examples [7]. For electrostatic problems, Kim *et al.* combined topological sensitivity with shape sensitivity for smooth boundary optimization [8].

When considering TS, the problem domain is composed of either real material or empty space (no intermediate materials), which is well suited to an ON/OFF method. Thus, TS can be used as the on/off criteria for each cell, and a rapid change of material distribution is possible. However, due to limited design space, the fluctuations of the objective function can occur when TS alone is used for material update.

In this paper, we propose a new hybrid ON/OFF method for the fast and accurate solution of the electromagnetic inverse problems in the high frequency domain. The proposed method utilizes both MS and TS in its updating scheme, and is different from a conventional ON/OFF method where either one of MS or TS would be used for the cell update criterion. Also, the proposed method does not require an annealing loop or sorting of elements. The sensitivity analysis of this hybrid ON/OFF method is based on the variational approach, which means the sensitivity calculation does not depend on any specific analysis method. Thus, commercial EM simulation software can be used as long as the adjoint source can be applied accurately, whereas in the conventional methods using discrete sensitivity analysis, a custom EM solver is required to get the derivative of the system matrix (FEM or FDTD).

The structure of the paper is as follows. First, the concepts and formulations for material sensitivity and topological sensitivity in the high frequency domain will be reviewed. Next, a short comparison of the final forms of the MS and TS is given along with some comments. Then, the procedures for the hybrid ON/OFF method using both sensitivities will be explained in the context of their actual implementation. Next, two numerical examples will be presented for proof-of-concept, and the results of the hybrid ON/OFF method will be compared with those obtained by MS alone. Finally, some discussion of the results and conclusion will be given.

2. Sensitivity Formulations

2.1. Material sensitivity in the high frequency domain

Material sensitivity in the high frequency domain can be derived based on a variational approach and the augmented Lagrangian method (ALM) as follows [1].

Fig. 1 shows a design domain Ω defined on R^n ($n = 2$ or 3) with boundary Γ. Consider an objective function $\Psi_{obj}(\mathbf{H}, \mathbf{p})$ defined on the boundary Γ_g as follows:

$$\Psi_{obj}(\mathbf{H}, \mathbf{p}) = \int_{\Gamma_g} g(\mathbf{H}(\mathbf{p}))d\gamma \quad (1)$$

where g is a scalar function differentiable with respect to

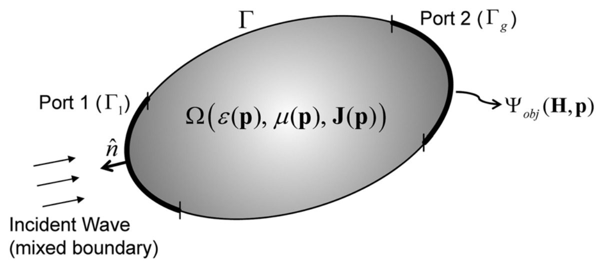

Fig. 1. Definition of the design domain Ω and boundary Γ (primary system).

$\mathbf{H}$, $\mathbf{H}$ is the magnetic field, which is also the state variable of the system and is itself a function of the design parameter vector $\mathbf{p}$, which defines the material properties such as permittivity ε, permeability μ, or current density $\mathbf{J}$ of the design domain. Using the ALM, a new objective function $\overline{\Psi}_{obj}$ is defined by adding the variational form of the vector wave equation to (1) as,

$$\overline{\Psi}_{obj}(\mathbf{H},\mathbf{p}) = \int_{\Gamma_g} g(\mathbf{H}(\mathbf{p}))d\gamma + \\ + \int_{\Omega} \lambda\cdot[-\nabla\times(\varepsilon_r^{-1}\nabla\times\mathbf{H})+k_0^2\mu_r\mathbf{H}]d\omega . \quad (2)$$

where λ is the Lagrange multiplier vector, which is, at the same time, interpreted as an adjoint variable vector. Manipulating (2) with vector calculus and assuming that a boundary condition of the third kind is defined on Γ_1 as,

$$\varepsilon_r^{-1}\hat{n}\times(\nabla\times\mathbf{H})+\gamma_h\hat{n}\times(\hat{n}\times\mathbf{H})=\mathbf{V} , \quad (3)$$

where γ_h and $\mathbf{V}$ are known parameters, we obtain,

$$\Psi_{obj}(\mathbf{H}, \mathbf{p}) = \int_{\Gamma_g} g(\mathbf{H}(\mathbf{p}))d\gamma \\ +\int_{\Omega} [-\varepsilon_r^{-1}(\nabla\times\mathbf{H})\cdot(\nabla\times\lambda)+k_0^2\mu_r\mathbf{H}\cdot\lambda]d\omega \\ + \int_{\Gamma_1} [\gamma_h(\hat{n}\times\lambda)\cdot(\hat{n}\times\mathbf{H})+\lambda\cdot\mathbf{V}]d\gamma. \quad (4)$$

Assuming λ is the solution of the adjoint system, which is given in a variational form as,

$$\int_{\Omega}[-\varepsilon_r^{-1}(\nabla\times\overline{\lambda})\cdot(\nabla\times\lambda)+k_0^2\mu_r\overline{\lambda}\cdot\lambda]d\Omega \\ - \int_{\Gamma_1}[\gamma_h(\hat{n}\times\overline{\lambda})\cdot(\hat{n}\times\lambda)+\overline{\lambda}\cdot\mathbf{g}_{\mathbf{H}}]d\Gamma=0 , \quad (5)$$

where $\mathbf{g}_{\mathbf{H}}\equiv\partial g/\partial\mathbf{H}$ is the adjoint source and $\overline{\lambda}$ is an arbitrary vector, then the material sensitivity equation is finally given as,

$$\frac{d\overline{\Psi}_{obj}}{d\mathbf{p}} = \mathrm{Re}\int_{\Omega}\left[-\frac{\partial(\varepsilon_r^{-1})}{\partial\mathbf{p}}(\nabla\times\mathbf{H})\cdot(\nabla\times\lambda)+k_0^2\frac{\partial\mu_r}{\partial\mathbf{p}}\mathbf{H}\cdot\lambda\right]d\Omega . \quad (6)$$

2.2. Topological sensitivity in the high frequency domain

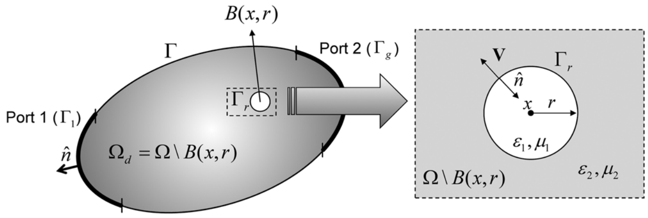

Fig. 2. A design domain $\Omega_d = \Omega \backslash B(x,r)$ with a small hole $B(x, r)$ and an enlarged figure of $B(x, r)$.

Consider a small hole $B(x, r)$ with boundary Γ_r in a design domain Ω, where x is the location of the hole center and r is the radius of the hole (Fig. 2). The topological gradient $G(x)$ is then defined as,

$$G(x) = \lim_{r \to 0} \frac{\Psi_{obj}(\Omega \backslash B(x,r)) - \Psi_{obj}(\Omega)}{\delta(\Omega)} \tag{7}$$

where $\Omega \backslash B(x,r)$ is the domain excluding the small hole B, and $\delta(\Omega)$ is the volume of B with a negative sign. If we introduce a scalar function $J(r) = \Psi_{obj}(\Omega \backslash B(x,r))$, then its derivative with respect to the hole radius r can be calculated from the shape sensitivity formula based on a variational approach as [9],

$$dJ(r)/dr = -\int_{\Gamma_r} L(\mathbf{H},\lambda)\hat{n}d\Gamma \tag{8}$$

$$L(\mathbf{H},\lambda) = \mathrm{Re}\{-\omega^2 \varepsilon_0(\varepsilon_2 - \varepsilon_1)(E_{1n}E_{2n}(\lambda) - E_{1t}E_{2t}(\lambda)) \\ -k_0^2(\mu_{r2} - \mu_{r1})(H_{1n}\lambda_{2n} - H_{1t}\lambda_{2t})\}, \tag{9}$$

where λ is the adjoint field from the same adjoint system as in equation (5), the material sensitivity case. There is a minus sign in (8) because the normal vector $\hat{n}$ and velocity vector $\mathbf{V}$ are in the opposite direction, as shown in Fig. 2.

Now, to obtain the limit of $J(r) - J(0)$ as r approaches zero, local asymptotic expansions of $\mathbf{H}$ and λ at x are used along with (8) and (9), which yields the topological expansion in the 3-D case as,

$$J(r) - J(0) = \\ 4\pi r^2 \mathrm{Re}\left\{ \frac{\varepsilon_{r2} - \varepsilon_{r1}}{\varepsilon_{r2}(\varepsilon_{r1} + 2\varepsilon_{r2})} (\nabla \times \mathbf{H}(x)) \cdot (\nabla \times \lambda(x)) \right\} + o(r^3), \tag{10}$$

where $o(r^3)$ is the error term, and terms relating to the permeability difference are ignored [7]. Substituting (10) into (7), the topological sensitivity $G(x)$ is finally derived as,

$$G(x) = -\mathrm{Re}\left\{ \frac{\varepsilon_{r2} - \varepsilon_{r1}}{\varepsilon_{r2}(\varepsilon_{r1} + 2\varepsilon_{r2})} (\nabla \times \mathbf{H}(x)) \cdot (\nabla \times \lambda(x)) \right\}. \tag{11}$$

2.3. Comparison of material and topological sensitivities

Upon examining (6) and (11), it can be seen that the material and topological sensitivities have very similar forms to each other when the permeability difference terms are ignored. This is often the case in high frequency domain electromagnetic inverse problems. The essential $(\nabla \times \mathbf{H}) \cdot (\nabla \times \lambda)$ terms are common to the two sensitivities, and they can be calculated directly from the same primary and adjoint fields ($\mathbf{H}$ and λ). The main difference comes from the permittivity coefficients. For MS, the $-\partial(\varepsilon_r^{-1})/\partial\mathbf{p}$ term can be calculated from the equation that defines the relationship between the permittivity and the design parameter $\mathbf{p}$. In this paper, the relative permittivity in a cell $\varepsilon_{r_{\text{cell}}}$ is given as follows [1],

$$\varepsilon_{r_{\text{cell}}} = \varepsilon_{r_{\min}} (\varepsilon_{r_{\max}} / \varepsilon_{r_{\min}})^p, \quad (0 \le p \le 1) \tag{12}$$

where $\varepsilon_{r_{\min}}$ and $\varepsilon_{r_{\max}}$ are the minimum and maximum value of relative permittivity in the design cell, respectively, and p can be interpreted as the normalized material density of the cell. For TS, the coefficient in (11) can be directly calculated from the relative permittivity values of the solid material and the hole.

The other difference is that the MS is calculated by the integration of $(\nabla \times \mathbf{H}) \cdot (\nabla \times \lambda)$ over each cell domain, whereas TS is given as a point-wise function at x. However, the integral in (6) can be calculated easily in most cases. Thus, MS and TS can be computed in one simple subroutine, which makes it relatively straightforward to implement the hybrid ON/OFF method that utilizes both sensitivities.

3. Hybrid ON/OFF Method

To improve the slow convergence of the deterministic optimization method when using MS alone, a hybrid ON/OFF method is proposed. The basic step-by-step procedure is as follows:

Step 1. Proceed with deterministic optimization method using MS, and assign continuous material properties to design cells,

Step 2. For every mth iteration, calculate $G(x)$ at the center of each cell,

Step 3. Create a hole at the location of the cell or fill the cell with the solid material according to $G(x)$,

Step 4. Check the convergence criteria and repeat from step 1.

For step 3, we present a more detailed explanation. As a result of prior iterations using MS, the material property of the design cell of interest, at the current iteration, can

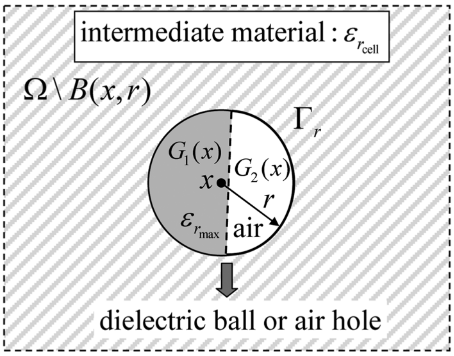

Fig. 3. Design cell with intermediate material properties (case 3).

be categorized into one of the following 3 cases,

Case 1: The cell is currently ON (filled with real dielectric material with $\varepsilon_r = \varepsilon_{r_{max}}$). Substituting $\varepsilon_{r1} = 1$ and $\varepsilon_{r2} = \varepsilon_{r_{max}}$ into (11), $G(x)$ is given by,

$$G(x) = -3\frac{(\varepsilon_{r_{max}}-1)}{\varepsilon_{r_{max}}(1+2\varepsilon_{r_{max}})}(\nabla\times\mathbf{H}(x))\cdot(\nabla\times\lambda(x)). \quad (13)$$

If $G(x) \geq 0$, $\Psi_{ojb}(\Omega)$ will decrease when a very small hole is created. Thus, an empty hole should be created (the cell is turned OFF) at the current location. Otherwise, current cell should remain as solid material (ON status).

Case 2: The cell is currently OFF (filled with air). Substituting $\varepsilon_r = \varepsilon_{r_{max}}$ and $\varepsilon_{r2} = 1$ into (11), $G(x)$ is given by,

$$G(x) = -3\frac{(1-\varepsilon_{r_{max}})}{(\varepsilon_{r_{max}}+2)}(\nabla\times\mathbf{H}(x))\cdot(\nabla\times\lambda(x)). \quad (14)$$

If $G(x) \geq 0$, the cell should be filled with a dielectric with $\varepsilon_r = \varepsilon_{r_{max}}$ (cell is turned ON). Otherwise, the current cell should remain as air (OFF status).

Case 3: The cell is currently filled with intermediate material ($\varepsilon_r = \varepsilon_{r_{cell}}$, $1 < \varepsilon_{r_{cell}} < \varepsilon_{r_{max}}$). For this case, 2 topological sensitivities should be calculated (Fig. 3),

$$G_1(x) = -3\frac{(\varepsilon_{r_{cell}}-\varepsilon_{r_{max}})}{\varepsilon_{r_{cell}}(\varepsilon_{r_{max}}+2\varepsilon_{r_{cell}})}(\nabla\times\mathbf{H}(x))\cdot(\nabla\times\lambda(x)),$$
$$G_2(x) = -3\frac{(\varepsilon_{r_{cell}}-1)}{\varepsilon_{r_{cell}}(1+2\varepsilon_{r_{cell}})}(\nabla\times\mathbf{H}(x))\cdot(\nabla\times\lambda(x)). \quad (15)$$

If $G(x) \geq 0$, the cell should be filled with a dielectric with $\varepsilon_r = \varepsilon_{r_{max}}$ (cell is turned ON). Otherwise if ($G_2(x) > 0$), the cell should be air (cell is turned OFF).

4. Numerical Examples

Two inverse problem examples of dielectric material reconstruction are investigated for verification of the feasibility and effectiveness of the proposed method. The objective function is given by,

$$\Psi_{ojb} = \sum_{j=1}^{2}\sum_{i=1}^{nf}\left[\int_{\Gamma_j}\left|S_{jj}^{(i)}-S_{jj0}^{(i)}\right|^2 d\Gamma\right] \quad (16)$$

where $S_{jj}^{(i)}$ is the calculated value, and $S_{jj0}^{(i)}$ is the measured or target value of the S-parameter for ith frequency point, Γ_j is the boundary of the jth port, and nf is the total number of the frequency points. The scattering parameters (S-parameters) are defined in relation to incident and reflected fields at the ports. For example, S_{11} can be defined as

$$S_{11} = \frac{\mathbf{H}_1^{ref}}{\mathbf{H}_1^{inc}} = \frac{\mathbf{H}_1^{tot}-\mathbf{H}_1^{inc}}{\mathbf{H}_1^{inc}} \quad (17)$$

where $\mathbf{H}_1^{inc}$, $\mathbf{H}_1^{ref}$, and $\mathbf{H}_1^{tot}$ are incident, reflected, and total magnetic fields at port 1, respectively.

For these examples, we use $nf = 27$ with frequency points between 0.3-3 GHz. Two source conditions are considered. These are dominant mode incident waves of unit magnitude at port 1 and 2, for the calculation of S_{11} and S_{22}, respectively. It should be noted that since the objective function is given in terms of S_{11} and S_{22}, there is no need to solve the adjoint system, and λ can be directly obtained from the primary system using the self-adjoint formulation given in [1]. This procedure can be summarized as follows:

(a) Solve the primary system and obtain the field solution vector $\mathbf{H}$.

(b) For the given goal function, (16), calculate the adjoint source term $\mathbf{g_H}$.

(c) Obtain the adjoint field λ by multiplying $\mathbf{H}$ by $|\mathbf{g_H}|/|\mathbf{V}|$.

(d) Calculate the sensitivity vector $d\Psi_{ojb}/d\mathbf{p}$ using (6).

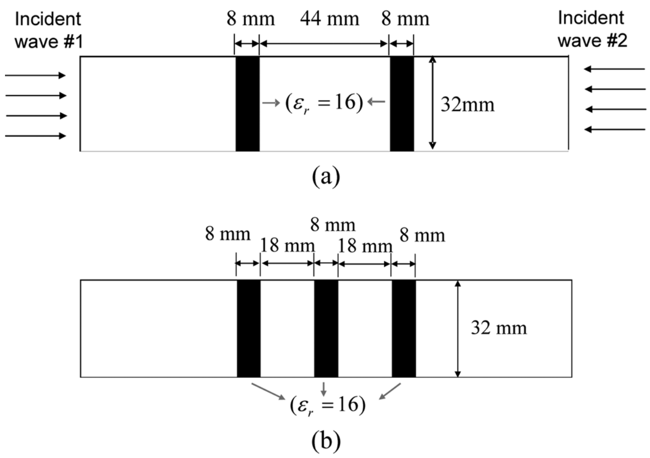

Fig. 4. Target dielectric distribution. (a) Model 1. (b) Model 2.

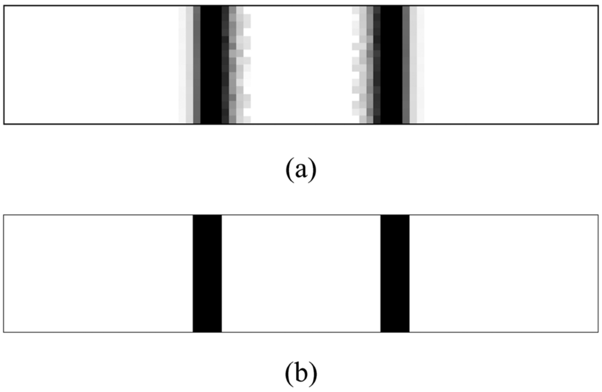

Fig. 5. Reconstruction results for model 1. (a) Material sensitivity method (50 iterations). (b) Hybrid ON/OFF method (20 iterations, exact reconstruction).

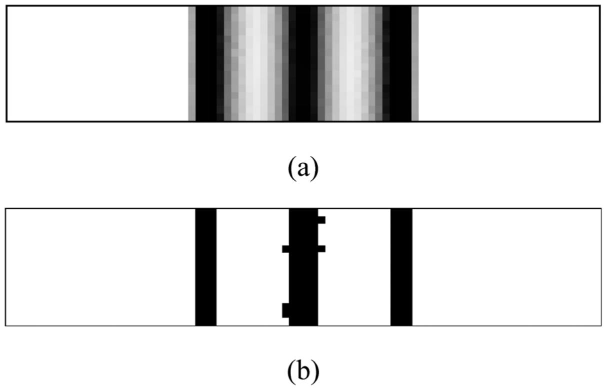

Fig. 6. Reconstruction results for model 2. (a) Material sensitivity method (200 iterations). (b) Hybrid ON/OFF method (5 iterations).

The target material distributions for two models are shown in Fig. 4. Model 1 has two dielectric walls with $\varepsilon_r = 16$ and model 2 has three walls. The TS interval m is 10 for model 1 and 5 for model 2. The reconstruction results are shown in Fig. 5 and 6. For model 1, the hybrid ON/OFF method was able to obtain the exact target dielectric distribution after 20 iterations, whereas the material sensitivity method still contains a lot of intermediate material cells after 50 iterations. Fig. 7 shows the convergence of the objective function for model 1. The material sensitivity method shows steady, but slow convergence, whereas the hybrid ON/OFF method shows 2 large drops of the objective function to the final value. These jumps occur at iterations where TS is used to determine the ON/OFF status of a cell (at the 10^{th} and 20^{th} iterations). The small ripples for the hybrid ON/OFF method after 20 iterations are caused by numerical errors.

For model 2, the inverse problem is much more difficult. Most of the incident waves are reflected by the side walls, and the field strength is very low near the center wall. As such, the center dielectric wall has a very low sensitivity value, as can be expected from looking at (6). Due to this, the material sensitivity method has very slow convergence for model 2, and there are still a lot of intermediate material cells even after 200 iterations (Fig. 6a). However, the hybrid ON/OFF method is able to achieve a reconstruction which is very close to the exact target after only 5 iterations (Fig. 6b). Unfortunately, the reconstructed distribution is not exactly symmetric, whereas the target distribution is. This difference can be attributed to numerical errors. Since the sensitivity is quite low near the center, small numerical errors can lead to an asymmetric distribution. It should be noted that the 5th iteration is the very first iteration when TS is used in model 2. From these results, it is clear that the proposed hybrid ON/OFF method can be used to obtain fast and accurate solutions of electromagnetic inverse problems in the high frequency domain.

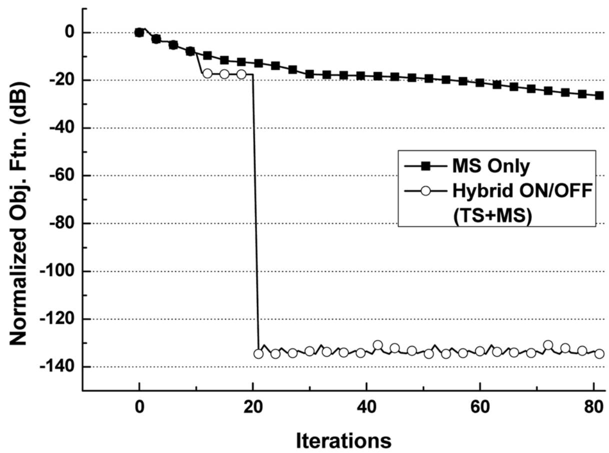

Fig. 7. Objective function convergence for model 1.

5. Conclusion

The proposed hybrid ON/OFF method utilizes the advantages of both MS and TS approaches, that is, the rapid change of material distribution with a smooth design space. Numerical examples show a considerable decrease in the number of iterations required when using our hybrid ON/OFF method. Iterations with TS acts as a 'jump' step after the MS method's convergence has slowed down, and even moving out of local minima seems possible in some cases. For future publication, the implementation of a TS formula that includes a hole with the PEC boundary seen in [10] is in progress, and it is expected to provide an effective solution for various practical inverse problems and optimal designs in the high frequency domain.

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