ISSN (Print) 1226-1750
ISSN (Online) 2233-6656
Journal of Magnetics 24(4), 577-582 (2019) https://doi.org/10.4283/JMAG.2019.24.4.577

Random Optimization Method to Reduce Cogging Torque of Interior Permanent Magnet Synchronous Motor

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(Received 17 June 2019, Received in final form 27 November 2019, Accepted 3 December 2019)

The interior permanent magnet synchronous motor (IPMSM) has many advantages like high torque density, high efficiency, and a wide range of operation area since the reluctance torque and magnetic torque can be applied simultaneously. However, cogging torque that is generated by difference in magnetic resistance due to salient rotor topology and its position is one of the most important factors to be considered while designing an IPMSM as it causes vibration and noise. This paper presents a method for reduction of cogging torque in IPMSM by using random optimization. Random optimization is a constrained stochastic approximation procedure that uses random-direction finite difference gradient estimates. Random optimization is advantageous in cases where the objective function is unclear or cannot be differentiated. In this paper, the use of improved random optimization technique than the existing one for optimal design of IPMSM with minimized cogging torque is proposed.

Keywords : Interior permanent magnet synchronous motor (IPMSM), cogging torque, random optimization, stochastic approximation

1. Introduction

Stochastic optimization is a method to determine the minimum or maximum point of an objective function in the presence of randomness. The way to deduce the minimum point of any function is to generalize the problem of geometric optimization, such as linear programming. Over the years, several ways to find the minimum point of a function have been devised. Many optimization methods aim to find a global minimum point rather than a local minimum point. Random optimization is an optimization method that uses randomness or probabilities. The key principle of random optimization is sampling a convex set with a random number. Until now, the following methods of random optimization have been studied random jumping method and random walk method. However, because these methods have their disadvantages, this paper proposes a random jumping and walk method that overcomes the limitations of the two existing methods [1, 2].

The interior permanent magnet synchronous motor (IPMSM) has many advantages, including high torque density, high efficiency, and a wide range of driving areas since the reluctance torque and magnetic torque can be applied simultaneously. However, because of the difference in magnetic resistance due to rotor position in IPMSM, a reluctance torque that acts as a torque ripple and cogging torque is generated. Since it causes vibration and noise, minimizing the cogging torque by optimally designing the magnetic circuit is essential in the design of IPMSM. There are many ways to reduce cogging torque. One of the ways is to alter the shape of the rotor and stator [3, 4].

This paper proposes an optimal design for the rotor and stator of an IPMSM using random optimization to minimize cogging torque. This is achieved by changing the offset angle of the rotor and chamfering angle of the stator.

2. Random Optimization

Random optimization is one of the methods to find the minimum point in non-linear problems by using random

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numbers. Because the computer libraries have random number generators, random optimization can be used quite conveniently.

The method of random optimization is considered when a function is non-differential, in which the exact relationship between the function value and a variable is not understood, or is determined by system measurement.

When there is no specific structure to confirm whether a local minimum point is actually a global minimum point, devising algorithms to calculate the global minimum point of a function is an arduous task. In general, as opposed to deterministic methods that guarantee asymptotic convergence to the optimization, random optimization ensures convergence in probability. Random optimization is advantageous because it can produce relatively accurate solutions quickly and easily [5].

Another advantage of random optimization is that it is easier to solve complex problems using it. Because it relies only on function values rather than on gradient or Hessian information, random optimization can be quickly coded and applied to a global minimum point [6].

Random optimization has the advantage of being applicable even when the objective function is discontinuous and non-differentiable, and can be used to find the global minimum when the objective function possesses several local minimum points.

Random optimization can be implemented using two methods: random jumping method and random walk method. However, because these methods have their own disadvantages, this paper propose a combination of the two methods, called random jumping and walk method.

2.1. Random Jumping Method

Random jumping method sets the minimum limit l and maximum limit u for the design variables x_i although the problem is unconstrained. It then generates n numbers evenly distributed between l and u.

$$l \le x_i \le u \ , \ i = 1, 2, ..., n \tag{1}$$

$$\mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} = \begin{Bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{Bmatrix} \tag{2}$$

where i is the number of iterations. The value of the function is evaluated at the point **X**. By generating a large number of random points in **X** and evaluating the value of the objective function at each of these points, the optimization can take the smallest value of $f(\mathbf{X})$ as the desired minimum point. Fig. 1 shows a flow chart of the random jumping method.

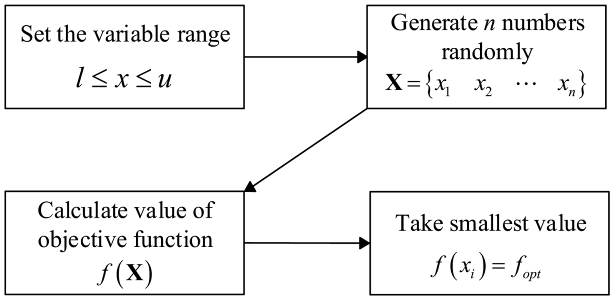

Fig. 1. Flow chart of random jumping method.

2.2. Random Walk Method

The random walk method generates a sequence of improved approximations to the minimum, which is derived from the preceding approximation. Thus if X_i is the approximation to the minimum obtained in the $(i\text{–}1)^{\text{th}}$ iteration, the new or improved approximation in the i^{th} stage is found from the following relation:

$$X_{i+1} = X_i + \lambda u_i \tag{3}$$

where λ is a prescribed scalar step length and u_i is a unit random vector generated in the i^{th} stage. The range of u_i is −1 to 1.

Fig. 2 shows a flow chart of the random walk method. The detailed process of the random walk method is described as follows [7]:

1. Start with an initial point X_1, a sufficiently large initial step length λ, a minimum allowable step length ε, and a maximum permissible number of iterations N.
2. Find the function value $f_1 = f(X_1)$
3. Set the iteration number as $i = 1$.
4. Generate a set of n random numbers each lying in the interval [−1, 1], and the unit vector **u**.

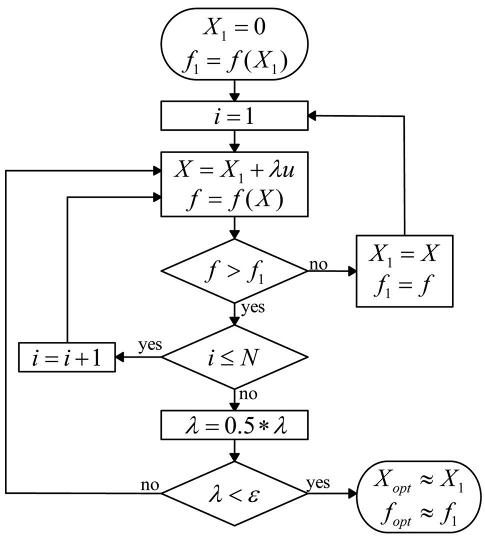

Fig. 2. Flow chart of random walk method.

$$\mathbf{u} = \begin{Bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{Bmatrix} \tag{4}$$

5. Compute the new vector and the corresponding function value as $X = X_1 + \lambda u$ and $f = f(X)$.

6. Compare the values of f and f_1. If $f < f_1$, set the new values as $X_1 = X$ and $f_1 = f$, and revert to step 3. If $f \geq f_1$, proceed to step 7.

7. If $i \leq N$, set the new iteration number as $i = i+1$ and revert to step 4. If not, then proceed to step 8.

8. Compute the new step length as $\lambda = \lambda / 2$. If $\lambda \leq \varepsilon$, proceed to step 9. Otherwise, revert to step 4.

9. Stop the process by taking $X_{opt} \approx X_1$ and $f_{opt} \approx f_1$.

2.3. Random Jumping and Walk Method

The random jumping and walk method is an optimization method that combines the random jumping and random walk method to address the limitations of the parent methods. If the number of iterations is insufficient, the disadvantage of the random jumping method is that even upon the application of the optimization method, the optimal value may not be close to the minimum point. The disadvantage of the random walk method is that, depending on the initial point, the optimal value is a local minimum point, and not a global minimum point.

The first step of the random jumping and walk method is that deduces the optimal value using the random jumping method. It then sets the optimal value to the initial point of the random walk method. Finally, by using the random walk method based on this initial point, it deduces the global minimum point. Fig. 3 shows a flow chart of the random jumping and walk method.

2.4. Test Function

A test function is set up to verify the proposed method of random optimization. The test function has one global minimum point and an additional four local minimum points.

$$f(x, y) = -|10xye^{-x^2-y^2+0.5x}| + xye^{-x} \tag{5}$$

Fig. 4 shows the 3D plot of a two-variable test function.

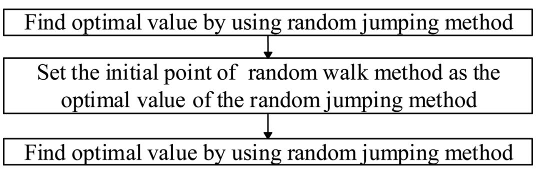

Fig. 3. Flow chart of random jumping and walk method.

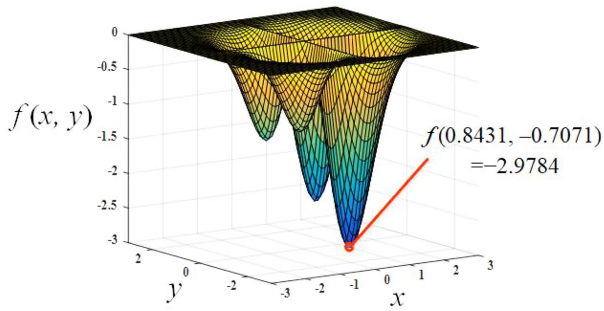

Fig. 4. (Color online) Two-variable test function.

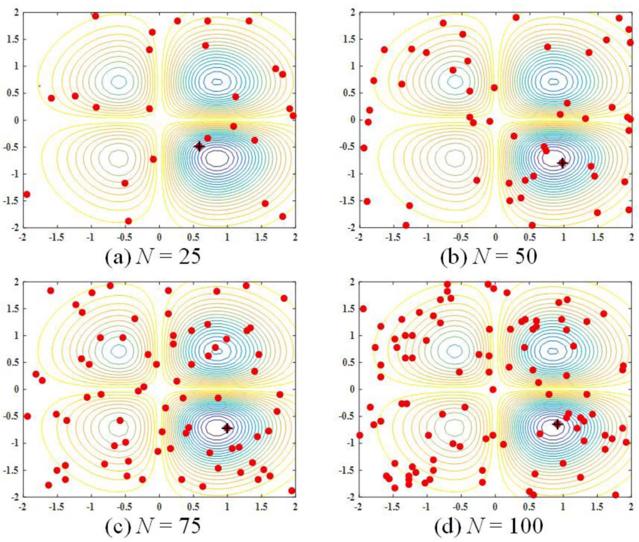

Fig. 5. (Color online) Optimization results of random jumping method for the test function.

Fig. 5 and Table 1 show the optimization results of the random jumping method for different number of iterations: 25, 50, 75, and 100. The plus marker indicates the optimal value. These results show that if the number of iterations is insufficient, the optimal values do not converge on the global minimum point.

Fig. 6 and Table 2 show the optimization results of the random walk method for the test function for different initial points: (1, −1), (1, 1), (−1, −1), and (−1, 1). The results show that the optimal value depends on the initial point. The optimal value of (d) converges on the global minimum point. However, the optimal values of the other

Table 1. Optimization results of random jumping method.

	Iteration	x	y	Optimal value
(a)	25	0.59	–0.48	–2.3396
(b)	50	0.97	–0.79	–2.8548
(c)	75	0.99	–0.72	–2.8736
(d)	100	0.90	–0.64	–2.9346

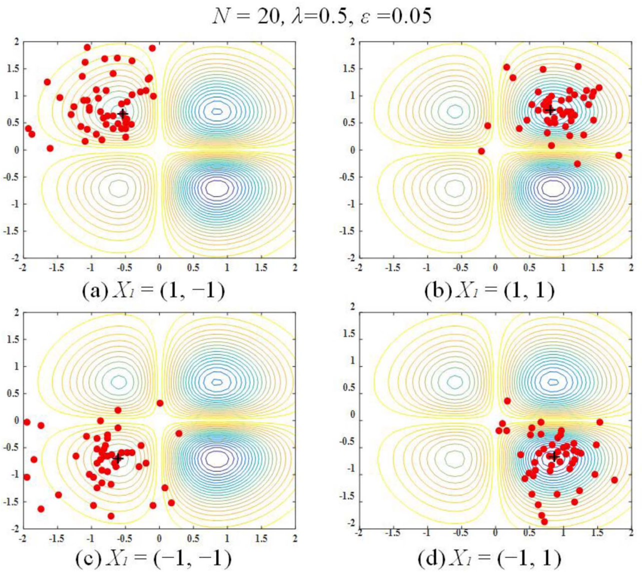

Fig. 6. (Color online) Optimization results of random walk method for the test function.

Table 2. Optimization results of random walk method.

	Initial point	x	y	Optimal value
(a)	(−1, 1)	−0.55	0.67	−1.2076
(b)	(1, 1)	0.80	0.75	−2.4215
(c)	(−1, −1)	−0.61	−0.70	−1.1771
(d)	(1, −1)	0.86	−0.65	−2.8530

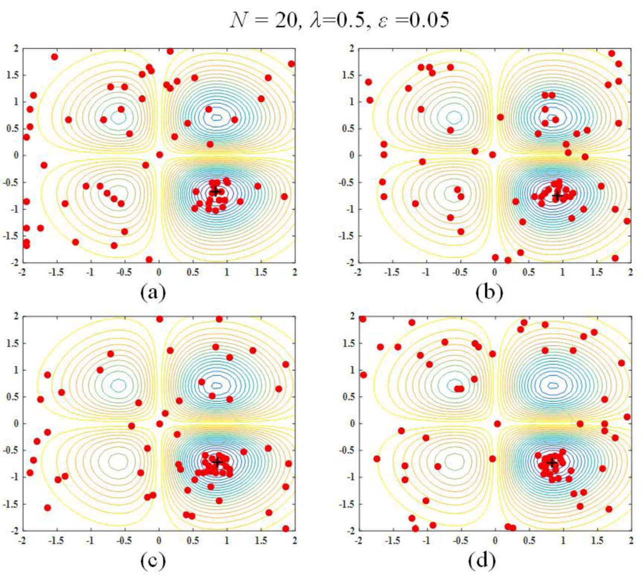

Fig. 7. (Color online) Optimization results of random jumping and walk method for the test function.

cases converge on the local minimum point.

Fig. 7 shows the optimization results of the random jumping and walk method for the test function. The number of iterations in random jumping method and random walk method is set up to 50. These results show that, in any case, the optimal value converges on the global minimum point.

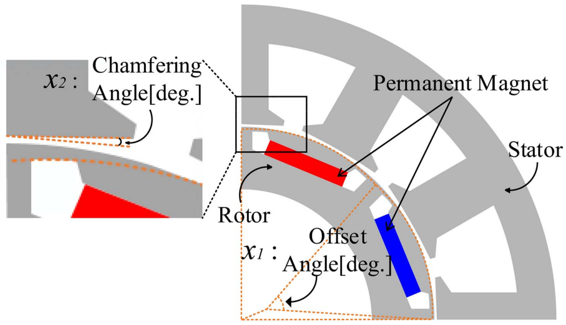

Fig. 8. (Color online) IPMSM with optimization variables.

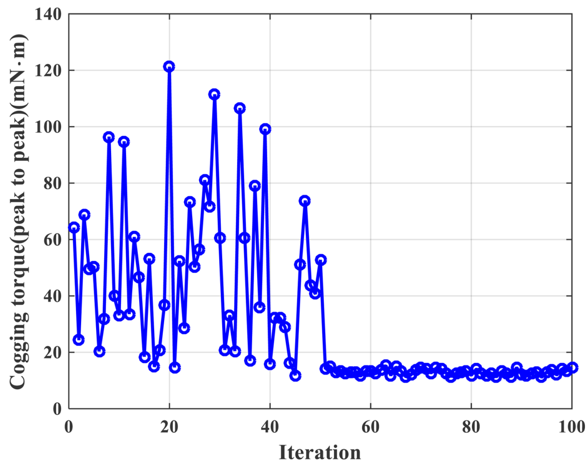

Fig. 9. (Color online) Cogging torque versus number of iterations.

3. Optimization Result

To apply the random jumping and walk method, an IPMSM consisting of 12-slot stator and 8-pole PMs in its rotors is selected. Fig. 8 shows the IPMSM with all optimization variables. x_1 is the offset angle of the rotor and x_2 is the chamfering angle of the teeth. For the optimization, the range of x_1 is 45-60 deg., the range of x_2 is 0-6 deg., and the number of iterations of both the random jumping method and the random walk method is 50. Fig. 9 shows the cogging torque of the IPMSM according to the number of iterations. It shows the cogging torque obtained by the random jumping method until the 50th iteration. The next 50 iterations since then show the cogging torque obtained by the random walk method. Table 3 shows the reduction in the cogging torque obtained by the new random jumping and walk method. The cogging torque of the initial model is 98.55 mN-m. The

Table 3. Cogging torque by random jumping and walk method.

Iteration	x_1	x_2	Cogging torque (peak to peak) [mN-m]
0	45	0	98.55
45	54.20	3.92	11.90
64	54.05	3.93	11.82
67	54.34	3.89	11.32
85	54.34	3.89	11.30
94	54.29	3.90	11.12

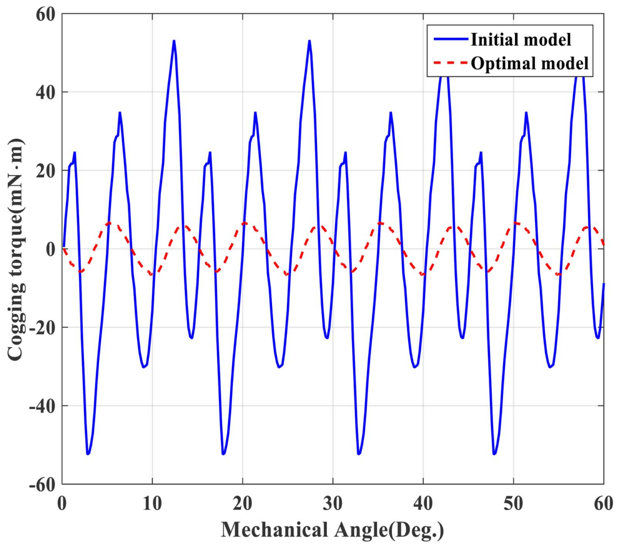

Fig. 10. (Color online) Cogging torque waveform of initial model and optimal model.

initial values of the random walk method are set as x_1 = 54.20 and x_2 = 3.92. At this time, the cogging torque is 11.90 mN-m. It can be observed that the cogging torque

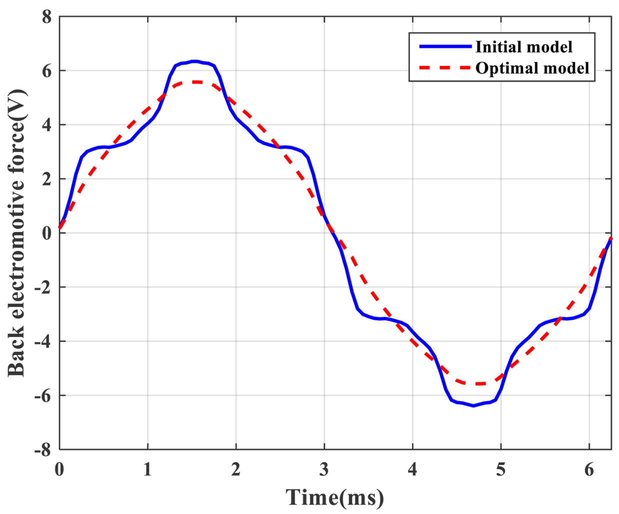

Fig. 11. (Color online) Back-EMF waveform of initial model and optimal model.

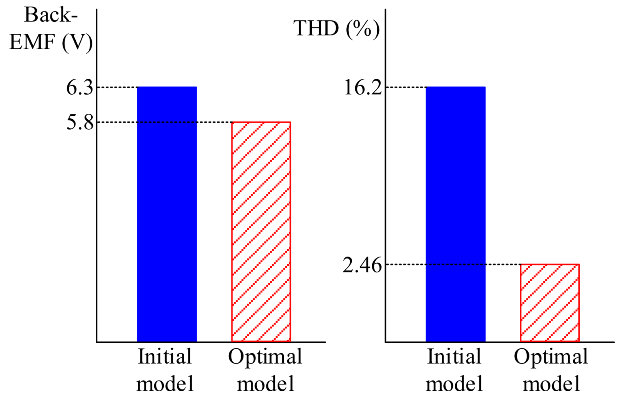

Fig. 12. (Color online) Comparison of back-EMF and THD of initial model and optimal model.

decreases as the optimization proceeds. As a result, the optimal value close to the minimum point is found at the 94th iteration. Fig. 10 shows the comparison of the cogging torque waveform between the initial and the optimal models. It can be observed that the optimal model reduces the cogging torque by 88 %. In addition, Fig. 11 shows the back-EMF (electro-motive force) waveform of the initial model and optimal model and Fig. 12 shows comparison of the back-EMF and THD (total harmonic distortion) of the initial model and optimal model. The back-EMF is decreased from 6.3[V] to 5.8[V], and THD is decreased from 16.2[%] to 2.46[%] by using the optimal model.

4. Conclusions

In this paper, random jumping and walk method is proposed that overcomes the disadvantages of the random jumping method and random walk method. Firstly, the proposed method is applied to a test function and it has been verified. The characteristics of the IPMSM, such as cogging torque and back-EMF are compared between the basic model and optimal model, which is designed based on the random jumping and walk method. The proposed random jumping and walk method is expected to be very useful for multivariable function optimization as well as motor shape design.

Acknowledgments

This work was supported by the research fund of Chungnam National University.

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