Journal of Magnetics 27(2), 141-146 (2022)
ISSN (Print) 1226-1750
ISSN (Online) 2233-6656
https://doi.org/10.4283/JMAG.2022.27.2.141

Effect of Direction of Applied Magnetic Field on Magnetization of Coupled Superconducting Filaments

Thitipong Satiramatekul[1], Varunya Attasena[1]*, and Frédéric Bouillault[2]

[1]*Faculty of Engineering at Kamphaeng Saen, Kasetsart University, Nakhon Pathom, 73140, Thailand*
[2]*GeePs, CNRS UMR8507, CentraleSupélec, Université Paris-Saclay, Gif sur Yvette, 91190, France*

(Received 1 December 2021, Received in final form 28 March 2022, Accepted 4 April 2022)

Numerical modeling by using the finite element method is developed for the prediction of magnetization. In this work, we modeled a superconducting strand made up of seven coupled filaments. We studied the influence of the deformation of the filaments on magnetization. We also studied the effect of the direction of the applied magnetic field on magnetization. We could calculate the magnetization of coupled superconducting filaments. We could also add the dependence of the current density according to the magnetic induction. We found that in the case where superconducting filaments are coupled, for a given spacing and whatever the direction of the applied magnetic field, the total magnetization of the strand does not depend on the deformation of the filaments in the strand if this strand is made up of several filaments.

Keywords : magnetic field, magnetization, superconducting filaments

1. Introduction

One of the main applications of low T_C superconductor has so far concerned the production of coils for obtaining intense magnetic fields for large physical instruments. However, in the case of accelerator magnets, the diamagnetic currents linked to the penetration of the magnetic field in the superconducting filaments lead to significant disturbances of the field in the opening of the magnets. In monitoring the fabrication of superconducting strands for the Large Hadron Collider (LHC) under construction at CERN, magnetization measurements indicated that these are very sensitive to variations in spinning and designing parameters. The deformations of the filaments can induce high variations in magnetization that exceed the limits imposed for the specifications. A better understanding of the magnetization increase in deformed superconducting filaments by a finite element simulation would therefore be a useful contribution for better manufacturing control. A strong coupling between filaments having been observed by the magnetization measurements in the superconducting strands at low magnetic fields, the numerical model must therefore integrate this effect.

*Corresponding author: Tel: +66-34-351-897
Fax: +66-34-351-404, e-mail: varunya.a@ku.ac.th

In practice, a real superconducting cable is made up of several superconducting strands. Each strand is made up of several superconducting filaments. If the currents can flow from one filament to the other at the end of strand [1] as shown in Fig. 1, the filaments are said to be coupled. In this situation, each filament does not carry its own outward and return current. To approach this reality, we have worked in the case of several superconducting filaments in which the coupling effect between filaments is taken into account. We, therefore, calculated the magneti-

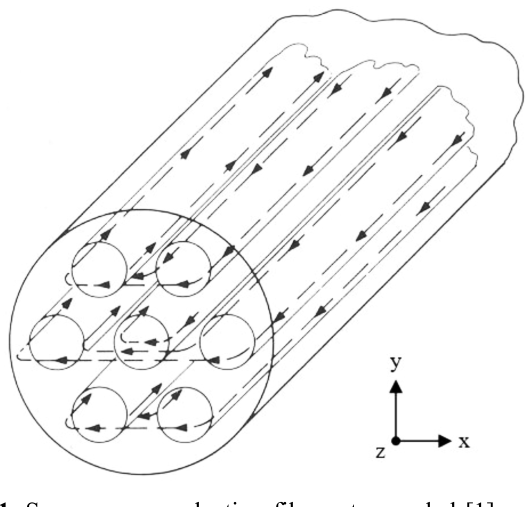


Fig. 1. Seven superconducting filaments coupled [1].

zation of several filaments by studying the influence of the deformation of the filaments on the magnetization. At first, we assumed that there is no deformation of the superconducting filaments and that all the filaments are cylindrical, as in [2, 3]. Then we studied the case of a mixture of elliptically deformed and undeformed filaments. In this work, the effect of the direction of the applied magnetic field on magnetization has been studied too.

2. Problem Presentation

We consider seven superconducting filaments immersed in a magnetic induction (or magnetic field) perpendicular to the axis of the filament (or z-axis) and directed along the y-axis, as in Fig. 1. Due to the symmetries of the problem, we only modeled a quarter of the study domain. We then implicitly assumed that certain currents circulating in the filaments loop in others due to the coupling. In practice, this coupling can be greatly reduced by twisting the filaments together [4].

We applied sinusoidal magnetic induction B of frequency f (Hz) and amplitude B_{max} (T). Bean's model [5] is insensitive to the effect of frequency, the magnetization cycles obtained are the same regardless of the frequency. The value of B_{max} is chosen in such a way that it is greater than the value of the penetration induction B_p to have complete penetration.

In the manufacture of superconducting strands where each strand is made up of several layers of the superconducting filaments [6], we observe that some filaments are deformed. The simple consideration of the shape of those filaments is elliptical [7]. A study of the influence of the deformation of the filaments and the direction of the applied magnetic field on the magnetization would therefore be a useful contribution for better manufacturing control.

2.1. Constant Current Density

Due to the manufacturing process, we are interested to consider the domain in Fig. 2(a). It consists of three cylindrical filaments located in the middle of the bundle of filaments and four elliptical filaments located at the left and right ends of this bundle. The area of the cross-section of each filament is identical, that is 38 μm^2. For this, we used the radius of the cylindrical filament (r) equal to 3.5 μm and the lengths of the axes of the ellipse along the x-axis and along the y-axis (a and b) equal to 2.8 μm and 4.4 μm respectively.

To reduce the computational time, looking at the symmetries of the problem, we modeled a quarter of the study domain as in Fig. 2(b).

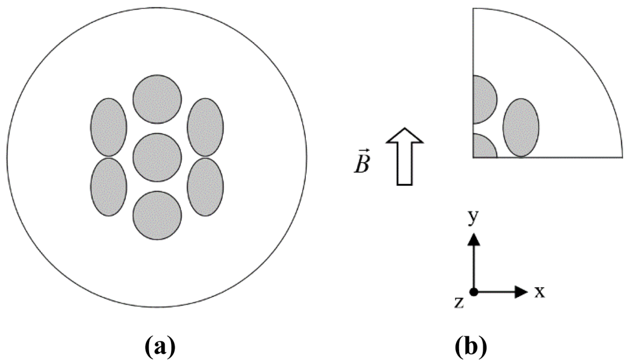


Fig. 2. (a) Study domain and (b) modeled domain.

2.2. Current Density as a function of Magnetic Induction

In the previous part, the current density was assumed to be constant. In other words, the critical current density J_c is independent of the value of the magnetic induction. To take this dependence into account, we have integrated Kim's law [8] into our model.

In practice, the critical current density is not constant, it decreases monotonically with the modulus of the magnetic induction. The solution proposed by Kim to evaluate the dependence of the critical current density as a function of the magnetic induction in the critical state is to hypothesize a local dependence of $J_c(B)$ with the monotonic decrease of J_c as a function of B as follows:

$$J_c(B) = J_c(0)/(1+|B|/B_0) \quad (1)$$

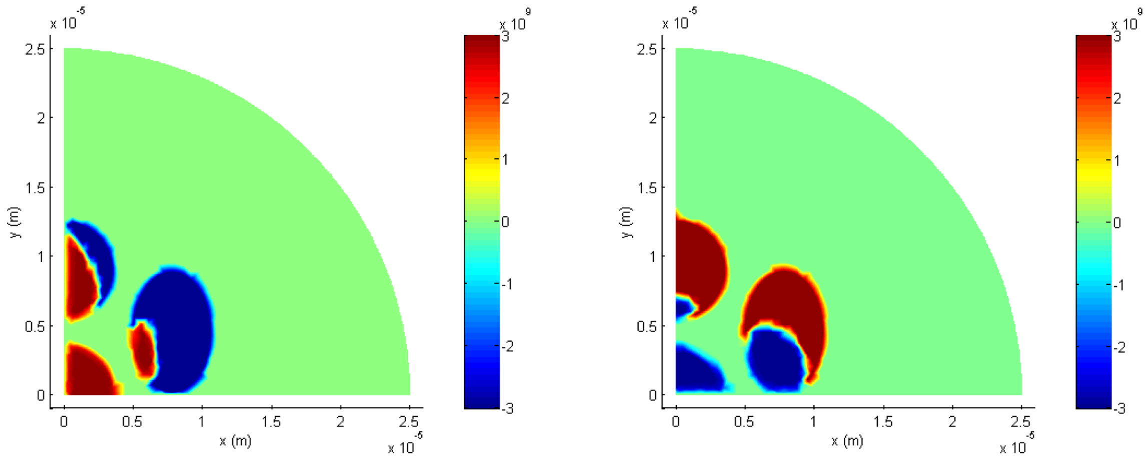
where $J_c(0)$ is the critical current density when the magnetic induction is zero and B_0 is a constant which depends on the material [2].

3. Result and Discussion

At the LGEP (now the GeePs) in France, a finite element program was developed for predicting the value of the magnetization. We proposed the methods for solving the problem of coupled superconducting filaments in [2-4, 6, 7, 9, 10].

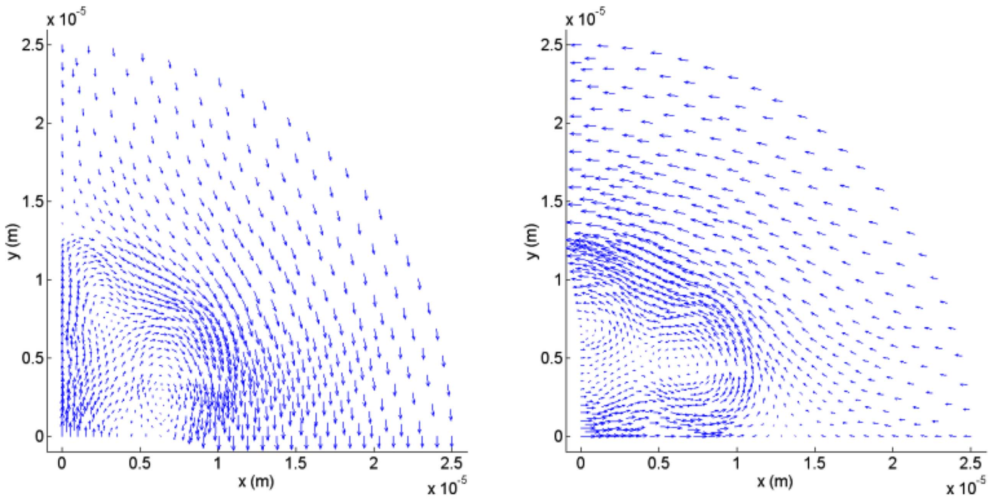
3.1. Constant Current Density

In the first step, magnetic induction was applied in the direction of the major axis of the ellipse. It was applied vertically. Its amplitude and frequency are 0.04 T and 50 Hz respectively. Furthermore, the critical current density of Bean's model (J_c) was taken to be 3×10^9 A/m^2. In this case, the spacing between the filaments (d) is 2 μm. Fig. 3(a) shows the simulation result of the current density distribution in the modeled domain at time $t = 65\times T/160$. We can see the current shell in the superconducting



(a) **(b)**

Fig. 3. (Color online) Current density distributions at time t = 65×T/160. (a) Case of vertical magnetic induction. (b) Case of horizontal magnetic induction.



(a) **(b)**

Fig. 4. (Color online) Magnetic induction distributions at time t = 85×T/160. (a) Case of vertical magnetic induction. (b) Case of horizontal magnetic induction.

filaments. Fig. 4(a) shows that the external magnetic field has changed direction instantly. On the other hand, the magnetic field in the cylindrical filaments is trapped in the opposite direction to that of the external magnetic field. We find that these results are very similar to those observed for a strand made up of seven cylindrical filaments.

In the second step, we are interested in the study of the magnetization of superconductors subjected to a variation of the magnetic field which does not have the same direction as the initial dipole moment. For that, we kept the same arrangement of the filaments (Fig. 2), but this time the applied magnetic induction was rotated by 90°. In other words, it was applied horizontally. Its amplitude and frequency were taken equal to the same values as those used in the case of vertical magnetic induction. Fig. 3(b) shows the simulation results such as the current density distributions. We see again the current shell in the superconducting filaments. Moreover, these distributions present anti-symmetry concerning the x-axis and symmetry concerning the y-axis. In Fig. 4(b), we represent the distribution of magnetic induction in the modeled domain at time t = 85×T/160. In this case, we see that the magnetic inductions are indeed horizontal.

In addition, it is interesting to compare the magnetization cycles obtained in this case (horizontal magnetic induction) with those obtained in the case of vertical magnetic induction. We present in Fig. 5 this comparison for

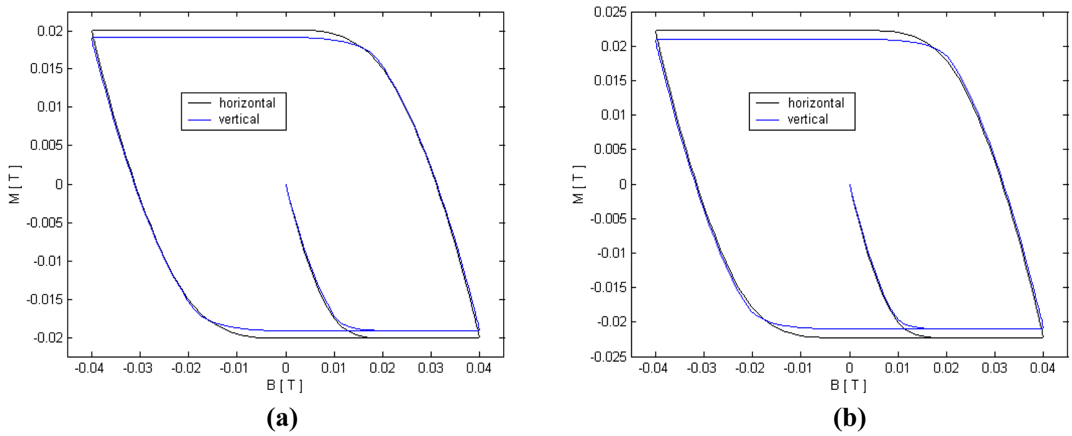


Fig. 5. (Color online) Comparisons of the magnetization cycles of elliptical filaments for (a) d = 2 μm and (b) d = 3 μm.

different values of the spacing between the filaments. We find that by rotating the applied magnetic induction 90° concerning the direction of the major axis of the ellipse, the value of the saturation magnetization increased by 5 % for d = 2 μm and 6 % for d = 3 μm. The magnetization cycles shown in Fig. 5 were obtained from the values of the magnetization in the direction of the applied magnetic induction. However, it is the magnetization along the y-axis for the case of vertical magnetic induction that is calculated, and it is the magnetization along the x-axis for the case of horizontal magnetic induction.

To explain the result obtained on the magnetization cycles presented in Fig. 5, we consider the current density distributions in the superconducting filaments at saturation in the case of vertical magnetic induction, in Fig. 6(a) and (b). We analytically calculated the magnetization of the filaments located on the left or the right of the strand [10]:

$$\vec{M} = \int_L \int_S (\vec{r}_1 + \vec{r}_2) \cdot \vec{j} \, ds \, dz \qquad (2)$$

where L is the length of the strand, S is the area of the cross-section of the filament, the vectors r_1 and r_2 are defined in Fig. 6.

The total magnetization per unit volume of superconductor at saturation obtained in both cases is as follows:

$$M = 4\mu_0 J_c r^2 (r + 4x_1\pi) / S \qquad (3)$$

where x_1 is defined in Fig. 6. In both cases, this value is the same because the area of the cylindrical or elliptical filaments is the same.

Using the explanation given previously in the case of vertical magnetic induction, the total magnetization per unit volume of superconductor at saturation obtained in the two cases in Fig. 6(c) and (d) is, therefore, the same. Finally, we obtain that:

$$M = 4\mu_0 J_c r^2 (r/3 + 4y_1\pi + 2y_2\pi) / S \qquad (4)$$

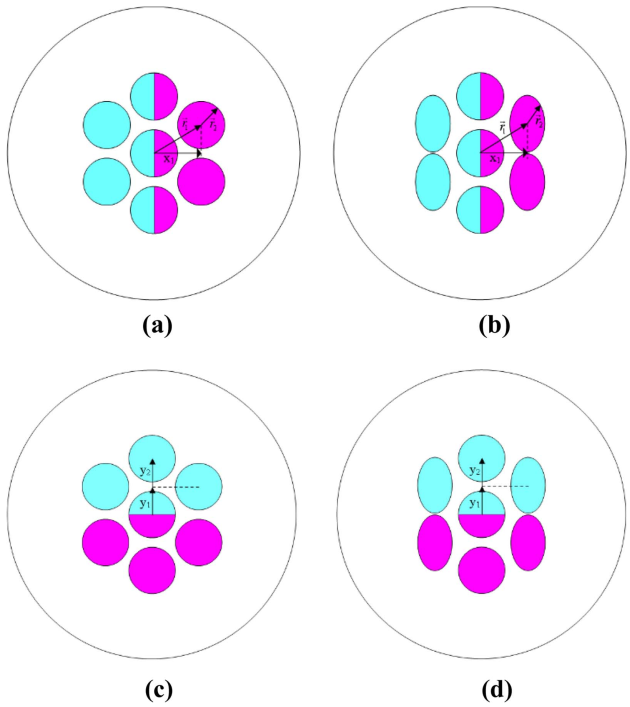


Fig. 6. (Color online) Current density distributions at saturation for the cases studied. (a) and (b) Case of vertical magnetic induction. (c) and (d) Case of horizontal magnetic induction.

where y_1 and y_2 are defined in Fig. 6.

Hence we find that the value of the total magnetization per unit volume of superconductor at saturation obtained in the case of horizontal magnetic induction is greater than that obtained in the case of vertical magnetic induction whatever the layout (with or without the deformed filaments).

To verify our remarks, we compared in Fig. 7 the magnetization cycle of seven cylindrical filaments (d = r) obtained in the case of horizontal magnetic induction with that obtained in the case of vertical magnetic induction. It

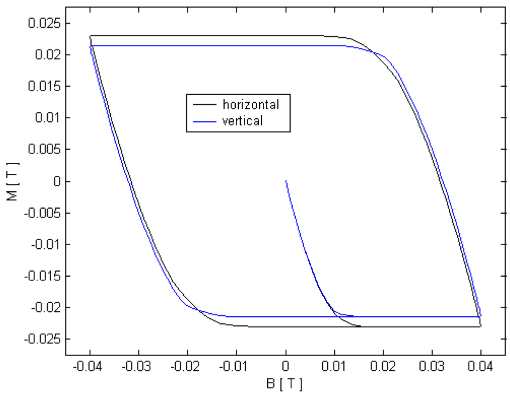


Fig. 7. (Color online) Comparison of the magnetization cycles of cylindrical filaments.

is found that the value of the magnetization at saturation obtained in the case of horizontal magnetic induction is greater than that obtained in the case of vertical magnetic induction. This result confirms our results presented in Fig. 5. This difference is observed for a strand made up of seven filaments (one layer). After doing the simulations for a strand made up of several filaments (n layers), this difference is not important. For a real strand, considering a large number of filaments, the value of the magnetization at saturation will be the same regardless of the direction of the magnetic induction.

3.2. Current Density as a function of Magnetic Induction

The same arrangement of the filaments in Fig. 2 and the same applied magnetic induction (vertical and horizontal) were studied but this time with Kim's model: $J_c(0)$ = 3×10^9 A/m^2 and B_0 = 3×10^{-3} T. In Fig. 8, we see the different color levels in the filaments in both cases of magnetic induction. This means that the critical current density is not constant but is a function of magnetic induction.

When the applied magnetic induction is rotated by 90°

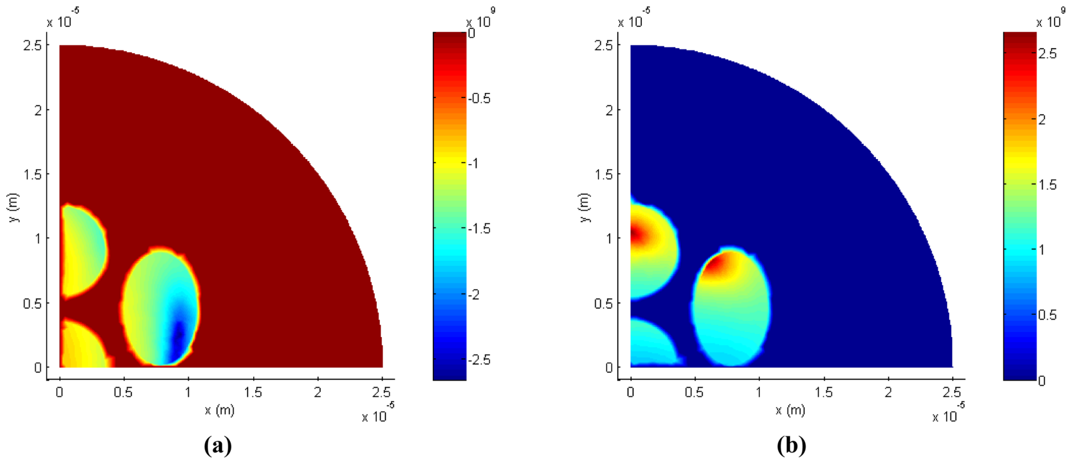


Fig. 8. (Color online) Current density distributions at time t = T/2. (a) Case of vertical magnetic induction. (b) Case of horizontal magnetic induction.

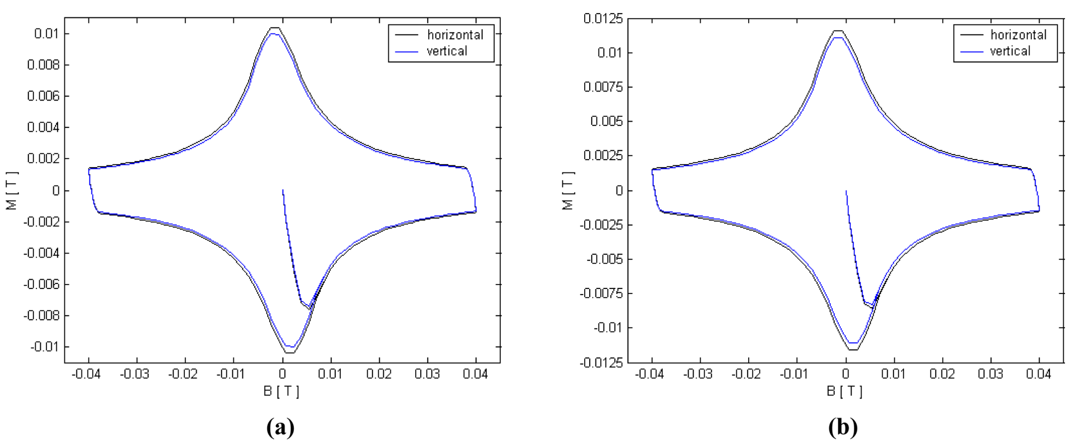


Fig. 9. (Color online) Comparisons of the magnetization cycles of elliptical filaments for (a) d = 2 µm and (b) d = 3 µm.

concerning the direction of the major axis of the ellipse, we find that the maximum magnetization increased by 4 % for d = 2 μm and 5 % for d = 3 μm. In both cases of magnetic induction, it should be noted that the magnetization cycles shown in Fig. 9 were obtained from the values of the magnetization in the direction of the applied magnetic induction.

4. Conclusion

We carried out numerical modeling by the finite element method of a strand made up of superconducting filaments. In this case, we have assumed that these filaments are coupled to each other. In this research, we modeled a strand made up of seven coupled filaments. We studied the influence of the deformation of the filaments on magnetization. For this, we modeled a strand made up of filaments deformed into an ellipse and undeformed. To study the effect of the direction of the applied magnetic field on the magnetization, at first, we worked with a magnetic induction that was applied in the direction of the major axis of the ellipse (vertical). Next, we rotated this magnetic induction 90° relative to the direction of the major axis of the ellipse (horizontal). We compared the magnetization cycles obtained in both cases of magnetic induction. Furthermore, the dependence of the current density on magnetic induction has been taken into account. In conclusion, after doing the simulations for a strand made up of one filament, seven filaments (one layer), and several filaments (n layers), in the case where superconducting filaments are coupled together, for a given spacing and whatever the direction of the applied magnetic field, the total magnetization of the strand does not depend on the deformation of the filaments in the strand if this strand is made up of several filaments.

References

[1] M. N. Wilson, Superconducting Magnets, Oxford University Press, UK (1983).
[2] T. Satiramatekul and F. Bouillault, IEEE Trans. Magn. **41**, 3751 (2005).
[3] T. Satiramatekul, F. Bouillault, and L. Santandrea, IEEE Trans. Magn. **42**, 887 (2006).
[4] T. Satiramatekul, F. Bouillault, A. Devred, and D. Leroy, IEEE Trans. Appl. Supercond. **17**, 3737 (2007).
[5] C. P. Bean, Phys. Rev. Lett. **8**, 250 (1962).
[6] T. Satiramatekul and F. Bouillault, IEEE Trans. Magn. **50**, 393 (2014).
[7] T. Satiramatekul, F. Bouillault, A. Devred, and L. Bottura, IEEE Trans. Appl. Supercond. **15**, 3680 (2005).
[8] Y. B. Kim, C. F. Hempstead, and A. R. Strand, Phys. Rev. **129**, 528 (1963).
[9] T. Satiramatekul and F. Bouillault, IEEE Trans. Appl. Supercond. **20**, 1634 (2010).
[10] T. Satiramatekul and F. Bouillault, IEEE Trans. Magn. **46**, 3229 (2010).