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Magnetometric Demagnetization Factors for Hollow Cylinders

Jae-Hyeon Parq[1,2]*

[1]*Computational Science and Technology, Seoul National University, Seoul 08826, Republic of Korea*
[2]*Research Institute of Oceanography, Seoul National Univeristy, Seoul 08826, Republic of Korea*

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I estimated magnetometric demagnetization factors of hollow cylinders. Demagnetization factors of extreme hollow cylinders are obtained analytically, and axial demagnetization factors of finite hollow cylinders are calculated numerically to find that the approximation designed for the axial demagnetization factors [Nam et al., J. Appl. Phys. 111, 07E347 (2012)] is valid as long as the hollow ratio is not close to one, depending on susceptibility and the aspect ratio. I also discuss the transverse demagnetization factor and the sum rule of the demagnetization factors for a finite hollow cylinder. Dependence on susceptibility being related to the hollow ratio is a feature of magnetometric demagnetization factors of finite hollow cylinders and hollow spheres. A thin closed hollow shape might be the better candidate than open hollow shapes if one wants to maximize susceptibility dependence of the magnetometric demagnetization factor of magnetic material.

Keywords : demagnetizing field, demagnetization tensor, nanoparticle

1. Introduction

The 'demagnetization factor' is a convenient and useful concept. The demagnetizing field $\mathbf{H}_d$ in a diamagnetic or paramagnetic body of a uniform scalar susceptibility χ, generated with the magnetization $\mathbf{M}$ when a uniform external magnetic field $\mathbf{H}_0$ is applied, can be simply expressed as $\mathbf{H}_d = -\tilde{N}\mathbf{M}$ (thus total magnetic field $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_d$). The demagnetization tensor $\tilde{N}$ is diagonalized if three coordinate axes x, y, and z are chosen along the principal axes, and its eigenvalues are three demagnetization factors N_x, N_y, and N_z corresponding to the axes [1]. Once the demagnetization factors are calculated for a base set of geometric shapes, they can be applied to any diamagnetic or paramagnetic bodies of a geometric shape in that set because they depend only on the geometric shape and the susceptibility χ. For $\chi = 0$, further, they obey the famous sum rule,

$$N_x + N_y + N_z = 1 \tag{1}$$

which means that at least one demagnetization factor can be easily obtained. This rule was rigorously proven for magnetometric demagnetization factors [2, 3]. A magnetometric demagnetization factor in the x direction is defined as $N_x = -\langle H_{d,x}\rangle / \langle M_x\rangle$ where $\langle \cdots \rangle$ denotes average over the body volume and is corresponding to magnetometric measurement which enables the magnetic moment of the entire body to be obtained [4].

The magnetometric demagnetization factors for long hollow cylinders and hollow spheres are interesting because they can be greatly dependent on susceptibility. If an infinitely long hollow cylinder has outer radius R and inner radius $r = \alpha R$ with a uniform scalar susceptibility χ, its transverse magnetometric demagnetization factor [5] is

$$N_t = \frac{1}{2}\left(1 - \alpha^2 \frac{\chi}{\chi + 2}\right) \tag{2}$$

When α, called the hollow ratio, is close to 1, since $-1 \le \chi < \infty$, N_t can approach either one or zero, *i.e.*, the range of N_t depending on χ is almost the largest range that a demagnetization factor can have. Likewise, the magnetometric demagnetization factor of a hollow sphere with outer radius R and inner radius $r = \alpha R$ [6],

$$N = \frac{1}{3} - \frac{2\alpha^2 \chi}{6\chi + 9} \tag{3}$$

has a range with respect to χ from zero to one at the limit $\alpha \to 1$. Moreover, the sum of its demagnetization factors is

*Corresponding author: Tel: +82-2-878-7855
Fax: +82-2-871-3269, e-mail: parkq2@snu.ac.kr

$$N_x + N_y + N_z = 3N = 1 - \frac{\alpha^2 \chi}{\chi + 3/2} \tag{4}$$

whose infimum and supremum are 0 and 3, respectively. The sum can be largely deviated from one if α and χ are far from zero.

Magnetic material hollow cylinders can be used for superconducting [7, 8] and nanoparticle research [9], but calculations of their demagnetization factors are limited although there are plenty of literature for demagnetization factors of solid cylinders [2, 10-13]. For the transverse factor, equation (2) is the only published analytical calculation result, to my knowledge. For the axial factor, exact analytical expression is developed for zero susceptibility [14, 15] (the latter reference is more useful.), and an approximate relation between the axial demagnetization factor for the hollow cylinder N_a^{hollow} and that for the solid cylinder N_a^{solid} [9],

$$N_a^{hollow} \cong (1 - \alpha^2) N_a^{solid} \tag{5}$$

which was compared with results from the analytical expression but has not been confirmed for nonzero susceptibility. Summarizing the above, there are still no reliable values of the transverse demagnetization factors of the finite hollow cylinder and the axial demagnetization factors for nonzero susceptibility.

In this paper, I first find the magnetometric demagnetization factors of extremely long or short hollow cylinders. Next, accuracy of the approximation (5) is investigated, and an approximate form is devised for transverse magnetometric demagnetization factors of finite hollow cylinders. I also approximate the sum of magnetometric demagnetization for finite hollow cylinders. Finally, I discuss comparison with the magnetometric demagnetization factor of the hollow sphere.

2. Analysis for Extreme Hollow Cylinders

According to the exact analytical result [15], demagnetization factors for hollow cylinders with zero susceptibility are independent of the hollow ratio α when the cylinders are infinitely long or extremely short. The axial demagnetization factor goes to zero as the length becomes infinite, while its extremely short limit is one. This also means that from the sum rule (1), the transverse demagnetization factor is one for the infinitely long limit and zero for the extremely short limit. However, all these are under the zero susceptibility condition. For nonzero susceptibility, the transverse magnetometric demagnetization factor for an infinitely long cylinder follows equation (2). So it is necessary to check the others.

It is possible to find demagnetization factors of hollow cylinders under the extreme geometric conditions by estimating the fictitious surface magnetic poles. Inside a paramagnetic or diamagnetic body, there are no volume magnetic poles, defined as $-\nabla \cdot \mathbf{M}$, since $\nabla \cdot \mathbf{B} = 0$. Thus the magnetic field inside the body is determined by the external magnetic field and the surface magnetic poles. For an infinitely long hollow cylinder, whose top and bottom surfaces are ignored, only the magnetic poles on its inner and outer side surfaces need to be found, which must be zero under a uniform axial external magnetic field due to the symmetry along the axis. Consequently, their axial demagnetization factors are zero regardless of susceptibility. For a flat ring (an extremely short hollow cylinder), whose side surfaces are ignored, what matter are its top and bottom surfaces, which determine the magnetization of the body because the surface magnetic poles are defined as $\mathbf{M} \cdot \mathbf{n}$ ($\mathbf{n}$ is the unit vector perpendicular to the surface). When a uniform transverse external magnetic field is applied to a flat ring, the magnetic poles on the top and bottom surfaces become zero because the magnetic poles on the top surface must be the same as those on the bottom surface, and therefore the transverse demagnetization factor is zero even if the susceptibility is not zero. Under a uniform axial external magnetic field, the demagnetizing field $\mathbf{H}_d$ in a flat ring equals to $-\mathbf{M}$, and therefore the axial demagnetization factor is one.

The above arguments can be rigorously confirmed by solving the Laplace equations in cylindrical coordinates (ρ, θ, z). For problems with a uniform axial external magnetic field, the Laplace equation of a vector potential $\mathbf{A}$ is acquirable from the Coulomb gauge ($\nabla \cdot \mathbf{A} = 0$) and $\nabla \times \mathbf{H} = 0$ with no free current condition, and the vector potential can be set as

$$\mathbf{A} = \frac{1}{2} \mu_0 H_0 f(\rho, z) \hat{\boldsymbol{\theta}} \tag{6}$$

with help of the axial symmetry [12,16]. Here H_0 is strength of the uniform external magnetic field, and $\hat{\boldsymbol{\theta}}$ is the azimuthal unit vector. Likewise, for the problems with a uniform transverse external magnetic field, the Laplace equation of a scalar potential ϕ can be obtained with $-\nabla\phi = \mathbf{H}$, $\nabla \cdot \mathbf{B} = 0$, and no free current condition, and the scalar potential should be set as

$$\phi = -H_0 f(\rho, z) \cos\theta \tag{7}$$

Boundary conditions are generated by the continuity of $\mathbf{A}$, the continuity of the normal component of $\mathbf{B}$, and the continuity of the tangential component of $\mathbf{H}$, and they can be applied only to the side surfaces for infinitely long hollow cylinders and only to the top and bottom surfaces for flat rings. Also, $f(\rho, z)$ must be finite at $\rho = 0$ and $f(\rho, z) \to \rho$ as $\rho \to \infty$ or $z \to \infty$. Solving the Laplace equations for hollow cylinders with extreme length is not a difficult task because $f(\rho, z) = f(\rho)$ inside the cylinders, and I just write down the solutions of $f(\rho)$ in the hollow cylinder body region. The solution for the infinitely long cylinder under a uniform axial external magnetic field is

$$f(\rho) = (1+\chi)\rho - \chi\frac{r^2}{\rho} \tag{8}$$

where r is the inner radius, and for the flat ring, the solution is

$$f(\rho) = \rho \tag{9}$$

either under a uniform transverse external magnetic field or under a uniform axial external magnetic field although it gives different results depending on whether it goes into (6) or (7). These solutions give the same magnetometric demagnetization factors as those obtained from the surface magnetic poles.

Now we can obtain the sum rules of the magnetometric demagnetization factors for infinitely long hollow cylinders and flat rings. Because $N_x = N_y = N_t$ due to the axial symmetry, setting $N_z = N_a$, we get, from (2) and the above results,

$$2N_t(\alpha,\chi) + N_a(\alpha,\chi) = 1 - \alpha^2\frac{\chi}{\chi+2} \tag{10}$$

for infinitely long hollow cylinders and

$$2N_t(\alpha,\chi) + N_a(\alpha,\chi) = 1 \tag{11}$$

for flat rings. While the sum rule for flat rings is the same with that of zero susceptibility (1), the sum rule for infinitely long hollow cylinders is variable in α and χ, and its infimum and supremum are 0 and 2, respectively. The supremum, 2 is one less than that of the sum of the magnetometric demagnetization factors for hollow spheres, but both the sums (4) and (10) reach its suprema when $\chi = -1$ and $\alpha \to 1$. The magnetometric demagnetization factors of infinitely long hollow cylinders also disagree with the sum rule of the magnetometric demagnetization factors for solid cylinders [12],

$$2N_t\left(\beta, \frac{-\chi}{\chi+2}\right) + N_a(\beta,\chi) = 1 + \delta(\beta,\chi) \tag{12}$$

where

$$\begin{aligned}\delta(\beta,\chi) &= \frac{C(\beta)N_a(\beta,\infty)\chi}{1+N_a(\beta,\chi)\chi}, \ \chi > 0 \\ &= \frac{C(\beta)N_a(\beta,\infty)\chi(1+\chi)}{1+N_a(\beta,\chi)\chi}, \ \chi < 0\end{aligned} \tag{13}$$

and β is the aspect ratio L/2R (L: length of cylinder, R: radius) and $C(\beta)$ is a coefficient. This suggests that the sum rule (12) should not be applied to finite hollow cylinders as well.

3. Approximation for Finite Hollow Cylinders

The first task is to ascertain whether the approximation (5), for the axial demagnetization factor of a hollow cylinder (see its schematic geometry in the inset of Figure 1), is valid for nonzero susceptibility, but before then, it is necessary to check the dependence of its accuracy for zero susceptibility on the hollow ratio α and the aspect ratio β. My result for infinitely long hollow cylinders shows that the difference between the values from the approximation (5) and the exact values for hollow cylinders must be zero at the long cylinder limit. The exact analytical result of axial demagnetization factor for zero susceptibility hollow cylinders [15] shows asymptotic behavior at the long cylinder limit ($\beta \to \infty$) as

$$\begin{aligned}N_a^{hollow} &= \frac{4}{3\pi\beta}\left(1+\frac{\alpha^2}{1-\alpha}\right) - \frac{\alpha^2\,{}_2F_1(-\frac{1}{2},\frac{1}{2},2,\alpha^2)}{\beta(1-\alpha^2)} \\ &\quad - \frac{1-\alpha^2}{8\beta^2} + O(\beta^{-3}) = (1-\alpha^2)N_a^{solid} + \frac{4\alpha^2}{3\pi\beta}\left(\frac{2-\alpha}{1-\alpha}\right) \\ &\quad - \frac{\alpha^2\,{}_2F_1(-\frac{1}{2},\frac{1}{2},2,\alpha^2)}{\beta(1-\alpha^2)} + O(\beta^{-3})\end{aligned} \tag{14}$$

The difference between the approximation (5) and the analytical expression has order of β^{-1} when the hollow cylinder is long. So its absolute error converges to zero as $\beta \to \infty$, but its relative error does not. If we define the relative error of the approximation as

$$\eta_{approx} = \frac{\left|(1-\alpha^2)N_a^{solid} - N_a^{hollow}\right|}{N_a^{hollow}} \tag{15}$$

then its limit for infinite β and zero susceptibility is

$$\lim_{\beta\to\infty}\eta_{approx} = 1 - \frac{(1-\alpha^2)^2}{1-\alpha^2-(3\pi/4)\alpha^2\,{}_2F_1(-\frac{1}{2},\frac{1}{2},2,\alpha^2)} \tag{16}$$

This limit reaches its supremum 1 as $\alpha \to 1$. η_{approx} for $\beta \to 0$ limit also approaches 1 as $\alpha \to 1$ since all the demagnetization factors in (15) become one. From the asymptotic behavior of the analytical expression [15] near $\alpha = 1$, $N_a^{hollow} \sim (1-\alpha)\log(1-\alpha)$, one can find that η_{approx} for zero susceptibility always converges to 1 (*i.e.*, 100 %) as $\alpha \to 1$, regardless of β. Since η_{approx} must converge to 0 as $\alpha \to 0$, its dependence on α appears to be more important than its dependence on β.

I performed a numerical calculation of the axial

Table 1. Numerically calculated axial magnetometric demagnetization factors of hollow cylinders of susceptibility $\chi = -0.5$, for various hollow ratios α and aspect ratios β.

α	$\beta = 2$	3	5	10	20	50
0.1	1.939E-1	1.383E-1	8.792E-2	4.603E-2	2.357E-2	9.569E-3
0.2	1.865E-1	1.329E-1	8.447E-2	4.420E-2	2.263E-2	9.183E-3
0.3	1.753E-1	1.249E-1	7.928E-2	4.145E-2	2.121E-2	8.605E-3
0.4	1.610E-1	1.146E-1	7.263E-2	3.793E-2	1.939E-2	7.864E-3
0.5	1.439E-1	1.022E-1	6.465E-2	3.370E-2	1.722E-2	6.978E-3
0.6	1.239E-1	8.773E-2	5.539E-2	2.881E-2	1.470E-2	5.955E-3
0.7	1.008E-1	7.118E-2	4.480E-2	2.326E-2	1.185E-2	4.796E-3
0.8	7.280E-2	5.218E-2	3.277E-2	1.701E-2	8.652E-3	3.497E-3
0.9	3.816E-2	2.916E-2	1.859E-2	9.710E-3	4.925E-3	1.987E-3

Table 2. Numerically calculated axial magnetometric demagnetization factors of hollow cylinders of susceptibility $\chi = 1$, for various hollow ratios α and aspect ratios β.

α	$\beta = 2$	3	5	10	20	50
0.1	1.652E-1	1.141E-1	6.988E-2	3.525E-2	1.766E-2	7.067E-3
0.2	1.596E-1	1.103E-1	6.760E-2	3.413E-2	1.711E-2	6.848E-3
0.3	1.507E-1	1.043E-1	6.400E-2	3.235E-2	1.623E-2	6.499E-3
0.4	1.390E-1	9.629E-2	5.918E-2	2.996E-2	1.504E-2	6.028E-3
0.5	1.247E-1	8.641E-2	5.319E-2	2.698E-2	1.356E-2	5.439E-3
0.6	1.077E-1	7.467E-2	4.604E-2	2.340E-2	1.178E-2	4.727E-3
0.7	8.788E-2	6.099E-2	3.766E-2	1.919E-2	9.669E-3	3.884E-3
0.8	6.472E-2	4.507E-2	2.790E-2	1.427E-2	7.202E-3	2.896E-3
0.9	3.576E-2	2.599E-2	1.618E-2	8.351E-3	4.219E-3	1.698E-3

Table 3. Numerically calculated axial magnetometric demagnetization factors of hollow cylinders of susceptibility $\chi = 100$, for various hollow ratios α and aspect ratios β.

α	$\beta = 2$	3	5	10	20	50
0.1	1.401E-1	9.192E-2	5.081E-2	2.084E-2	8.187E-3	2.546E-3
0.2	1.359E-1	8.915E-2	4.927E-2	2.023E-2	7.964E-3	2.486E-3
0.3	1.290E-1	8.460E-2	4.676E-2	1.922E-2	7.592E-3	2.387E-3
0.4	1.194E-1	7.825E-2	4.325E-2	1.781E-2	7.072E-3	2.246E-3
0.5	1.071E-1	7.013E-2	3.875E-2	1.600E-2	6.401E-3	2.062E-3
0.6	9.211E-2	6.020E-2	3.326E-2	1.379E-2	5.577E-3	1.831E-3
0.7	7.429E-2	4.845E-2	2.676E-2	1.117E-2	4.594E-3	1.548E-3
0.8	5.387E-2	3.476E-2	1.923E-2	8.135E-3	3.438E-3	1.200E-3
0.9	2.934E-2	1.903E-2	1.057E-2	4.624E-3	2.055E-3	7.550E-4

magnetometric demagnetization factors of hollow cylinders by using the Vuillermet's numerical integral method [17], and its results are shown in Tables 1-3. My program is written in Fortran with aid of LAPACK [18]. The number of volume elements amounts to $(1 - \alpha) \times \beta \times 270{,}000$ although the actual matrix size is much smaller thanks to the axial symmetry. Numerical errors of my results depend on degree of discretization and the LAPACK function's error. The discretization error can be obtained by variation of discretization, and the maximum numerical errors of my results for $\chi = -0.5$, $\chi = 1$, and $\chi = 100$ were estimated below 0.6 %, 0.4 %, and 0.1 %, respectively. I tested my zero susceptibility calculation with the parameter χ = 0.0001, though its results are not shown here, by comparing it with the exact analytical calculation [15], and the maximum error of my results was found to be less than 0.5 %.

From my numerical results and the analytical expression for $\chi = 0$ [15], I estimated the relative errors of the approximation η_{approx}. I used the axial magnetometric demagnetization factor values of solid cylinders numerically calculated previously [13] to calculate values from the approximation (5). Figure 1 shows hollow ratio dependence of the maximum η_{approx} among values for

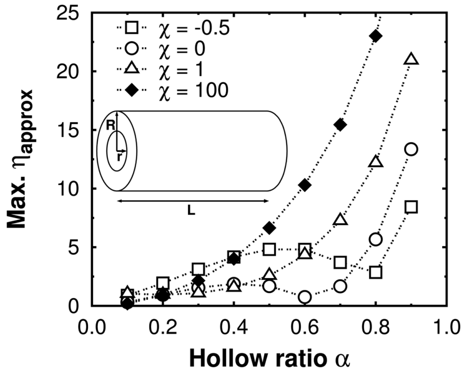

Fig. 1. Maximum relative error of the approximation among values of $\beta = 2$, 3, 5, 10, 20, and 50, for each hallow ratio α and susceptibility χ. Those for $\chi = 0$ were calculated with values from the formula of Ref. [9]. The inset shows the geometry of a hollow cylinder ($\alpha = r/R$ and $\beta = L/2R$).

various aspect ratios, which rapidly increases near $\alpha = 1$ as expected while it reduces small near $\alpha = 0$ (implying our numerical calculation approaches that of solid cylinder of Ref. 13). When $\alpha < 0.6$, every η_{approx} appears below about 10 %. So I restricted $\alpha \leq 0.7$ or $\alpha \leq 0.5$ and found the maximum η_{approx} value in that α range for each aspect ratio and susceptibility, whose results are shown in Fig. 2. Accuracy of the approximation (5) looks decreased when χ is far from zero and β is large. If the maximum η_{approx} has to be less than 10 %, an example of the appropriate region of (α, β, χ) is ($\alpha \leq 0.7$, $2 \leq \beta \leq 20$, $-0.5 \leq \chi \leq 100$). If η_{approx} below 5 % is desired, one can restrict the region of (α, β, χ) as ($\alpha \leq 0.5$, $2 \leq \beta \leq 20$, and $-0.5 \leq \chi \leq 100$) or ($\alpha \leq 0.5$, $2 \leq \beta \leq 50$, and $-0.5 \leq \chi \leq 1$). Therefore, it is evident that the approximation (5) can be valid even for nonzero susceptibility, but in certain accuracy with appropriate conditions of α, β, and χ.

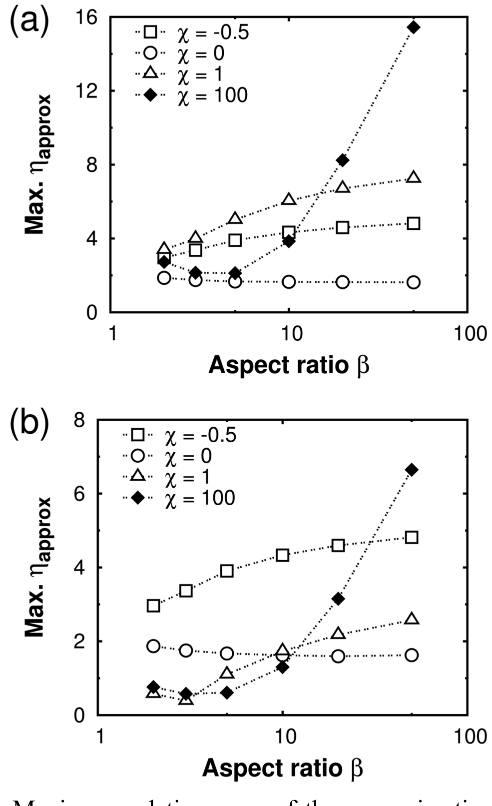

Fig. 2. Maximum relative error of the approximation in the range of (a) $0 \leq \alpha \leq 0.7$ and (b) $0 \leq \alpha \leq 0.5$, for each aspect ratio β and susceptibility χ. Those for $\chi = 0$ were calculated with values from the formula of Ref. [9].

The next task is to estimate an approximate form for transverse magnetometric demagnetization factors of finite hollow cylinders. Since $0 < \alpha < 1$, it is possible to set

$$N_t^{hollow}(\alpha,\beta,\chi) = N_t^{solid}(\beta,\chi) + h(\alpha,\beta,\chi) \tag{17}$$

where $h(\alpha,\beta,\chi) = 0$ for $\alpha = 0$, and $N_t^{solid}(\beta,\chi)$ can be obtained from previous literature [12, 13]. Applying the sum rule (1) for $\chi = 0$, one can get

$$h(\alpha,\beta,\chi) = \tfrac{1}{2}\left[N_a^{solid}(\beta,0) - N_a^{hollow}(\alpha,\beta,0)\right] + O(\alpha\chi)\,. \tag{18}$$

$N_a^{solid}(\beta,0)$ and $N_a^{hollow}(\beta,0)$ can be obtained from the analytical expression [15]. However, if one applies the demagnetization factors of extreme hollow cylinders discussed in the previous section to (17), the above function does not satisfy the limit $\beta \to \infty$ although it does the limit $\beta \to 0$. If β is very large, another term should be added to $h(\alpha, \beta, \chi)$ by using the equation (2) for the transverse magnetometric demagnetization factor of an infinitely long hollow cylinder.

$$h(\alpha,\beta,\chi) = \frac{1}{2}\left[N_a^{solid}(\beta,0) - N_a^{hollow}(\alpha,\beta,0) - \alpha^2\frac{\chi}{\chi+2}\right] + O\left(\frac{\alpha\chi}{\beta}\right) \tag{19}$$

Looking at (18) and (19), I propose an approximation of $h(\alpha, \beta, \chi)$ as

$$h(\alpha,\beta,\chi) \approx \frac{1}{2}\left[N_a^{solid}(\beta,0) - N_a^{hollow}(\alpha,\beta,0) - \alpha^2\frac{\chi}{\chi+2}g(\beta)\right] \tag{20}$$

where $g(\beta) = 0$ at the limit $\beta \to 0$ and $g(\beta) = 1$ at the limit $\beta \to \infty$. This approximation is exact when $\chi = 0$ or $\alpha = 0$ or β is at extremes. Higher order terms of α and χ are ignored in (20), but it is highly probable that higher orders of α less influence the result since $0 < \alpha < 1$. $N_t^{solid}(\beta,\chi)$ and $N_a^{solid}(\beta,\chi)$ appear to be monotonous functions of β, and the analytical result [15] shows that $N_a^{hollow}(\beta,0)$ is a monotonous function of β. This is natural because $N = -1/\chi(1 + H_0/H_d)$ and the demagnetizing field must continuously increase or decrease as β increases. Likewise, it is reasonable to presume that $N_t^{hollow}(\alpha,\beta,\chi)$ and $N_a^{hollow}(\alpha,\beta,\chi)$ are monotonous functions of β, and therefore $g(\beta)$ is approximately monotonous since a linear combination of monotonous functions is monotonous. The last term in the approximation (20) is therefore not dominant since $0 < \alpha < 1$, $0 < g(\beta) < 1$, and χ is close to 0. Although accuracy of the approximation (20) still depends on $g(\beta)$, I stop here and leave its exact form for future work. Nevertheless, qualitative behavior of $N_t^{hollow}(\alpha,\beta,\chi)$ can be found, and it decreases as χ increases, like $N_a^{hollow}(\alpha,\beta,\chi)$, but whether it is a decreasing function of α or not depends on χ, unlike $N_a^{hollow}(\alpha,\beta,\chi)$.

Combining (5), (12), (17), and (20) gives an approximate sum rule of magnetometric demagnetization factors of a finite hollow cylinder,

$$2N_t^{hollow}\left(\alpha,\beta,\frac{-\chi}{\chi+1}\right)+N_a^{hollow}(\alpha,\beta,\chi) \approx 1+\delta(\beta,\chi)+\alpha^2\left[N_a^{solid}(\beta,0)-N_a^{solid}(\beta,\chi)+\frac{\chi}{\chi+2}g(\beta)\right] \tag{21}$$

where $\delta(\beta,\chi)$ is defined in (13). Note that this approximation is valid when χ is close to 0. Because $C(\beta)$ in (13) is so small [12], $\delta(\beta,\chi)$ is negligible if absolute value of χ is small. According to the numerical result [13], $N_a^{solid}(\beta,0)-N_a^{solid}(\beta,\chi)$ is minus when $\chi < 0$, and plus when $\chi > 0$. Therefore, the sum rule (21) is larger than one if $\chi > 0$, and smaller than one if $\chi < 0$, and its discrepancy from one increases as α increases. It seems that sum rules of magnetometric demagnetization factors for hollow geometries generally depend on susceptibility and the hollow ratio.

Now let's compare χ dependence of magnetometric demagnetization factors of hollow cylinders and their sum. When α increases and β is far from zero, χ dependence of the transverse magnetometric demagnetization factor will probably increase because the last term in (20) appears to reinforce χ dependence of $N_t^{hollow}(\alpha,\beta,\chi)$ by $N_t^{solid}(\beta,\chi)$ in (17), so does χ's dependence on the sum of the magnetometric demagnetization factors. This χ dependence related to α is also the feature of the magnetometric demagnetization factor of a hollow sphere. Conversely, the χ dependence of the axial magnetometric demagnetization factor reduces as α increases so long as the cylinder is finite, which can be found from Tables 1-3 or if one combines the approximation (5) and the numerical values of N_a^{solid} [13]. Due to behavior of $g(\beta)$, the χ dependence related to α of the transverse magnetometric demagnetization factor is more prominent for longer hollow cylinders. As $\beta \to \infty$, the magnetic flux passing through the hollow region and the side surfaces increases and the magnet flux passing through the top and the bottom surfaces decreases. This appears related to the fact that χ dependence connected with α of the transverse magnetometric demagnetization factor of the long hollow cylinder is similar to that of the hollow sphere. Therefore, if one wants to maximize susceptibility dependence of the magnetometric demagnetization factor of magnetic material, closed hollow shapes are probably better than open hollow shapes and the thickness should be very small.

4. Conclusion

I estimated magnetometric demagnetization factors of hollow cylinders. The axial demagnetization factor of an infinitely long hollow cylinder and the transverse demagnetization factor of a flat ring are zero while the axial demagnetization factor of a flat ring is one, regardless of the hollow ratio, α. I calculated the axial demagnetization factors of finite hollow cylinders numerically to find that the approximation designed for them is valid as long as the hollow ratio is not close to one, depending on susceptibility and the aspect ratio. I devised an approximate form of the transverse demagnetization factor and approximated the sum rule of the demagnetization factors for a finite hollow cylinder. Their susceptibility dependence is closely associated with the hollow ratio.

Dependence on susceptibility being related to the hollow ratio is a feature of the magnetometric demagnetization factor of the hollow sphere. This feature is more prominent for transverse magnetometric demagnetization factors of longer hollow cylinders. Comparison of magnetometric demagnetization factors for hollow cylinders and hollow spheres reveals that a thin closed hollow shape might be the best candidate if one wants to maximize susceptibility dependence of the magnetometric demagnetization factor of magnetic material.

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